Discrete physical space. Discrete structure of black holes, white holes and dark matter

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Abstract

Using the methods and results obtained in digital topology, we construct discrete two- and three-dimensional physical spaces. The structure of discrete spaces is free from constraints and can be chosen depending on external conditions. We develop local and global structure of discrete physical spaces and construct and investigate discrete black and white holes. We hypothesize that the points forming the discrete space have certain properties that turn them into sources of dark matter.

Key words: Discrete Physical Space; Digital Space; Dimension; Graph; Black Hole; White Hole; Dark Matter

1. Introduction

Since the late 1990s, various approaches have been developed suggesting that physical spacetime is discrete (see e.g. [1-9]). These approaches are widely used in various fields of physics, especially in gravity. It should be noted that in these papers, geometric models of discrete physical space were not proposed.
Meanwhile, Einstein, in his letter to his former student in 1916, stated the need to use discrete models of physical space and regretted the lack of such models in mathematics. This letter was translated and published by J. Stachel in [10].
Einstein wrote: "But you have correctly grasped the drawback that the continuum brings. If the molecular view of matter is the correct (appropriate) one, i.e., if a part of the universe is to be represented by a finite number of moving points, then the continuum of the present theory contains too great a manifold of possibilities. I also believe that this 'too great' is responsible for the fact that our present means of description miscarry with the quantum theory. The problem seems to me how one can formulate statements about a discontinuum without calling upon a continuum (space-time) as an aid; the latter should be banned from the theory as a supplementary construction not justified by the essence of the problem, which corresponds to nothing 'real'. But we still lack the mathematical structure, unfortunately. How much have I already plagued myself in this way!"
In the presented paper, we use digital topology methods to construct geometric models of discrete physical spaces and the structure of black and white holes. The structure of a discrete physical space resembles the structure of the crystal lattice in a crystalline solid.
Digital topology methods are used in many fields, including medicine, biology, industry etc. In digital topology, a continuous object is replaced by a finite set of elements that form a digital model of this continuous object. In works [11-15], the methods of replacing continuous two-dimensional surfaces with a finite set of two-dimensional cells were studied. These methods preserve the basic mathematical

characteristics of two-dimensional surfaces. In papers [16-24], an algebraic approach to constructing discrete models of continuous spaces using coverings was studied and developed. In this approach, a discrete model of continuous space is considered as a simple undirected graph with a certain structure. An important feature of this approach is the similarity of the properties of a discrete model with the properties of its continuous analog in terms of algebraic topology.

2 Preliminaries

2.1 Discrete (Digital) Space

In this part, we present the main definitions and properties concerning digital spaces. Our approach for constructing digital spaces was introduced and studied in works [19-32]. Note that digital spaces are called simple undirected graphs in graph theory.
Definition 2.1.1

> A ‘digital space G=(V,W) is a pair of sets V and W. $V=\{v_1,v_2,...v_n,\ldots\}$ is a finite or countable set of points. W is a set of edges. Each edge (v_pv_q) connects two different points, $v_p\neq v_q$. Two edges (v_pv_q) and (v_qv_p) are the same. Two points can only be connected by one edge: $W=\{\ (v_pv_q),|\ v_p,\ v_q\in V,\ v_p\neq v_q, (v_pv_q)=(v_qv_p)\}$ (see fig. 1-2).

We will use the terminology of graph theory whenever it is convenient. For an edge

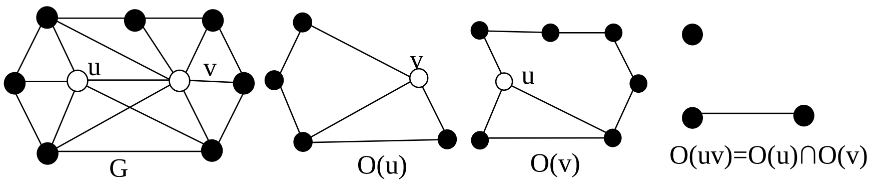

Figure 1. O(u) and O(v) are the rims of points u and v. O(uv) is the joint of points u and v.

(uv) of G, the points u and v are called its endpoints and u and v are incident with (uv). Points u and v are called adjacent or neighbors if they are the endpoints of an edge (uv). Such notions as the connectedness, the adjacency, the dimensionality and the distance on G are completely defined by sets V and W (e.g. [19, 22, 25, 29]).
Definition 2.1.2

> The digital space H=(P,S) is called the subspace of the digital space G=(V,W), if $P\subseteq V$ and H is induced by the set of points P.

According to this definition, H is obtained by removing from G points that are not contained in P together with their incident edges. The space G and subspaces O(u), O(v) and O(uv) are shown in fig. 1.
Definition 2.1.3

> Let G and v be a digital space and a point of G. The subspace O(v) induced by the set of points of G that are adjacent to v (without v) is called the nearest neighborhood or the rim of point v in G, (fig. 1). The subspace $O(v)\cup v=U(v)$ is called the ball of v. The joint rim $O(uv)=O(u)\cap O(v)$ of points u and v is a subspace, each point of which is adjacent to both the point u and the point v.

2.2 Contractible Discrete (Digital) spaces

Contractible digital spaces were defined and studied in [19-21, 26]. To define contractible digital spaces, we use the inductive definition.
Definition 2.2.1

The one-point digital space K(1)=v is the contractible space.

Let G be a contractible space containing n-points, |G|=n and H be a contractible subspace of G, H⊆G. Then the space P=G∪x, that is obtained by attaching a point x to G in such a way that the rim O(x)=H, is a contractible space.

Contractible digital spaces with a number of points less than or equal to 4 are depicted in fig. 2.

Definition 2.2.2

A point v of the digital space G is called simple if the rim O(v) is a contractible space.

Let points v and u of G are adjacent and the joint rim O(vu)=O(v)∩O(u) is a contractible space. Then the edge (vu) is called simple.

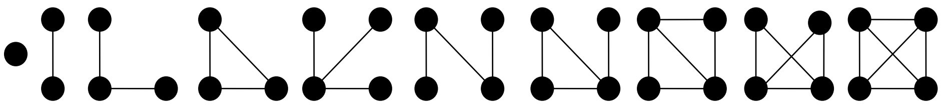

Figure 2. Contractible spaces with the number of points n<5.

Proposition 2.2.1

A contractible digital space G can be transformed into a point of G by sequentially removing simple points and edges.

Definition 2.2.3

Digital spaces G and H are called homotopy equivalent if G can be obtained from H by a sequence of gluing and removing simple points and edges.

Gluing and removing simple points or edges are called contractible transformations.

Properties of the Euler characteristic and the homology groups of digital spaces were studied in [16-21, 25]. It was shown that the Euler characteristics and the homology groups of homotopy equivalent digital spaces G and F are equal.

2.3 Discrete (Digital) n-dimensional Manifolds.

This part contains definitions of n-dimensional digital spaces and transformations of these spaces. There is abundant literature devoted to the study of different approaches to digital lines, surfaces, and spaces used by researchers. Just mention some of them

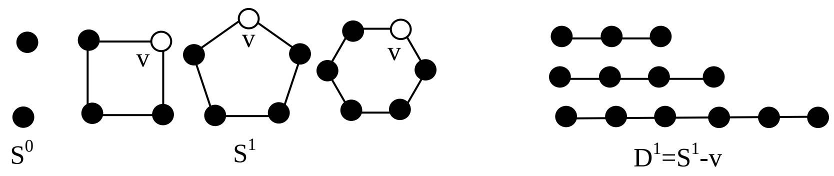

Figure 3. Zero- and one-dimensional spheres S^0 and S^1 and one-dimensional disks D^1.

[11-20]. A digital n-manifold is a special case of a digital n-surface defined and investigated in [20, 22, 24, 28]. First, we will define discrete n-dimensional spheres and n-dimensional disks.

Definition 2.3.1

- A 0-dimensional sphere, S^0, is a disconnected digital space with just two points: a and b (fig. 3).
- A connected digital space, S^1, with a finite number of points is called a 1-dimensional sphere, if the rim of any point v is a 0-dimensional sphere $O(v)=S^0$ (fig. 3).

- A contractible space, $D^1=S^1$-v, is called a digital one-disk (fig. 3) with the (spherical) boundary $\partial D^1= S^0$ =O(v) and the interior Int$D^1=D^1$-∂D^1.
- A connected digital space S^n with a finite number of points is called n-dimensional sphere, if the rim of any point v is (n-1)-dimensional sphere O(v)= S^{n-1} and $D^n=S^n$-v is a contractible space. $D^n=S^n$-v is called a digital n-disk (fig. 3-4) with the (spherical) boundary $\partial D^n= S^{n-1}$=O(v) and the interior Int$D^n=D^n$-∂D^n (fig. 4).

Definition 2.3.2

A connected digital space M with a finite number of points is called a closed n-dimensional manifold, n>0, if the rim O(v) of any point v is an (n-1)-dimensional sphere.

Obviously, if a closed n-manifold M is not an n- sphere, then for any point v belonging to M, the subspace M-v is not a contractible space.

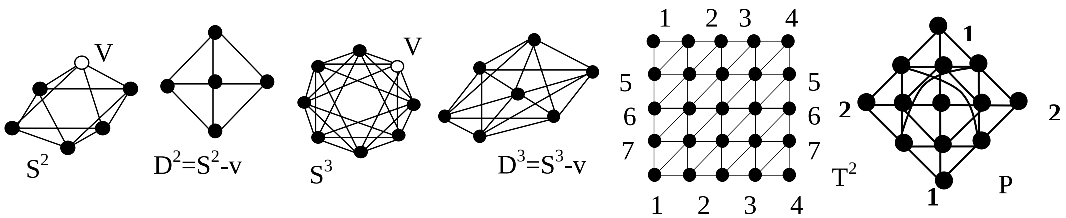

Figure 4. Minimal 2- and 3-dimensional spheres S^2 and S^3 and disks D^2 and D^3. Discrete 2-dimensional torus T^2 and discrete 2-dimensional projective plane P^2.

For example, a discrete 2-dimensional torus T^2 and a discrete2-dimensional projective plane P^2 are shown in fig. 4. It is easy to check directly that for any point v, subspaces T^2-v and P^2 – v are not contractible.

2.4 Transformations of Discrete (Digital) n-dimensional Manifolds.

Digital models of continuous spaces can be obtained using LCL coverings of these spaces [25, 27, 29]. Intersection graphs of LCL coverings are digital models of these spaces with the same mathematical characteristics as the continuous spaces themselves including the dimension, the Euler characteristic, the homology groups and so on.

Digital spaces can be transformed from one space to another by various types of transformations. One type of transformation models the connection between homotopically equivalent continuous spaces in classical topology, the other type of transformation models the homeomorphism between spaces of classical topology. Consider transformations that translate one digital space into another digital space with the same mathematical characteristics and properties. At the same time, the number of points and edges can vary arbitrarily [26].

Definition 2.4.1

Let M be a digital space and (vu) be the edge in M. Remove the edge (vu) from M and glue the point z to M, where z adjoins points u, v and all points in the subspace O(uv)=O(u)∩O(v), O(z)=v∪u∪O(vu). This pair of contractible transformations is called the replacement of an edge by a point or R-transformation, R: M→N. The obtained space N is denoted by N=RM=(M-(vu))∪z (fig. 5).

Obviously, the R-transformation increases the number of points in the digital space M.

Definition 2.4.2

Let M be a digital space, (vu) be the edge in M and any point x belonging to O(u)-O(v) is not adjacent to any point y belonging to O(v)-O(u).
Remove the points u and v from M and glue the point z to M, such that z is adjacent to all points in the subspace O(u)∪O(v), O(z)= O(u)∪O(v).
This pair of contractible transformations is called the contraction of points u and v or C-transformation. CM=(M∪z)-{u,v} and the points u and v are called a simple pair {u,v} of points (fig. 5).

Obviously, the C-transformations reduce the number of points in the digital space M.

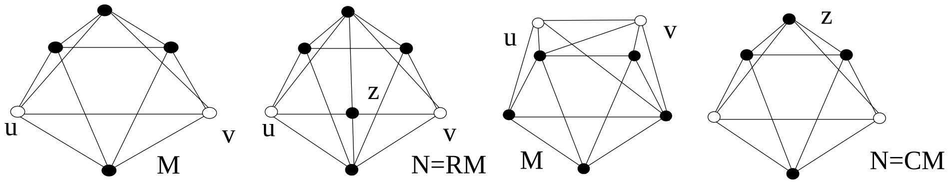

Figure 5. The replacement of the edge (uv) by the point z or R-transformation N=RM.
The contraction of points u and v to point z or C-transformation N=CM.

The properties of R- and C-transformations have been studied in a number of papers. In paper [28], the contraction of simple pairs of points was applied for classification of digital n-manifolds. The following result can be used to study properties of closed digital n-manifolds.

Proposition 2.4.1

Let M be a closed digital n- manifold, n>0 and N=CM (N=RM) be the space obtained from M by C-transformations (R-transformations). Then N and M are homeomorphic (N is closed digital n- manifold with the same mathematical properties as M).

3 Discrete models of continuous Euclidean spaces

In classical physics, the mathematical model of physical space is Euclidean space. In the beginning it was the three-dimensional Euclidean space.
As we have already mentioned, Einstein wrote about the need to use discrete models of physical space and regretted the absence of such models in mathematics.
He believed that the concept of continuity of space should be excluded from consideration when creating a quantum theory, as a construction that does not correspond to anything real.
As Rylov writes in [8], “The hypothesis on discreteness of the space-time geometry appears to be more fundamental, than the hypothesis on quantum nature of microcosm. Discrete space-time geometry allows one to describe quantum effects as pure geometric effects”. In mathematics, the study of discrete spaces began only at the end of the 20th century in a branch of mathematics called digital topology.
Consider the construction of discrete models of continuous spaces using digital topology methods. Nowadays, there are many methods of discretization of continuous objects that are widely used in various fields in science, technology, medicine and so on. We use a method developed in a number of works (see e.g. [25]). A continuous object is covered by a collection of elements called an LCL cover. The intersection graph of this collection is a discrete model for this continuous object. The structure of a discrete physical space resembles the structure of the crystal lattice
of a crystalline solid.

3.1 Two-dimensional Euclidean Discrete Space (discrete Euclidean plane)

Let the LCL set W of the unit squares be a covering of the plane (fig. 6).
Let's construct the intersection graph of this cover, W. We replace each square with a

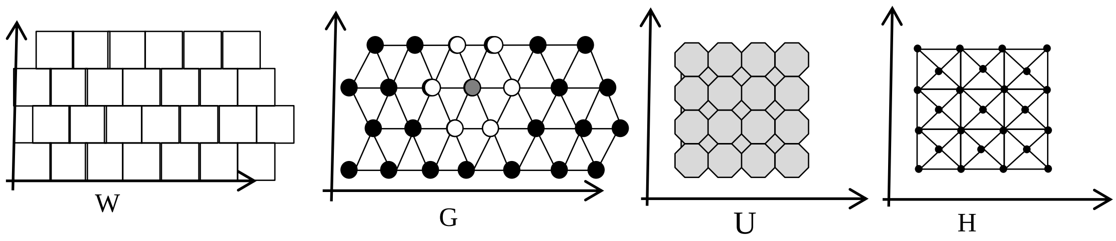

Figure 6. W is the LCL covering of the plane. G is a discrete model of the plane. U is the LCL covering of the plane. H is a discrete model of the plane.

point, and if two squares have common elements, then we connect the corresponding points with an edge. We get a discrete model of the plane. This is a discrete two-dimensional space, since the rim of each point is a discrete one-dimensional sphere containing six points. Figure 6 depicts the LCL covering of the plane with hexagons and construct the intersection graph of this covering, which forms a discrete model of the plane. As you can see, the discrete plane in this case has the same structure as in the previous case.

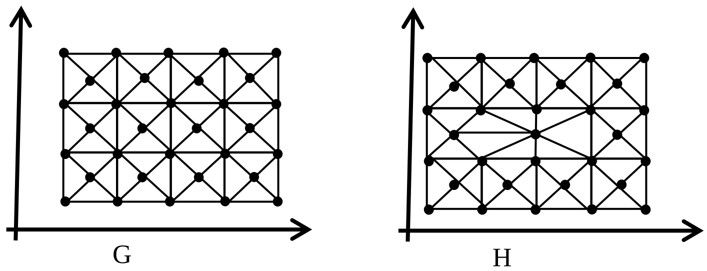

Figure 7. Discrete planes G and H. Some points in these spaces have different rims. H is obtained from G using C-transformations, H=CB.

Using different coverings, we obtain different models of the discrete plane. The rim of each point is always a one-dimensional discrete sphere; however, different points may have rims containing different numbers of points. Figure 7 shows possible variants of such discrete planes.

3.2 Three-dimensional Euclidean discrete space

As in the case of the plane, the intersection graph of the LCL covering of a three-dimensional continuous Euclidean gives us a three-dimensional Euclidean discrete space. The process of constructing the LCL covering can be carried out in various ways. In particular, this can be done using three-dimensional cubes. (see [25, 27]). Note that in a discrete three-dimensional Euclidean space, the rim of any point must be a discrete two-dimensional sphere. A three-dimensional discrete Euclidean space can be constructed in many ways. Let's construct a discrete space using identical

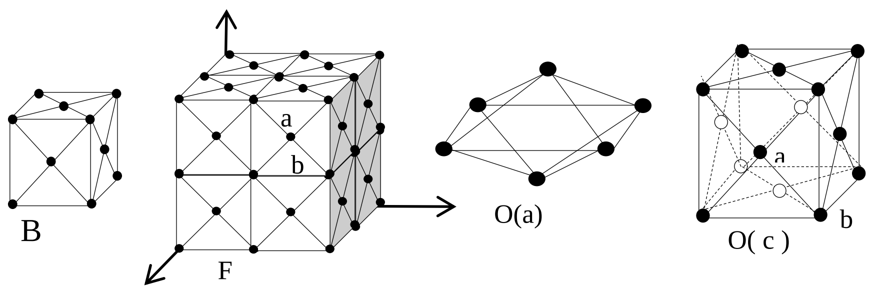

Figure 8. The cube B contains 15 points. Discrete 3-d space F and the rims of points a and c.

cubes B (fig. 8) that stick together along their side faces in three-dimensional continuous Euclidean space and cover this space. The cube B contains 14 points that lie on its surface O(c) and one point {c} located in the middle of B and adjacent to all the other points (fig. 8). The resulting space is a discrete three-dimensional space F (fig. 8), since the rim of each point is a discrete two-dimensional sphere. F contains three types of points: points a, b, and c. It is easy to check that the rim of point a contains six points, the rim of point c contains 14 points, and the rim of point b contains 26 points.

4 Discrete models of black and white holes

The properties of space-time and black holes have been studied in loop quantum gravity. This theory predicts that space and time are made of discrete pieces (see for example [3]). However, in this theory it was not possible to describe the geometric structure of discrete black holes.

4.1 The geometric structure of a discrete black hole.

First, for clarity, let's consider a black hole in a two-dimensional discrete Euclidean space. Let's choose a two-dimensional discrete disk A (fig. 9), containing five points. Using R-transformations, replace edges {02} and {04} with points. We obtain disk B with three points inside. Then, by replacing edges {01} and {03} with points in B, we obtain disk C with five points inside.
Repeating the same R-transformations the required number of times, we get a two-dimensional discrete disk with a boundary that is a one-dimensional sphere containing

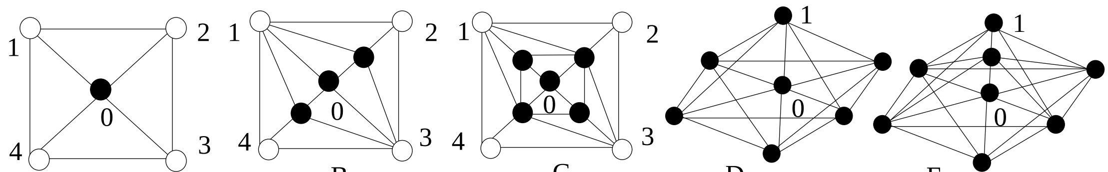

Figure 9. A is a 2-d discrete disk.. The disk B is obtained by replacing the edges {02} and {04} with points in A. The disk C is obtained by replacing the edges {01} and {03} with points in B. D is a 2-d discrete disk containing seven points. The disk E is obtained by replacing the edge {01} with point in D.

only four points and an arbitrary number of points inside this disk.
Exactly the same transformations can be applied to a three-dimensional discrete disk in a discrete three-dimensional physical space.
A discrete three-dimensional disk D is shown in fig. 9. The disk consists of a disk boundary which is a discrete two-dimensional sphere containing six points {1}-{6}, and one point {0} inside this sphere. Using R-transformations, replace the edge {01}, with a point. We get a disk E with two points inside. Repeating the same R-transformations the required number of times we get a discrete three-dimensional disk with an arbitrarily large number of points inside the sphere. Let's define now black holes in a discrete physical space.
Definition A

> A black hole in a discrete physical space is a three-dimensional discrete disk with an arbitrarily large number of points inside and a small number of points on the disk boundary.

4.2 The geometric structure of a discrete white hole.

In 1964, I. Novikov first suggested the possibility of the existence of white holes in the universe [33]. Since then, the structure and properties of white holes have been studied in a significant number of research papers (see e.g. [34-37]).

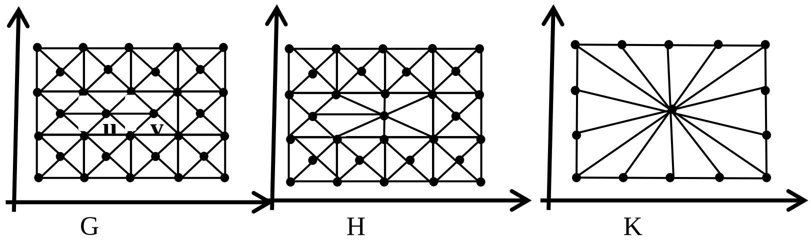

Figure 10 Discrete 2-d disks G, H and K. H is obtained from G using the contraction of points u and v. K is obtained from G using the contraction of all interior points of G.

The properties of white and black holes in a standard continuous physical space are significantly different. Obviously, the structure of a discrete white hole should be the opposite of the structure of a discrete black hole. Based on this, we give the following definition of a discrete white hole.

Definition B

> A discrete white hole in a discrete physical space is a discrete three-dimensional disk with an arbitrarily large number of points on the boundary of this disk, which is a two-dimensional discrete sphere, and a small number of points located inside this sphere.

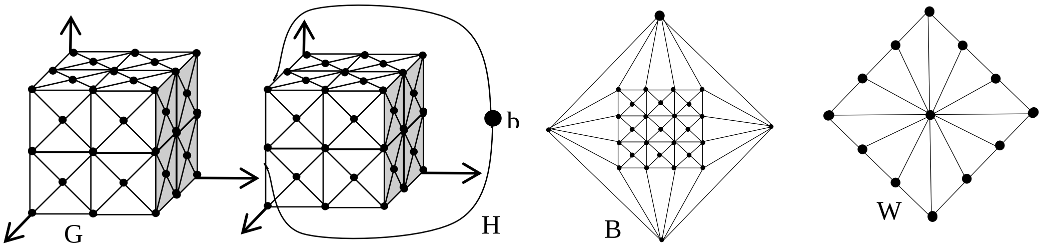

Figure 11. G is a 3-d discrete disk with 14 points inside. H is a 3-d discrete disk with a single inner point b. H is a white hole in a discrete physical space. B is a 2-d discrete black hole. W is a 2-d discrete white hole.

As before, for clarity, let's consider a white hole in a two-dimensional discrete Euclidean space. A two-dimensional discrete disc G in fig. 10 contains 18 interior points. By contracting two adjacent interior points u and v we obtain a discrete two-dimensional disk H. Applying this operation to all interior points we get a two-dimensional discrete disc K with the same boundary as G and H and only one interior point. K is a white hole in a discrete physical space.

In fig. 11, G is a 3-d discrete disk. The boundary of this disk is a two-dimensional discrete sphere that contains all the points belonging to all the side faces of this disk. This disk contains many internal points. It is easy to verify that by applying C-transformations to the inner points of this disk, we get a 3-d discrete disk H with a single inner point b (fig. 11). H is a white hole in a discrete physical space. A clear difference between discrete black and white holes is shown in fig. 11. For convenience, we use a two-dimensional discrete space. B is a two-dimensional discrete black hole. Topologically, the boundary of B is a discrete one-dimensional sphere and contains a small number of points (four points). The interior of B contains a sufficiently large number of points (twenty-five points). W is a two-dimensional discrete white hole. Topologically, the boundary of W is a discrete one-dimensional sphere and contains a sufficiently large number of points (twelve points). The interior of W contains a small number of points (one point).

5 Dark matter

The presence of a hidden mass in the universe is evidenced by observations. Dark matter is matter that does not enter into electromagnetic interactions. It makes up a significant part of the mass of the universe.
Various experiments are being conducted to search for dark matter. Meanwhile, dark matter can be directly related to the discreteness of physical space.
In article [3], L. Smolin wrote: "If we could probe to size scales that were small enough, would we see atoms of space, irreducible pieces of volume that cannot be broken into anything smaller?"
Thus, the points contained in the structure of a discrete physical space can be called atoms of space. It is natural to assume that these atoms of space contained in the structure of a discrete physical space form dark matter.

Concluding remarks

What needs to be done:
1. Reformulation of physical theories, taking into account the discreteness of physical space and time and properties of points of physical discrete space related to dark matter.
2. Analysis of the results of experiments and observations, taking into account the discreteness of physical space and time.
3. New experiments and observations that can confirm or refute the discrete structure of physical space and time.

Literature

[1] Bartolo C., Gambini R., Jorge Pullin J., Canonical quantization of constrained theories on discrete space-time lattices, http://arxiv.org/abs/gr-qc/0205123v1, 2002.
[2] Sorkin R., Causal Sets: Discrete Gravity, Notes for the Valdivia Summer School, 2002, https://doi.org/10.48550/arXiv.gr-qc/0309009, 2003.
[3] Smolin L., Atoms of Space and Time, Scientific American 290(1):66-75, DOI:10.1038/scientificamerican0104-66, Source PubMed., 2004.
[4] Gambini R., Pullin J., Discrete space-time, arXiv:gr-qc/0505023v1, 2005.
[5] Dowker F., Causal Sets and the Deep Structure of Space-Time., arxiv.org/abs/gr-qc/0508109v1, 2005.
[6] Henson J., Discovering the Discrete Universe, arXiv:1003.5890v1 [gr-qc], 2010.
[7] Peretto P. Physics in discrete spaces (A): Space-Time organization, https://arxiv.org/abs/1012.5925, 2010..
[8] Rylov Y., Physics geometrization in microcosm: discrete space-time and relativity theory, https://doi.org/10.48550/arXiv.1006.1254, 2011.
[9] Krajnik1 Z., Ilievski1 E., Prosen1 T. and Pasquier V., Anisotropic Landau-Lifshitz Model in Discrete Space-Time, arXiv:2104.13863v3 [cond-mat.stat-mech], 2021.
[10] Stachel J., Einstein and the quantum: Fifty years of struggle, In ed. R. Colodny, From Quarks to Quasars, Philosophical Problems of Modern Physics, p. 379. U. Pittsburgh Press, 1986.
[11] G. Herman, Geometry of Digital Spaces, Birkhäuser, Boston, 1998

[12] V. Kovalevsky, A Topological Method of Surface Representation, Proceedings of the 8th International Conference on Discrete Geometry for Computer Imagery, March 1999, Pages 118–135
[13] J-O. Lachaud, A. Montanvert, Continuous analogs of digital boundaries: A topological approach to iso-surfaces, Graphical Models and Image Processing 62, 2000, 129-164.
[14] R. Ayala, E. Dominguez, A.R. Frances, A. Quintero, J. Rubio, On surfaces in Digital Topology, in Proc. 5th International Workshop on Discrete Geometry for Computer Imagery, Clermont–Ferrand, France, 1995, pp. 271-276.
[15] G. Bertrand, M. Couprie, A model for digital topology, Discrete geometry for computer imagery, Springer Verlag, Vol. 1568, 1999, pp. 227-239
[16] Иващенко А.В., Топологические свойства молекулярных пространств, ВИНИТИ, Москва, № 6420-84, 1984.
[17] Иващенко А.В., Группы гомологий молекулярных пространств, ВИНИТИ, Москва, № 2346-85, 1985.
[18] Иващенко А.В., Геометрическое отображение графа, Кибернетика, 5, 120-121, 1988. (СССР).
[19] Ivashchenko A.V., Representation of smooth surfaces by graphs. Transformations of graphs which do not change the Euler characteristic of graphs, Discrete Mathematics, v. 122, pp. 219-233, 1993.
[20] Ivashchenko A.V., Graphs of spheres and tori, Discrete Mathematics, v. 128, pp. 247-255, 1994.
[21] Ivashchenko A.V., Homology groups on graphs, Transformations of graphs which do not change the homology groups of graphs, Discrete Mathematics, v. 126, pp. 159-170, 1994.
[22] Ivashchenko A.V., Yeong-Nan Yeh, Minimal graphs of a torus, a projective plane and spheres, Discrete Mathematics, v. 126, pp. 171-178, 1994.
[23] Evako A.V., Dimension on discrete spaces, International Journal of Theoretical Physics, v. 33, pp. 1553-1568, 1994.
[24] Evako A., Kopperman R., Mukhin Y., Dimensional properties of graphs and digital spaces, Journal of Mathematical Imaging and Vision, v. 6, pp. 109-119, 1996.
[25] Evako, A.V., Topological properties of closed digital spaces. One method of constructing digital models of closed continuous spaces by using covers, Computer Vision and Image Understanding, 102, (2006), 134-144.
[26] Evako, A.V., Characterizations of simple points, simple edges and simple cliques in digital spaces. One method of topology-preserving transformations of digital spaces by deleting simple points and edges, Graphical Models, 73(1), (2011), 1-9.
[27] Evako, A.V., Topology preserving discretization schemes for digital image segmentation and digital models of the plane. Open Access Library Journal, (January 2014) 01(04):1-10.
[28] Evako, A.V., Classification of digital n–manifolds, Discrete Applied Mathematics, 181, (2015), 289-296 (See also arXiv:1412.0134, 2014).
[29] Evako, A.V., Topology-preserving digitization of n-dimensional objects by constructing cubical models, arXiv:1506.02678, 2015.
[30] Evako, A.V., Graph theoretical models of discrete spaces with locally non-spherical topology, Applied Mathematics and Physics, 5(2), (2017), 47-52.
[31] Evako, A.V., On digital simply connected spaces and manifolds: a digital simply connected 3-manifold is the digital 3-sphere, arXiv:1307.1371, 2013.
[32] Evako, A.V., Graph Theoretical Models of Closed n-Dimensional Manifolds: Digital Models of a Moebius Strip, a Projective Plane, a Klein Bottle and n-Dimensional Spheres, Applied Mathematics and Physics, 2017, Vol. 5, No. 1, 19-27.

[33] I. Novikov, Физическая энциклопедия (in Russian). Vol. 1. Советская энциклопедия. 1988. p. 180.
[34] D. M. Eardley, Death of White Holes in the Early Universe, Phys. Rev. Lett. 33 (1974), 442-444 doi:10.1103/PhysRevLett.33.442.
[35] R. M. Wald and S. Ramaswamy, Particle production by white holes, Phys. Rev. D 21 (1980), 2736-2741 doi:10.1103/PhysRevD.21.2736.
[36] J. Macher and R. Parentani, Black/White hole radiation from dispersive theories, Phys. Rev. D 79 (2009), 124008 doi:10.1103/PhysRevD.79.124008 [arXiv:0903.2224].
[37] J. Olmedo, S. Saini and P. Singh, From black holes to white holes: a quantum gravitational, symmetric bounce, Class. Quant. Grav. 34 (2017) no.22, 225011 doi:10.1088/1361-6382/aa8da8 [arXiv:1707.07333].