# Primordial Cosmology from Self-Organized Criticality

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Abstract

Recent observations suggest that the dynamics of the primordial Universe unfolds as *complex evolution outside thermodynamic equilibrium*. Here we argue that a viable description of the primordial Universe must be built from concepts relevant to complex dynamics such as Self-Organized Criticality and Multifractal Geometry. This approach brings fresh insights into the early genesis of Dark Matter and into some outstanding challenges of standard cosmology.

**Key words**: Early-Universe cosmology, Complex dynamics, Far-from-Equilibrium phenomena, Self-Organized Criticality, Multifractals.

**Introduction and Motivation**

- The *Cosmic Web* (CW) denotes the spider-like blueprint of Dark Matter in the primordial to early Universe, as defined by $t \leq O(10^{-12})s$ elapsed time from the Big Bang singularity. The intricate topology of CW has been systematically mapped since the '80's after the first detection of *cosmic filaments* and *voids*. It is widely believed that, as the Universe cools off and expands, CW becomes the "scaffolding" platform for the creation and clustering of visible galaxies. Over the past decades, many 3-dimensional maps of galaxy distributions have been generated, using a wide range of probes from the *Sloan Digital Sky Survey* to the *Hubble* and the *James Webb Space Telescopes.*
- Conventional wisdom is that the formation of the CW and galactic clusters is governed by a *rich phenomenology* involving a time-dependent interplay between gravitation, turbulence, thermodynamics, and complex nonlinear processes.

- The key premise of our paper is the conjecture that Dark Matter is a relic CW structure left over from the *condensation of continuous spacetime dimensions* in the deep ultraviolet sector of field theory. Since continuous dimensions underlie the topology of fractal spacetime, this relic structure has been dubbed *Cantor Dust*. It can be argued that Cantor Dust replicates the behavior of hypothetical ultralight axions and Fuzzy Dark Matter models.
- By construction, Cantor Dust reflects the properties of *far-from-equilibrium phenomena*. Specifically,

  a) it contains the "seeds" of classical gravitation and particle physics in continuous dimensions,

  b) it emulates the multifractal structure of turbulence and strange attractors,

  c) acts as a thermodynamic model of collective interactions and,

  d) it stands out as a generic paradigm of complex dynamics.

As a result, Cantor Dust may be viewed as a **unifying concept** in the analysis and understanding of the primordial Universe.

- Pursuing this line of reasoning, we argue herein that a viable description of the primordial Universe must be built from methodology relevant to complex dynamics, in particular *Self-Organized Criticality* and its geometric formulation in terms of *Multifractals*. It is our belief that this approach brings fresh insights into the early genesis of Dark Matter and into some outstanding challenges of standard cosmology.
- *Self-organized criticality* (SOC) is a universal mechanism for self-sustained critical behavior in large-scale systems evolving outside equilibrium. The trademark signature of SOC is two-fold:

  a) it occurs in complex ensembles of interacting components,

  b) it is characterized by a power-law distribution of "avalanche" sizes.

Nowadays, SOC is considered a generic paradigm for a large variety of scale-invariant phenomena ranging from spin glasses, magnetic domains and turbulent flows to traffic jams, cardiac and neuronal activity, economic processes, earthquakes, percolation clusters, forest fires, cellular growth, weather patterns, social behavior patterns and so on.

The paper is divided into 3 sections in the following way:

1) **Primordial Dark Matter from SOC** The first section introduces the *sandpile model* of SOC and treats the formation of Cantor Dust as relaxation to a statistically stationary state at the end of the primordial Universe. The description is only "effective" as the sandpile model exclusively applies to slowly driven systems reaching a non-equilibrium steady state, whereby the average values of the grain influx and outflux coincide, that is, $< influx > = < outflux >$ [7].

2) **Multifractal Structure of the CW** It is known that multifractals underlie the geometry of strange attractors in the analysis of turbulence and nonlinear dynamical systems. This section details how multifractal geometry of SOC translates into the properties of the CW (see e.g., [2]).

3) **Standard Cosmology from SOC** The last section delves into the conundrums of standard cosmology and suggests how (at least some of) these challenges may be solved *outside* the inflationary paradigm. Emphasis is on the horizon and flatness problems, as well as on the $S_8$ tension of cosmology.

## 1. Primordial Dark Matter from SOC

### 1.1 The Sandpile Model as Prototype of SOC

The behavior of the one-dimensional pile of grains resembles the evolution of a *slowly driven sandpile* [1, 4 - 7]. The dynamical variable of the sandpile is the *local slope* defined as height difference between two adjacent sites. A grain of sand topples from a site to its nearest downward neighbor when the local slope exceeds a threshold value.

To fix ideas and with reference to Fig. 1, consider a linear lattice having $i = 1, 2, ..., L$ sites, limited by a vertical wall at the left boundary but open at the right boundary. The stack of grains $h_i$ at site $i$ counts the number of grains at that site, with $h_{L+1}=0$. As alluded to above, the local slope at site $i$ is defined as

$$z_i = h_i - h_{i+1}; \;\; i = 1, 2, \ldots, L \tag{1}$$

A *critical slope* occurs when the height difference between two adjacent sites exceeds one or two grains, i.e.,

$$z^c \in \{1, 2\} \tag{2}$$

The system is driven by adding a grain to a random site,

$$h_i \to h_i + 1 \tag{3}$$

If the slope at site $i$ exceeds the threshold slope, $z_i > z^c$, a grain topples to next site, that is,

$$h_i \to h_i - 1 \tag{4a}$$

and

$$h_{i+1} \rightarrow h_{i+1} + 1 \tag{4b}$$

The initial toppling has the effect of creating slopes larger than the critical slope at some sites in the lattice. As a result, these sites become unstable, topple in turn, and produce an *avalanche* that propagates downwards. The avalanche relaxes and comes to a halt until a *metastable configuration* is reached with $z_i \leq z^c$ for all sites $i$.

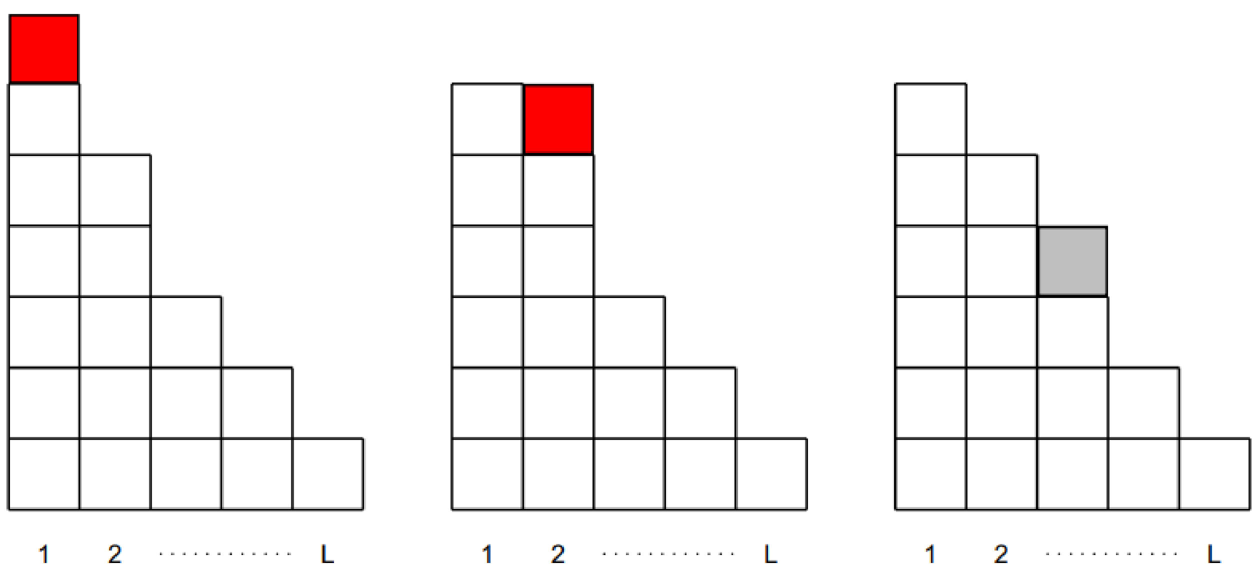


**Fig. 1** Relaxation dynamics of the one-dimensional sandpile [6-7]

In general, one can partition the overall set of *stable configurations* of the model ($S$) into a subset of *transient configurations* ($T$) and *recurrent* *configurations* $(R)$ [7]. After adding $n$ grains to the pile, the system passes through a series of transient configurations prior to evolving into a set of recurrent configurations according to the scheme,

$$T_0 \rightarrow T_1 \rightarrow .... \rightarrow T_{n-1} \Rightarrow R_n \rightarrow R_{n-1} \rightarrow ... \tag{5}$$

The sandpile model comes into a couple of embodiments, the original one being devised by Bak, Tang and Wiesenfeld (the *BTW model*), the other one being built on a purely stochastic basis (the *SSB model*).

**1.2 Dynamics of Dimensional Fluctuations as Sandpile Model**

Dimensional fluctuations on a fractal spacetime background are defined by scale-dependent deviations from four spacetime dimensions, i.e.

$$\varepsilon(\mu) = 4 - D(\mu) \propto O(m^2(\mu) / \Lambda_{UV}^2) \tag{9}$$

in which $\mu$ is the running scale, $m$ the mass of the objects under consideration and $\Lambda_{UV}$ the high-energy (ultraviolet) scale. Previous paragraph hints that "pixel-like" dimensional fluctuations can be interpreted as sand grains moving on a $i = 1, 2, 3, ...., L$ lattice and represented by

$$\boxed{(\delta\varepsilon)_i = [\delta(4-D)]_i \propto \Lambda_{UV}^{-2}\,\delta(m^2)_i} \tag{10}$$

The local amplitude of dimensional fluctuations denotes a stack "height" containing $k$ pixels as in

$$h_i = (\sum_k \delta\varepsilon_k)_i \tag{11}$$

Normalizing all pixels to unity,

$$\left[\delta\varepsilon_k\right] = 1 \tag{12}$$

enables a straightforward analogy of the sandpile model with the dynamics of dimensional fluctuations on the fractal spacetime background (9).

### 1.3 Dark Matter Clustering on a Square Lattice

Fig. 2 illustrates the morphology of a typical avalanche cluster of (a) the BTW and (b) the SSM models generated on an L = 128 square lattice [2]. Here, the underlying process of cluster formation is *percolation* of dimensional deviations (10)-(12) on the fractal spacetime background (9). It is important to note that percolation clusters in SSB simulations are examples of *statistical fractals*, characterized by fractal dimensions and scale invariance.

For comparison, Fig. 3 displays the results of an extended percolation analysis of the Cosmic Web detailed in [3].

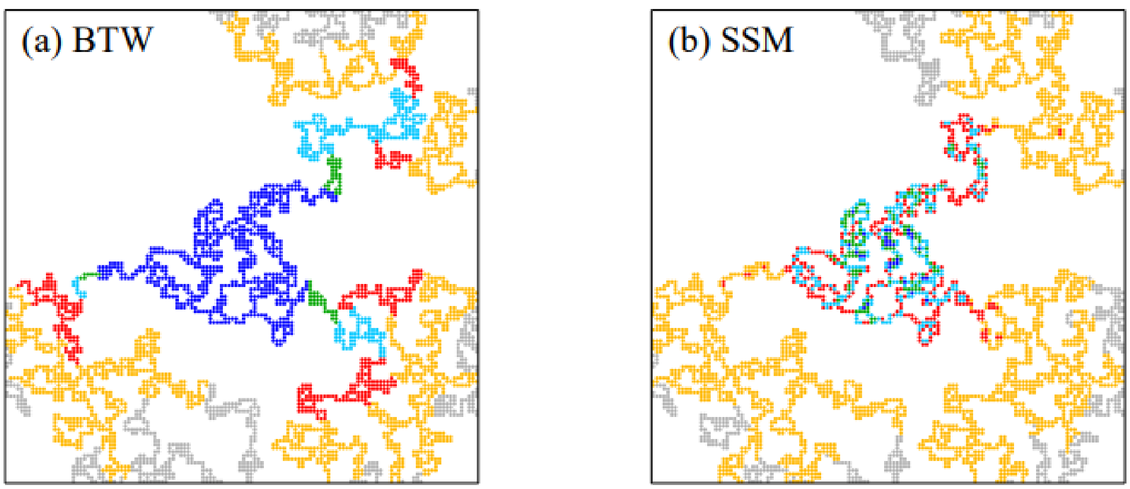


**Fig. 2** Avalanche clusters due to percolation in sandpile models [2]

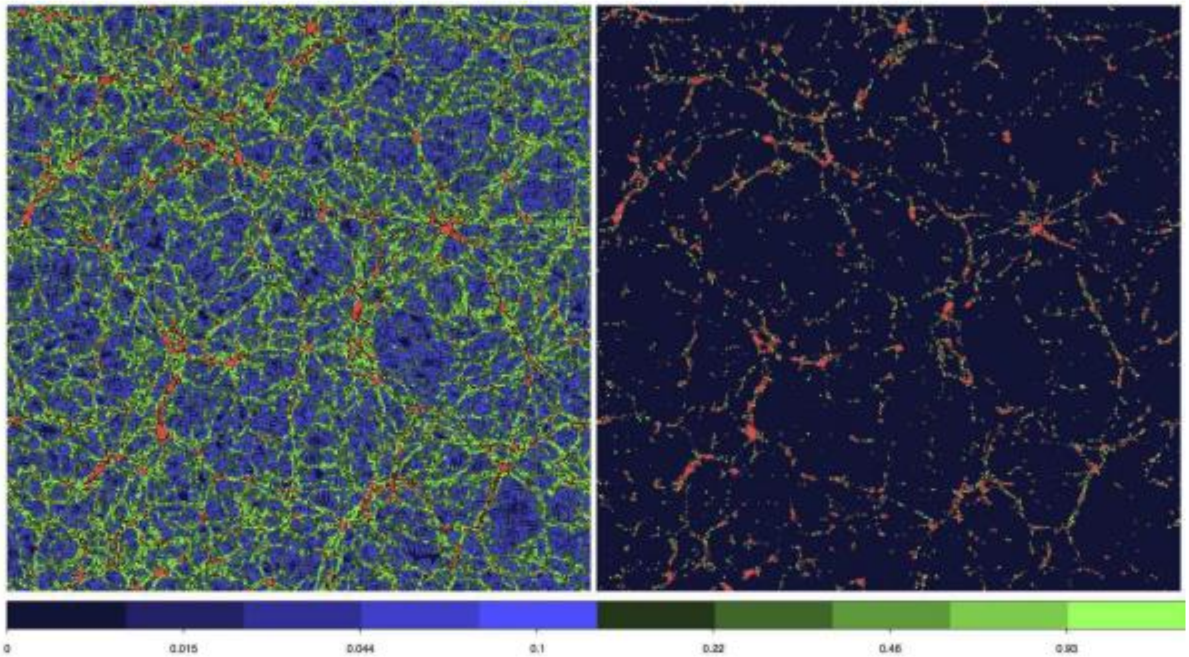


**Fig. 3** Extended **p**ercolation analysis of the Cosmic Web (CW) [3].

## References

1. https://www.hiskp.uni-bonn.de/uploads/media/sandpiles.pdf

2. https://arxiv.org/pdf/1905.04889.pdf

3. https://www.aanda.org/articles/aa/pdf/2018/08/aa33011-18.pdf

4. https://www.researchgate.net/publication/228674142

5. https://arxiv.org/pdf/cond-mat/9808047.pdf

6. Christensen K. and Moloney N. R., Complexity and Criticality, https://www.academia.edu/13641329/Complexity_and_criticality

7. Christensen K. and Moloney N. R, Complexity and Criticality, Imperial College Press, 2005.