A simple alternative explanation for dark matter

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Abstract

Dark matter is invoked in those astronomical scenarios where there is a discrepancy between the baryonic mass and the dynamical mass. Such scenarios include the rotation curves of disk galaxies and the high velocities of galaxies in galaxy clusters. For our alternative explanation we assume there is no dark matter and guess that the dynamical mass is a weighted sum of the baryonic mass. When we do this, it turns out that the required weighting function has a simple power law structure, and this is sufficient to explain both disk galaxies and galaxy clusters. The weighting function also enables us to predict the dynamical mass distribution from the baryonic mass distribution, so we can predict the rotation curves of disk galaxies and the velocities of galaxies in galaxy clusters. This basic observational result is difficult to ignore as it is so simple, and surpasses what dark matter and modified gravity theories can do. We make no use of dark matter and we make no changes to Newtonian gravity.

1 Introduction

Dark matter is a hypothetical type of matter that most astronomers believe exists and is invoked to explain a number of astronomical scenarios where the observed matter is insufficient to explain the observations. Dark matter is well described in the Wikipedia article ("Dark matter") and the peer-reviewed references contained therein. The problems with dark matter are well described in the book "The Dark Matter Problem" (Sanders, 2010) and the peer-reviewed references that it contains. The clearest observational evidence for dark matter comes from the rotation curves of disk galaxies, where the rotational velocity remains flat and does not show the decline expected for Newtonian gravitation (Lelli et al, 2016). A second example is provided by galaxy clusters, where the velocities of galaxy members are far too high to be supported by the observed mass (Li et al, 2023). In both cases, the addition of extra mass in the form of dark matter solves the problem.

When astronomers look at the rotation curves of spiral galaxies, they often find the curves are flat in the outer regions, i.e., the velocity is constant away from the galaxy centre. This is in disagreement with Newtonian gravity where a drop off in velocity is expected. The current way of explaining the observations is to postulate the existence of large amounts of dark matter in a spherical halo surrounding the galaxy. Given the distribution of baryonic matter across a spiral galaxy, astronomers cannot predict the shape of the rotation curve; they can only calculate the amount of dark matter that is required to explain the observed velocities (Sanders, 2010). So dark matter has explanatory power but no predictive power.

When astronomers look at galaxy clusters, they can estimate the amount of baryonic matter by observing the individual galaxy members and by observing the X-ray emission from the hot gas. Such observations show that the galaxies account for around 10% of the baryonic mass; the gas supplying the other 90%. Separately, astronomers can observe the velocities of the galaxies and apply the virial theorem to obtain the so-called dynamical mass. For undisturbed clusters, they can assume the X-ray gas is in hydrostatic equilibrium and obtain a second estimate of the dynamical mass. And they can also observe the gravitational lensing of remote galaxies by the whole cluster and obtain a third estimate of the dynamical mass. The different measures of the dynamical mass are in good agreement with one another but are around five times greater than the baryonic mass. As with spiral galaxies, the current way of explaining this discrepancy is to assume galaxy clusters have their own haloes of dark matter (Sanders, 2010). Again the addition of dark matter can explain the observations, but it cannot predict them.

No completely satisfactory answer has been found for the mass discrepancy observed in disk galaxies and galaxy clusters (and other scenarios), i.e. the discrepancy between the baryonic and dynamical masses. The problem is the subject of intense research activity amongst both astronomers and physicists. Currently, there are two classes of solution,

(a) dark matter. Some new form of non-baryonic matter exists beyond the particles in the standard model of particle physics. Example of such particles include: WIMPs; sterile neutrinos; axions. The matter discrepancy is solved by postulating the existence of one of these hypothetical particles and adding in sufficient quantities of them so that the problem goes away. Most astronomers and physicists believe that some form of dark matter exists.

(b) modification of the law of gravity. There is no dark matter, and the observed mass discrepancy arises because our law of gravity is incomplete in the regimes of disk galaxies and galaxy clusters. The best known of these hypotheses is MOND (Modified Newtonian Dynamics), first suggested by Milgrom in 1983 (Sanders, 2010). For MOND, the usual

Newtonian formula applies in high acceleration regimes, and a modified formula applies in low acceleration regimes. The crossover acceleration is around 1.0×10^{-10} $m.s^{-2}$.

We offer a third solution, namely,

(c) there is no dark matter, the observed matter is all there is, and Newtonian gravity applies. We guess that the dynamical mass is the weighted sum of the baryonic mass and we see where this guess takes us. Our guess leads naturally to the observed dynamical mass being larger than the observed baryonic mass, and so provides an alternative explanation for the dark matter problem.

In Section 2 we explain the terms baryonic mass and dynamical mass, and how they relate to the dark matter problem. Section 3 looks at weighted sums and Section 4 shows how a weighted sum of the baryonic mass leads to a measure of the dynamical mass. Section 5 shows how we can evaluate the weights from observations of the baryonic and dynamical masses. Section 6 shows the data for a few disk galaxies and Section 7 shows the data for a few galaxy clusters. Section 8 examines the linear relationship for the weighting function that drops out of the observations. Section 9 explains how the linear relationship for the weights can be used to predict the dynamical mass from the baryonic mass. Section 10 looks at the different forms of the gravitational acceleration for (a) dark matter, (b) modified gravity, and (c) our weighting function. Section 11 is a brief note on how we can also explain gravitational lensing. We end the paper in Section 12 with a discussion of what our result means.

Much of the material presented in this paper has already appeared in viXra article 2308.0030 (2023) "A linear relationship between the baryonic and dynamical masses of disk galaxies and galaxy clusters". However, the approach in that paper was somewhat more complicated than the more straightforward explanation put forward here.

In this paper we provide an alternative explanation to dark matter for galaxies, galaxy clusters, and gravitational lensing. Other areas invoking dark matter will be dealt with in a separate paper covering: physical cosmology; the acoustic peaks in the power spectrum of the cosmic microwave background; and structure formation.

2 Baryonic mass and dynamical mass

Physicists working on the Earth and within the solar system do not have a problem with mass. The different methods they use to measure masses, in particle accelerators, in laboratories, in satellites, and in space probes, are all in good agreement with one another. There are no inconsistences or discrepancies, and no need to introduce the concepts of baryonic mass and dynamical mass. It is the astronomers who have the problems. And it is the astronomers who have caused all the trouble by introducing the concepts of baryonic mass, dynamical mass, and dark matter. Dark matter cannot be made up of particles from the standard model of particle physics (Sanders, 2010), and is often referred to as "non-baryonic". By extension, this means normal matter is usually referred to as "baryonic".

Baryonic mass is simply the amount of normal matter in an object, be it a planet, a star, a galaxy, or a galaxy cluster. We give a couple of examples.

(a) The baryonic mass, M_{bar}, is what we get when we multiply the volume, V, of an object by its density, ρ.

$$M_{\text{bar}} = V \times \rho \tag{1}$$

(b) In astronomy we can estimate the baryonic mass by using photometry to measure the light output of an object, I , and multiplying that by the mass-to-light ratio, Y.

$$M_{\text{bar}} = I \times \Upsilon \tag{2}$$

Dynamical mass is the amount of matter an object must have to account for its dynamical properties. It is the mass that appears in Newton's laws of motion and Newton's law of gravity. A couple of examples should clarify this.

(a) The rotational velocity, v, of a disk galaxy, arising from the central mass, M_{dyn}, is given by

$$v^2 = \frac{G\, M_{\text{dyn}}}{r} \tag{3}$$

When we measure the velocity and use it to determine the mass, then that is the dynamical mass.

(b) The mass of a galaxy cluster, as determined by the virial theorem, is given by

$$M_{\text{dyn}} = \frac{V^2\, R}{G} \tag{4}$$

where V is the average velocity of galaxy members; R is the characteristic size of the cluster. Again the mass, M_{dyn}, is the dynamical mass.

We can see how see how baryonic mass and dynamic mass come together by considering the example of the rotation curve for disk galaxy NGC 4157, shown in Figure 1. The black diamonds are the measured velocities. The coloured lines are the contributions to the rotation curve from the different baryonic components, as measured by photometry: purple line for the central bulge; orange line for the stars; green line for the gas; blue line for the baryonic total. It is clear that the blue line, representing the total baryonic mass, falls a long way short of the observations.

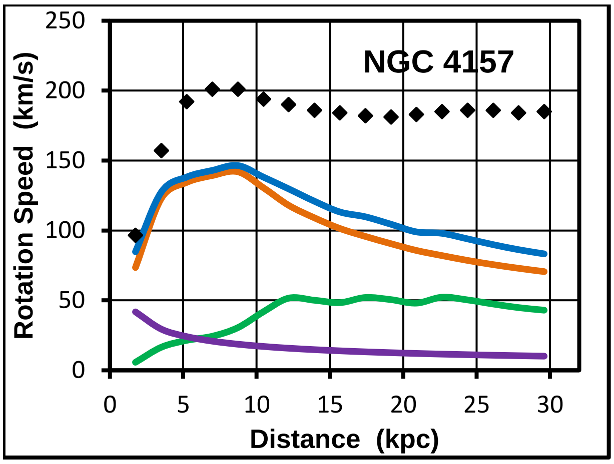


Figure 1. The rotation curve for disk galaxy NGC 4157. Black diamonds are the observations. The coloured lines are the contributions, measured by photometry, for the central bulge (purple), stars (orange), gas (green), and total (blue). The blue line (baryonic mass) falls a long way short of the observations (dynamical mass). Data from Lelli et al (2016).

Although not strictly true for disk galaxies, we can get at the cumulative mass, $\boldsymbol{M_{cum}}$, inside radius, $\boldsymbol{r}$, from the rotational velocity, $\boldsymbol{v}$, using

$$M_{\text{cum}}(r) = \frac{v^2\, r}{G} \tag{5}$$

This is shown for NGC 4157 in Figure 2. The black diamonds show the cumulative dynamical mass, corresponding to the black diamonds of Figure 1. And similarly, the blue line shows the cumulative baryonic mass, corresponding to the blue line of Figure 1. It is clear that the baryonic mass has converged by a radial distance of 10 kpc, with very little matter being added beyond that point. However, the dynamical mass continues increasing to the edge of the plot and shows no signs of slowing down.

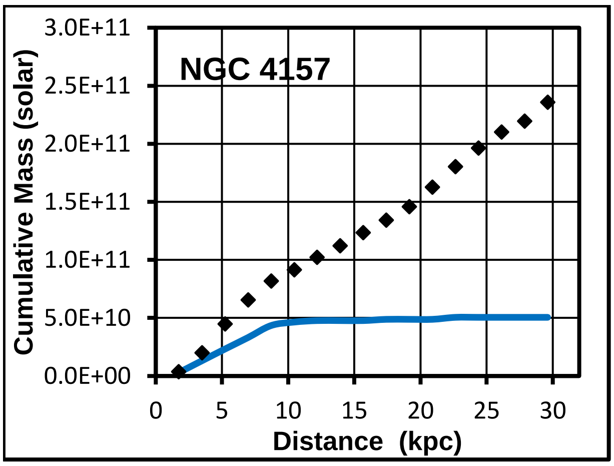


Figure 2. The cumulative mass distribution for disk galaxy NGC 4157, as derived from Figure 1. The black diamonds are the observations of dynamical mass; the blue line shows the observations of baryonic mass.

The usual explanation for the discrepancy between the masses, as shown in Figure 2, is that there is (on average) around five times as much non-baryonic dark matter as normal baryonic matter. The dark matter is presumed to exist as a spherical halo surrounding the galaxy.

3 A weighted sum

Our assumption is that there is no dark matter, and that the baryonic matter we observe is all there is. We make the guess that the required dynamical mass is some, as yet unspecified, weighted sum of the observed baryonic mass. In this section we examine what sort of weighted sum we need.

In many situations, where several identical items contribute to a total, we can obtain the total by simply adding up the individual contributions

$$\boldsymbol{M}_{\text{total}} = \sum_{i} \boldsymbol{m}_{i} \tag{6}$$

An example would be adding up gold coins to find out how much money we have.

In other situations, where items make different contributions, we need to take a weighted sum, with the individual weights, $\boldsymbol{w}_i$, defining the size of the contributions

$$\boldsymbol{M}_{\text{total}} = \sum_{i} \boldsymbol{w}_{i}\, \boldsymbol{m}_{i} \tag{7}$$

An example would be adding up coins made of gold, silver, or bronze, where we must take the value of the metal into account.

Equation (7) is not completely satisfactory, as the weights are not tied down or normalised. Also, unless the weights are pure numbers, then the units do not match either. A better way to get the total mass is to use something along the lines of

$$\boldsymbol{M}_{\text{total}} = \frac{\sum_{i} \boldsymbol{w}_{i}\, \boldsymbol{m}_{i}}{\sum_{i} \boldsymbol{w}_{i}} \tag{8}$$

where the weight appears in both the numerator and denominator. We want to do something similar in order to get at the dynamical mass from the baryonic mass.

There are a couple of points we need to consider when dealing with disk galaxies and galaxy clusters.
Firstly, for disk galaxies, each increment of mass can be considered to be a uniform ring of material centred on the galaxy centre. Binney & Tremaine (2008) have shown that, for disk galaxies, we can consider the mass as acting at the galaxy centre, without incurring significant error, i.e. like a spherical shell. And for galaxy clusters, the increment of mass is a uniform spherical shell, which acts as if the mass is at the cluster centre.
Secondly, we know from experiments on the Earth and within the solar system that, in our local neighbourhood, the dynamical mass equals the baryonic mass.

We can achieve our desired weighted mass as a slight modification of equation (8)

$$\boldsymbol{M}_{\text{dyn}}(\boldsymbol{r}) = \frac{\sum_{i} \boldsymbol{w}_{i}\, \boldsymbol{m}_{i}}{\boldsymbol{w}(\boldsymbol{r})} \tag{9}$$

where ***w(r)*** is the weight at ***r***. So, to get the dynamical mass, we take the individual baryonic masses, multiply them by their individual weights, add them up, and finally divide the sum by the weight at the location in question.

The weights, ***w(r)***, do not have absolute values; only relative values. This is clear from equation (9), where the weights appear in both the numerator and denominator. Perhaps a better way of writing equation (9) is as

$$\{w(r)\ M_{\mathrm{dyn}}(r)\} = \sum_i \{w_i\ m_i\} \tag{10}$$

Here it can be seen that all the masses are multiplied by their weighting function; so the symmetry is somewhat better.

As mentioned above, for disk galaxies and galaxy clusters, we can consider the mass of a ring or a shell as acting at the centre. For our weighted sum, the gravitational acceleration is then given by

$$g(r) = -\frac{G\,M_{\mathrm{dyn}}(r)}{r^2} = -\frac{G}{r^2}\,\frac{\sum_i w_i\,m_i}{w(r)} \tag{11}$$

This is the equation we are going to apply to the observed data for disk galaxies and galaxy clusters.

If we have a region where the weight is a constant, w_c, then equation (11) becomes

$$\frac{G\,M_{\mathrm{dyn}}(r)}{r^2} = \frac{G}{r^2}\,\frac{\sum_i w_c\,m_i}{w_c} = \frac{G}{r^2}\sum_i m_i = \frac{G\,M_{\mathrm{bar}}(r)}{r^2} \tag{12}$$

The weights in the numerator and denominator cancel out, and the dynamical mass equals the baryonic mass. So our weighted sum automatically satisfies the basic requirement that no difference between the baryonic mass and the dynamical mass will show up in any local experiment.

This also means we can make the prediction that no dark matter is needed for the Earth; the solar system, the solar neighbourhood, star clusters, galaxy centres, and galaxy cluster centres.

4 How the weights work

We have made the arbitrary decision to introduce hypothetical weights that act on the observed baryonic mass to give the observed dynamical mass. We need to understand what the benefits of this action are. From now on we use the symbol ξ for the weights, rather than ***w***; as this fits in with previous work.

Consider the following hypothetical example. We have a central mass of 100 units and a weighting function that decreases away from the centre. We look at what this means for massless test particles at various distances.

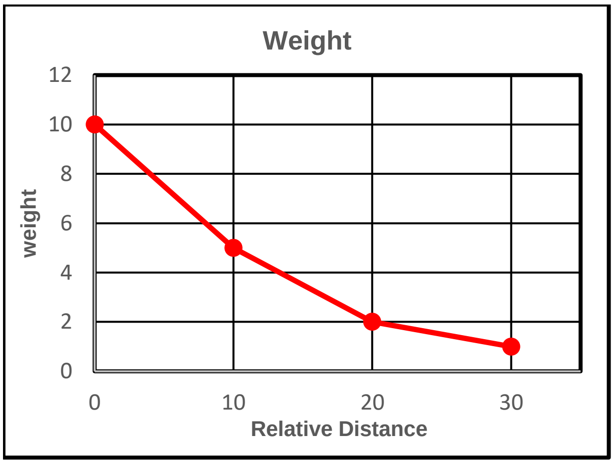


Figure 3. Weight function. An arbitrarily set of values for the weights, declining away from a central mass.

Figure 3 shows how the weighting function varies with distance. We have chosen four arbitrary values so that the weighting function decreases away from the central mass.

Figure 4 (below) shows the cumulative mass as measured by a remote observer. The green line is the cumulative baryonic mass, which remains constant at 100 units. This follows because we are working with a central mass with no additional mass away from the centre. It corresponds to what observers would measure for galaxies based on photometric data and a mass-to-light ratio. The blue line is the cumulative dynamical mass, which increases away from the centre in accordance with equation (10). This corresponds to what observers deduce from the rotational velocities of disk galaxies and from the velocities of galaxy members in a galaxy cluster. It is seen that even though there is no additional mass away from the centre, a variation in the weights can give rise to an increasing dynamical mass.
Figure 4 is illustrative of diagrams often presented for galaxy clusters (c.f. Figure 2 above).

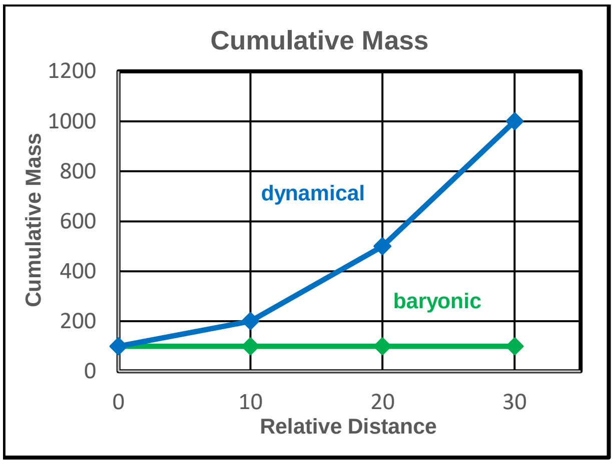


Figure 4. Cumulative mass. The green line is the cumulative baryonic mass as measured by a remote observer. The blue line is the cumulative dynamical mass as measured by a remote observer.

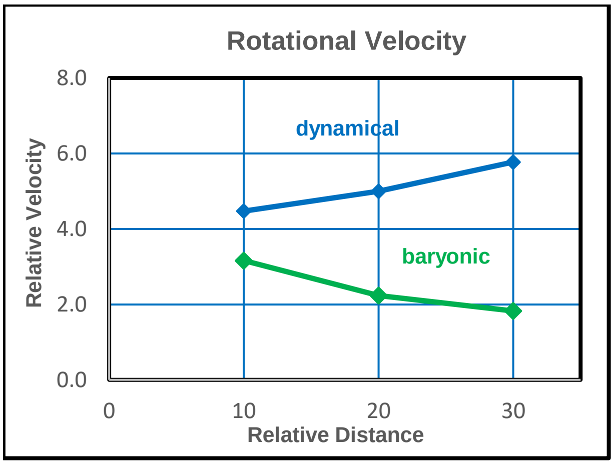


Figure 5. Rotation Curve. The green line (baryonic) is the expected curve for a central mass and Newtonian gravity. The blue line (dynamical) is what observers measure for the dynamical mass.

Figure 5 shows the rotation curve for our hypothetical values. The relative velocities have been calculated from the usual Newtonian formula

$$v = \sqrt{\frac{G\,M}{r}} \tag{13}$$

and setting G=1. The green line shows the usual Newtonian decline expected for a central mass; it is based on equation (13) where the mass is the baryonic mass. The blue line is what observers would see for massless test particles in a disk galaxy; it is also based on equation (13) where this time the mass is the dynamical mass. This figure shows how a weighting function can give rise to rotational velocities that are much higher than the expected velocities.

Figure 5 is illustrative of diagrams often presented for disk galaxies (c.f. Figure 1 above).

Distance	weight	baryonic mass	dynamical mass	baryonic velocity	dynamical velocity
0	10	100	100		
10	5	100	200	3.2	4.5
20	2	100	500	2.2	5.0
30	1	100	1000	1.8	5.8

Table 1. The hypothetical values used in Figures 3, 4 & 5.

For a disk galaxy, where the density falls off exponentially, we can assume the mass interior to a given radius is concentrated at the centre without incurring any significant errors (Binney & Tremaine, 2008). The rotational velocity, ***v(r)*** , at distance ***r*** is then given by.

$$v(r)^2 \;=\; \frac{G}{r\;\xi(r)} \int_0^r \xi(x)\; dM_{\text{bar}}(x) \tag{14}$$

where $\xi(r)$ is the value of the weighting function at ***r*** ; $\xi(x)$ is the value of the ξ-function at ***X***; $dM_{\text{bar}}(x)$ is the baryonic mass of the incremental shell at ***X***. So each incremental shell is weighted by the local value of ***ξ***, and the whole integral is then divided by the value of ***ξ*** at ***r*** .
This formula is used for explaining the rotation curves of disk galaxies as covered in the next section.

For a galaxy cluster, which is roughly spherical in shape, the total dynamical mass interior to radius ***r*** is given in terms of the baryonic mass by

$$M(r)_{\text{dyn}} \;=\; \frac{1}{\xi(r)} \int_0^r \xi(x)\; dM_{\text{bar}}(x) \tag{15}$$

where $M(r)_{\text{dyn}}$ is the cumulative dynamical mass interior to ***r*** ; $\xi(x)$ is the value of the ξ-function at ***X***; $dM_{\text{bar}}(x)$ is the baryonic mass of the incremental shell at ***X***.
This formula is used for explaining the relationship between the baryonic and dynamical masses of galaxy clusters.

We can construct more realistic diagrams by assuming a density distribution that drops off exponentially, so that the mass of an incremental shell is given by

$$dM_{\text{bar}}(x) \;=\; 4\,\pi\,\rho_c\, e^{-x/x_0}\, x^2\, dx \tag{16}$$

where ρ_c is the central density; x_0 is the characteristic distance.
We also choose, somewhat arbitrarily, the weighting function

$$\xi(x) = \frac{1}{1+x} \tag{17}$$

This behaves as ***1/x*** at large distances and also behaves normally at the centre. The following figures show the relative cumulative mass distributions and relative rotational velocities.

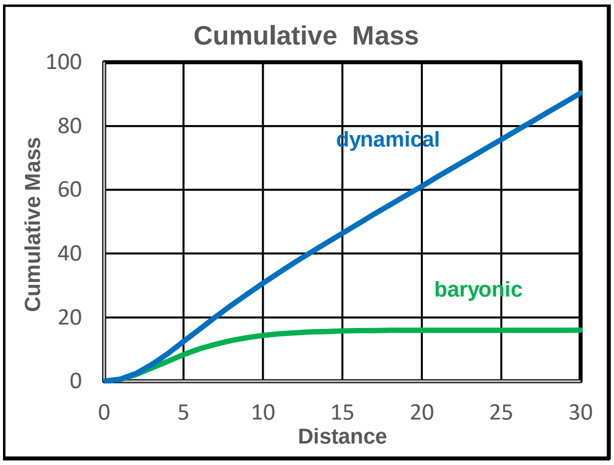


Figure 6. Cumulative mass distributions. The green line shows the relative cumulative baryonic mass distribution for the exponential density distribution of equation (16). The blue line shows the relative cumulative dynamical mass distribution after applying the weighting function of equation (17).

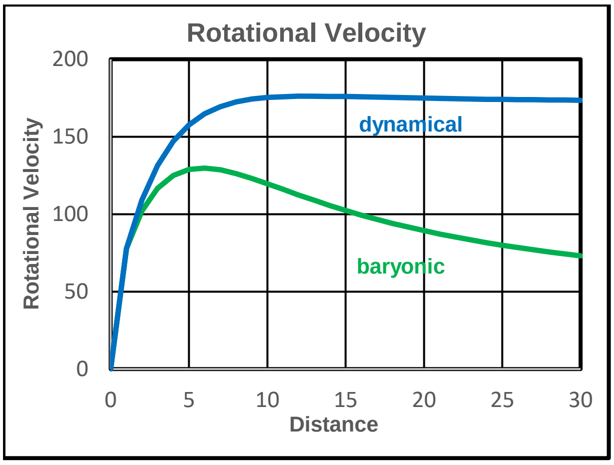


Figure 7. Rotation Curve. The green line is the rotation curve for the baryonic mass corresponding to the exponential density of equation (16). The blue line shows the rotation curve for the dynamical mass distribution after apply the weighting function of equation (17). The velocities and distances are relative.

Figure 6 is very similar to the observed cumulative mass distributions for disk galaxies. The baryonic mass has converged by relative distance 15, with little additional mass beyond this point. The dynamical mass continues to increase. The plot is also similar to the cumulative mass distributions for galaxy clusters. However, the observed cumulative baryonic mass tends to show a continuing increase, rather than a levelling off.

Figure 7 is very similar to the observed rotation curves of disk galaxies. The observed baryonic masses (stars and gas) lead to an expected rotation curve shown by the green curve. The observed rotation curves are very similar to the blue line which shows the derived dynamical mass.

5 Calculating the weights

So far we have made the arbitrary decision that there is no dark matter and that the observed dynamical mass is the weighted sum of the observed baryonic mass. We now need to calculate the values of the weights from the observations, and see where these might lead us, if anywhere.

If we know the baryonic & dynamical masses and the weights from the centre of an object out to distance ***r*** , then we know all of the values in equation (9), which can be written in incremental form as

$$\xi(r) \sum_{0}^{r} \Delta M_{\text{dyn}}(x) = \sum_{0}^{r} \xi(x)\, \Delta M_{\text{bar}}(x) \tag{18}$$

where $\Delta M_{\text{dyn}}(x)$ is an increment in the dynamical mass; $\Delta M_{\text{bar}}(x)$ is an increment in the baryonic mass.

For the next point further out, labelled ***(r+1)***, we have

$$\xi(\mathbf{r+1}) \sum_{0}^{r+1} \Delta M_{\text{dyn}}(x) = \sum_{0}^{r+1} \xi(x)\, \Delta M_{\text{bar}}(x) \tag{19}$$

$$= \sum_{0}^{r} \xi(x)\, \Delta M_{\text{bar}}(x) + \xi(\mathbf{r+1})\, \Delta M_{\text{bar}}(\mathbf{r+1})$$

or

$$\xi(\mathbf{r+1}) \left\{ \sum_{0}^{r+1} \Delta M_{\text{dyn}}(x) - \Delta M_{\text{bar}}(\mathbf{r+1}) \right\} = \sum_{0}^{r} \xi(x)\, \Delta M_{\text{bar}}(x) \tag{20}$$

which defines the weight at the next point out, $\xi(r+1)$. So if we know the weight at one point, then we can calculate the weight at the next point. This enables us to integrate outwards from the centre of an object to its periphery. We just need to know the starting value to kick the procedure off.

We recall that we expect the weight to have a constant value. ξ_c, across the object's centre, and that we only have relative values for the weights, not absolute. So, we are free to assign the weight a value of 1000 (say) at the centre. With just one data point for the centre, we have

$$M_{\text{dyn}}(0) = \frac{\xi_c\, M_{\text{bar}}(0)}{\xi(r_1)} = \frac{1000\ M_{\text{bar}}(0)}{\xi(r_1)} \tag{21}$$

where $M_{dyn}(0)$ is the observed dynamical mass of the centre; $M_{bar}(0)$ is the observed baryonic mass of the centre; r_1 is the radius of the centre; $\xi(r_1)$ is then our estimate of the weight at the edge of the centre. So we have moved from the centre out to point r_1 and can now move further outwards using equation (20).

Equation (20) guarantees that we can always find a positive value for the weight ξ at every point in a disk galaxy or galaxy cluster. In this sense, our weight is no better than dark matter, where we can always find an amount of dark matter that balances the dynamical mass with the baryonic mass. We appear to be substituting one arbitrary hypothesis (dark matter) for another arbitrary hypothesis (our weighting function ξ). What we need to do now is calculate the weights for a number of objects and see if there is any underlying pattern. We do this next for a sample of disk galaxies and galaxy clusters.

6 Disk galaxies

Mass models and rotation curves for 175 of disk clusters have been published by Lelli et al (2016). 70 of these were analysed in viXra paper 1903.0109 (Jo.Ke., 2019); the remainder were omitted because they had too few data points or only covered the central regions. The dynamical masses come from the observed rotation curves. The baryonic masses come from photometric observations in the infrared and radio wavelengths.
The data on 4 galaxies are presented in viXra paper 1903.0109.
The data on an additional 64 galaxies are presented in "SPARC galaxy rotation curves" (Jo.Ke., 2019).
The following pages show the data for three galaxies; the data for additional galaxies are available at the above locations.

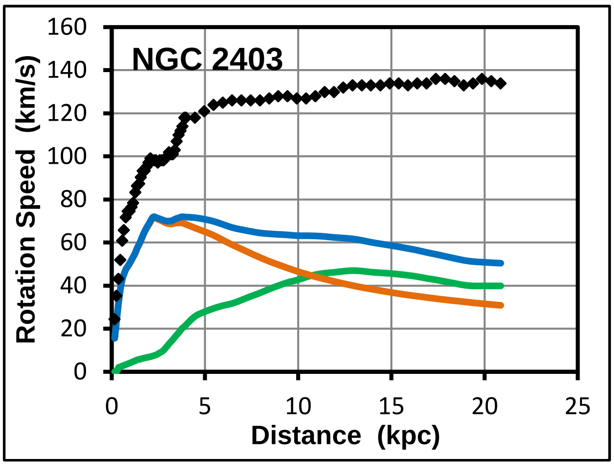
The upper left panel shows the rotation curves using the observed data of Lelli et al (2016). The black diamonds are observed velocities. The purple curve is the contribution to the velocity from the central bulge (if one exists); the orange curve from the disk of stars; the green curve from the gas. The blue curve is the expected velocity given by aggregating the other components.

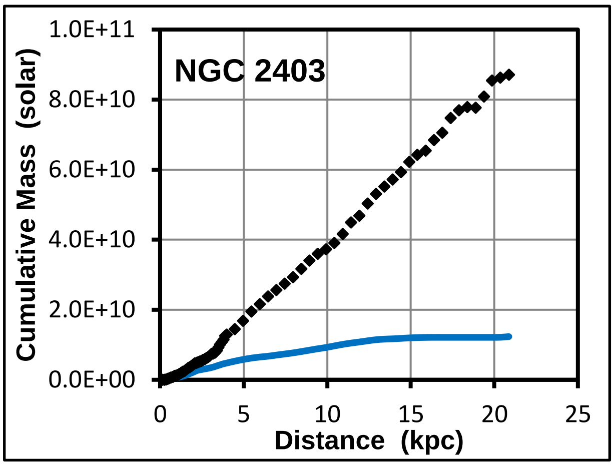
The top right panel shows the cumulative mass distribution corresponding to the velocities in the top left panel. The black diamonds give the observed total mass corresponding to the black diamonds in the top left panel. The blue line gives the normal matter mass corresponding to the blue line in the top left panel. This diagram shows whether the observed or expected masses are levelling off or are still increasing at the outer edge of the galaxy. The observed mass usually shows a continuing increase; the expected mass usually shows convergence.

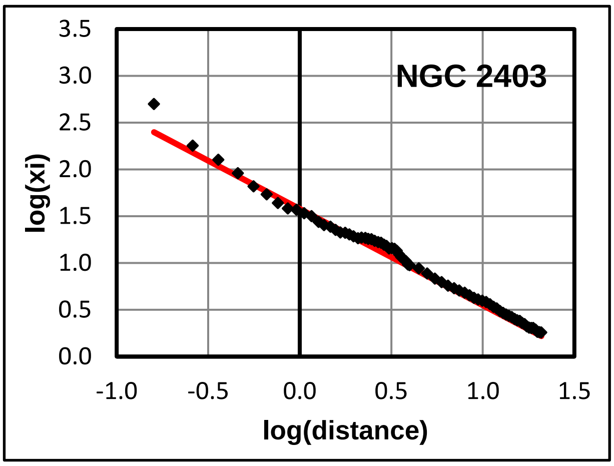
The lower left panel is a logarithmic plot of the ξ-function against the radial distance. The black diamonds are the values of the ξ-function and are based solely on the observed dynamical and baryonic masses. The near linear relationship away from the cluster centre is very clear. The red line is a straight line fit to the data, ignoring the first few data points. The linear relationship came as a surprise; it was completely unexpected.

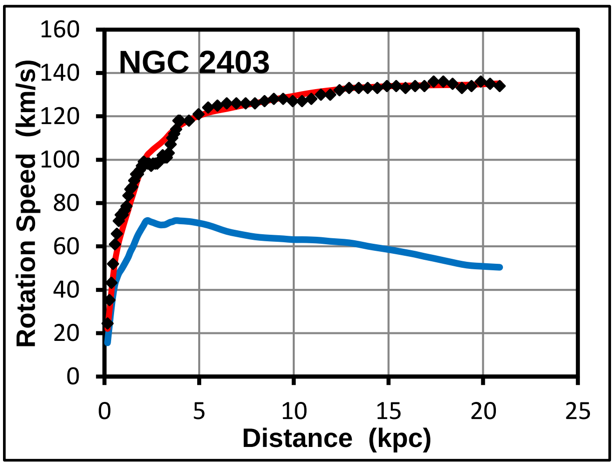
The bottom right panel shows the rotation curve again. The black diamonds are the same observed velocities as in the top left panel. Similarly, the blue line is the same expected velocities as in the top left panel. The red line is the fitted rotation curve derived by applying the red line from the bottom left panel for ξ-function to the blue line from the top right panel.

Disk Galaxy NGC 2403









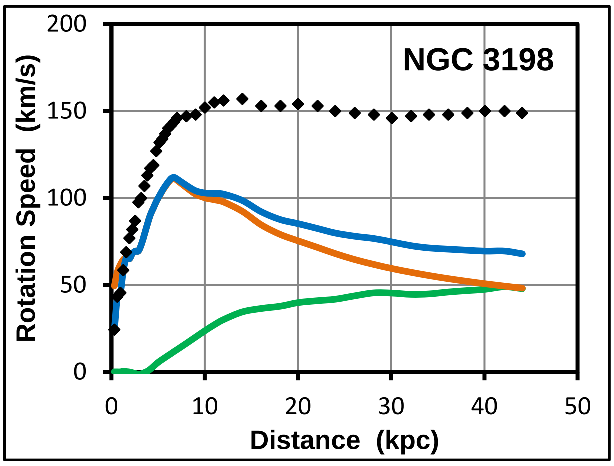
Slope of ξ-function: -1.03

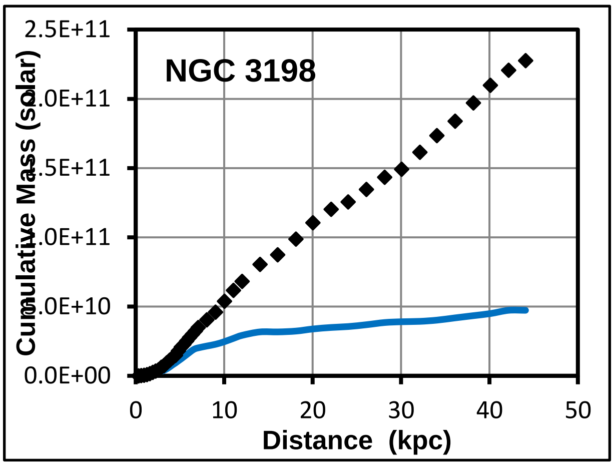
The red curve in the bottom right panel is the predicted shape of the rotation curve. It is a good fit to the observed rotation curve (black diamonds). It is based on the baryonic mass distribution (blue curve) and a straight line for the ξ-function, similar to that shown in the bottom left panel.

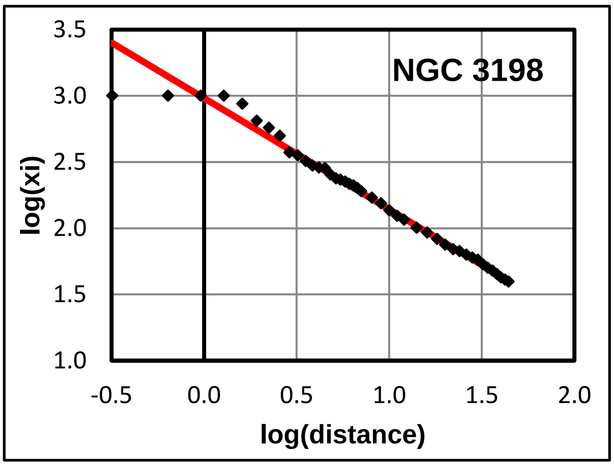
The upper right panel shows that the total baryonic mass (blue line) of the galaxy has converged by 15 kpc, whereas the total dynamical mass (black diamonds) continues to increase.

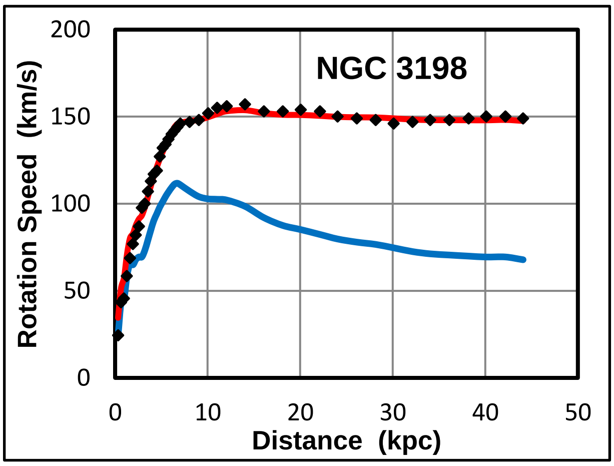
This galaxy is used as a typical disk galaxy throughout the book "The Dark Matter Problem" (Sanders, 2010).

Disk Galaxy NGC 3198







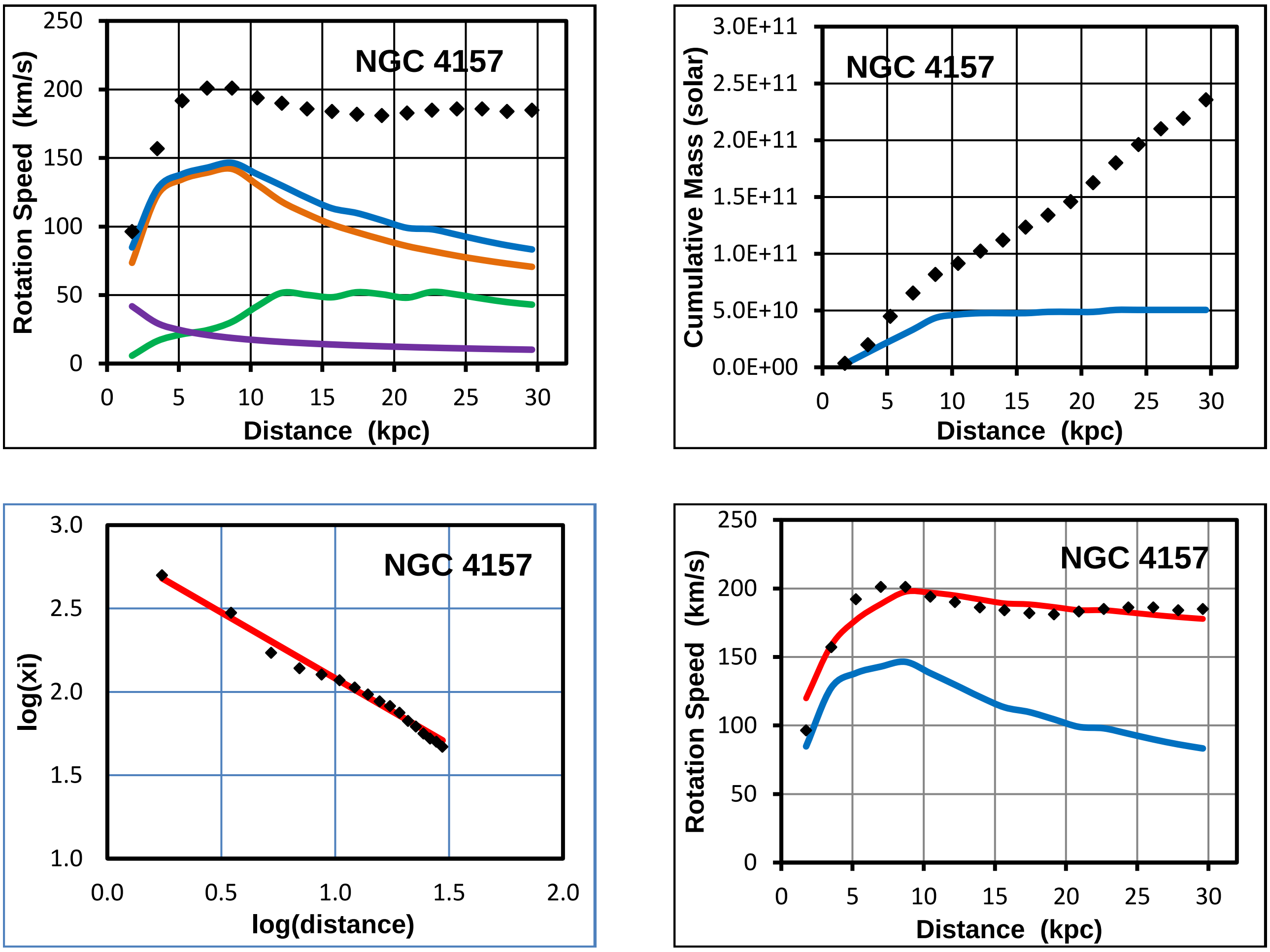


Slope of ξ-function: -0.84

The red curve in the bottom right panel is the predicted shape of the rotation curve. It is a good fit to the observed rotation curve (black diamonds). It is based on the baryonic mass distribution (blue curve) and a straight line for the ξ-function, similar to that shown in the bottom left panel.

The upper right panel suggests that the total baryonic mass (blue line) has converged just after 40 kpc, whereas the total dynamical mass (black diamonds) continues to increase and shows no sign of levelling out.

Disk Galaxy NGC 4157



Slope of ξ-function: -0.79

The red curve in the bottom right panel is the predicted shape of the rotation curve. It is a good fit to the observed rotation curve (black diamonds). It is based on the baryonic mass distribution (blue curve) and a straight line for the ξ-function, similar to that shown in the bottom left panel.

The upper right panel shows that the total baryonic mass (blue line) converges around 10 kpc, with little extra mass added beyond that. The total dynamical mass (black diamonds) continues to increase and shows no sign of levelling out.

7 Galaxy Clusters

Data on the baryonic and dynamical masses for a number of galaxy clusters have been published by Li et al (2023). We have analysed five of these clusters, omitting those that are disturbed and those with no X-ray data. The dynamical masses come from the velocity dispersion of galaxy members and the hydrostatic mass of the gas. The baryonic masses come from the X-ray gas, surface brightness fits, and the galaxy masses.

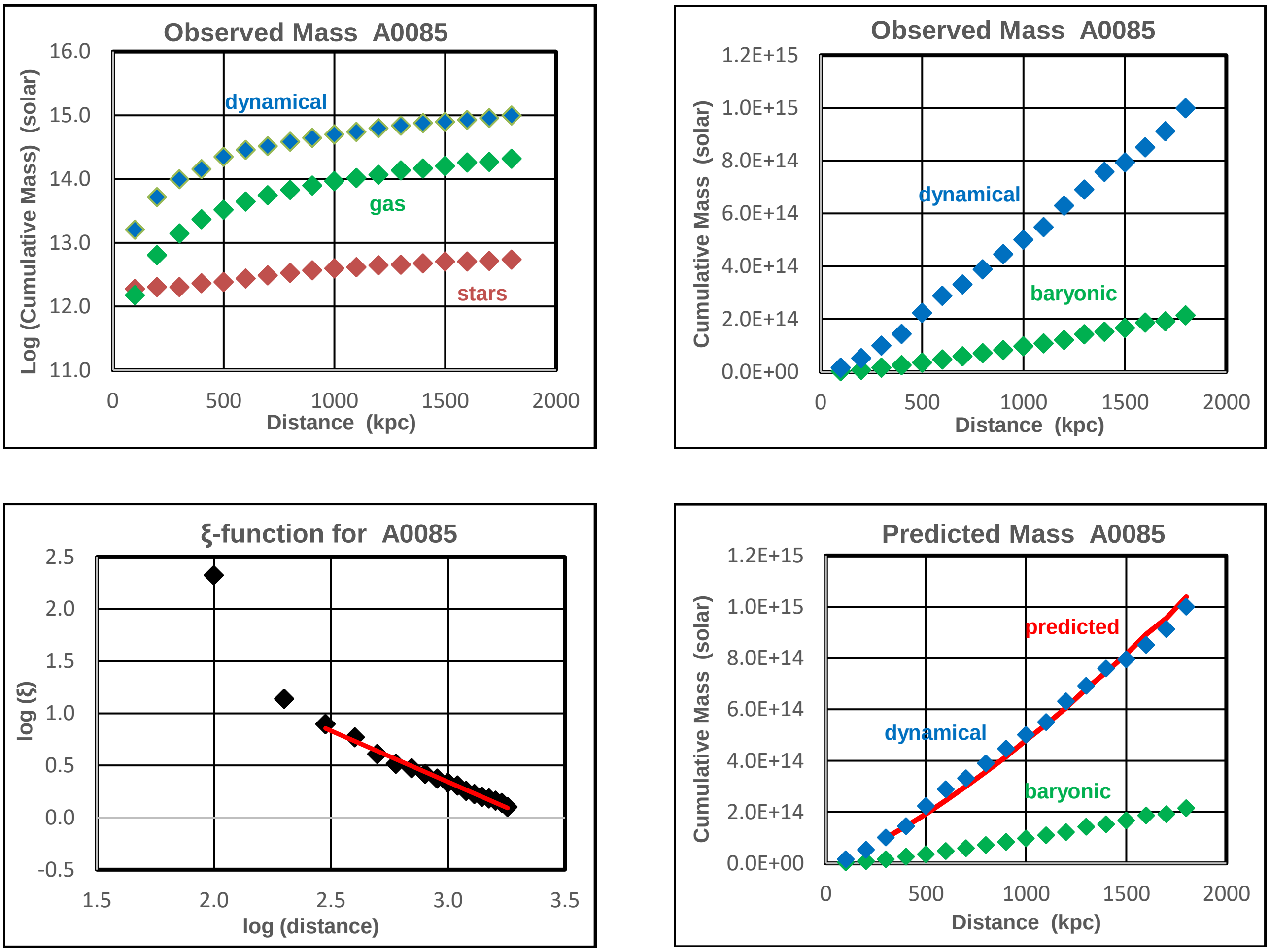
The upper left panel shows the data taken from Li et al (2023). The blue diamonds are the dynamical mass; green diamonds the mass of gas; orange diamonds the mass of stars. The dynamical mass is the average of the different values (velocity dispersion, hydrostatic mass, surface brightness). The gas mass is the average of the different values (gas mass profiles, surface brightness). The data comes as a logarithmic plot, which was measured manually. We did not have access to the linear (non-logarithmic) values or any tabular data.

The upper right panel is identical to the upper left diagram but with the data plotted on a linear scale. The blue diamonds are the dynamical masses; the green diamonds the baryonic masses. The baryonic mass is the sum of the gas and the stars, as shown in the upper left figure.

The lower left panel is a logarithmic plot of the ξ-function against the radial distance. The black diamonds are the values of the ξ-function and are based solely on the observed dynamical and baryonic masses. The near linear relationship away from the cluster centre is very clear. The red line is a straight line fit to the data, ignoring the first two data points. This is an observational result based on data from Li et al (2023) and our guess that the dynamical mass is a weighted sum of the baryonic mass. The linear relationship came as a surprise; it was completely unexpected.

The lower right panel is identical to the upper right diagram. The solid red line is the predicted dynamical mass based on the observed baryonic and a linear ξ-function similar to that shown in the lower left diagram. It is clear in all cases that the predicted dynamical mass (red line) is a good approximation to the observed dynamical mass.

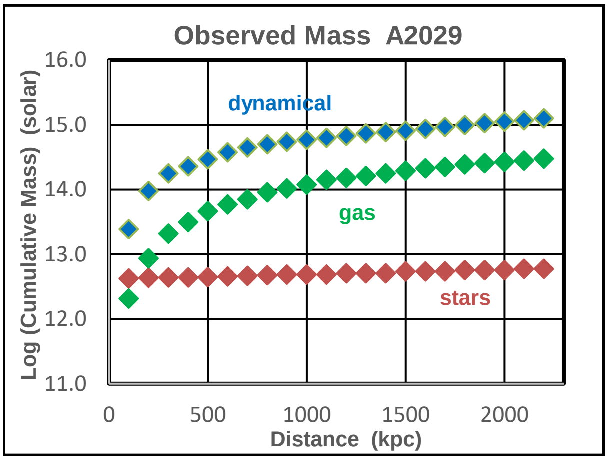
Galaxy Cluster A0085

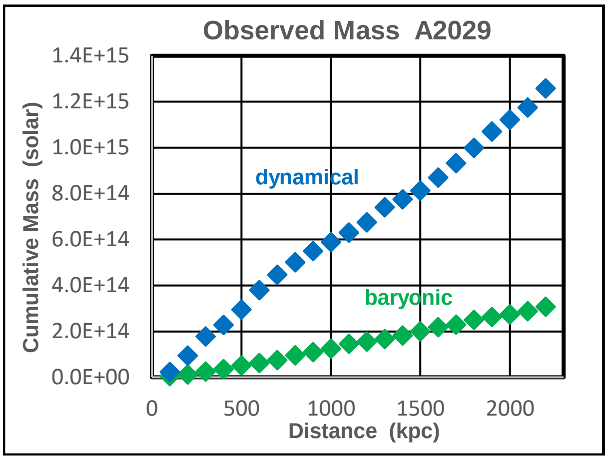


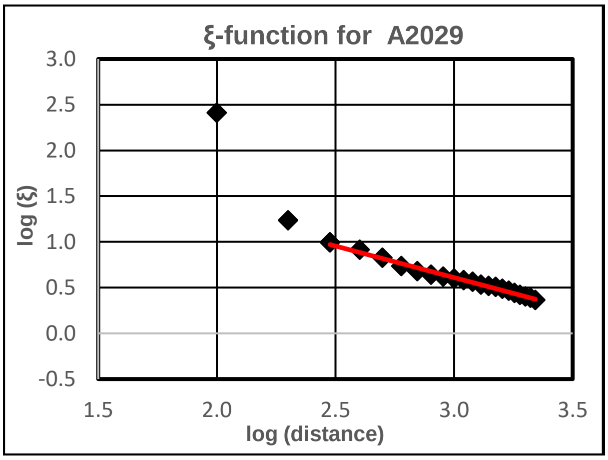
Slope of ξ-function: -1.05

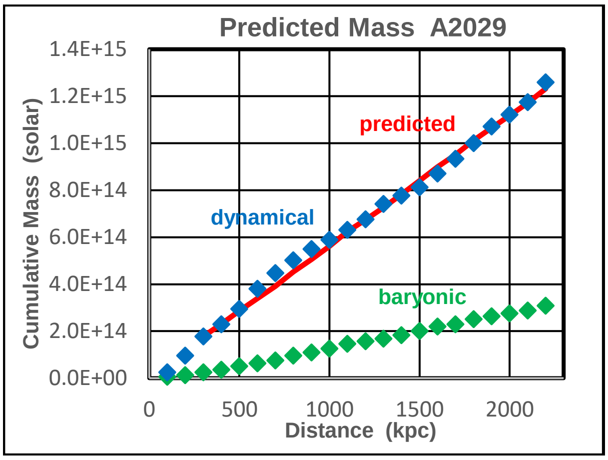
The red line in the bottom right panel is the predicted curve for the dynamical mass distribution. It is based on the baryonic mass distribution (green diamonds) and a straight line for the ξ-function, similar to that shown in the bottom left panel. The fit is normalised at the 3rd data point.

Galaxy Cluster A2029





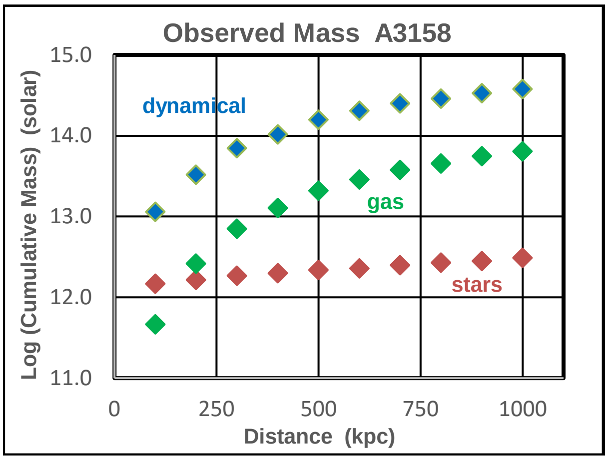


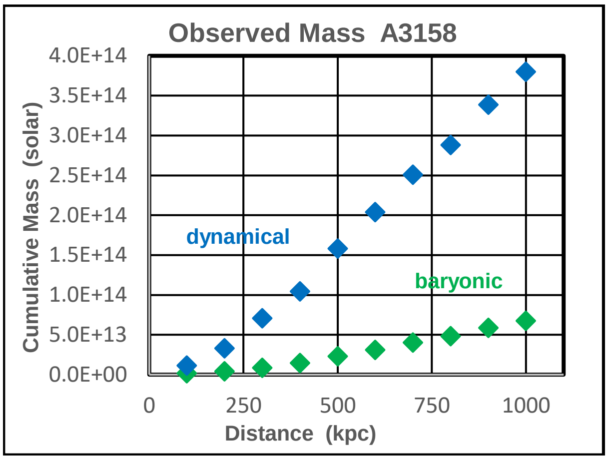


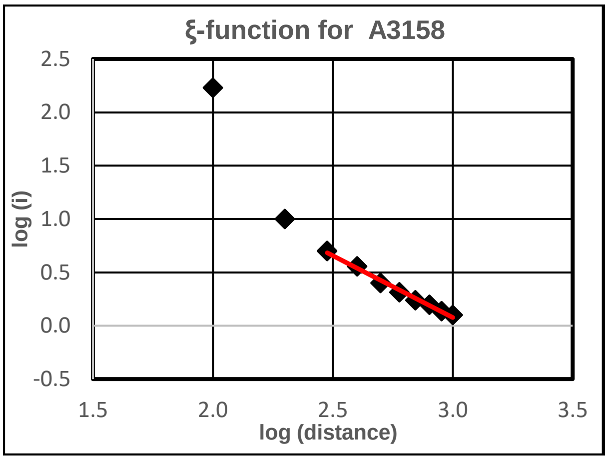
Slope of ξ-function: -0.73

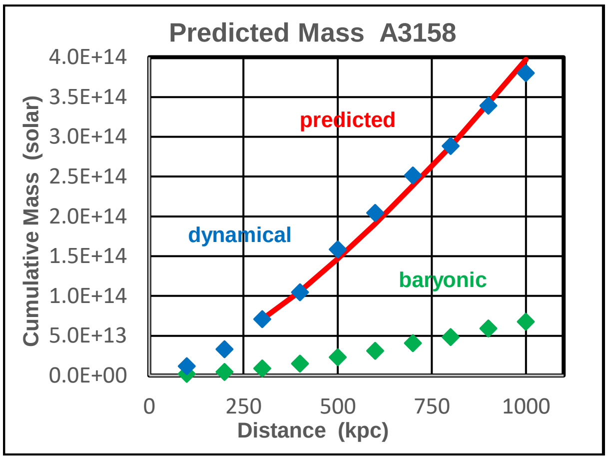
The red line in the bottom right panel is the predicted curve for the dynamical mass distribution. It is based on the baryonic mass distribution (green diamonds) and a straight line for the ξ-function, similar to that shown in the bottom left panel. The fit is normalised at the 3rd data point.

Galaxy Cluster A3158









Slope of ξ-function: -1.20

The red line in the bottom right panel is the predicted curve for the dynamical mass distribution. It is based on the baryonic mass distribution (green diamonds) and a straight line for the ξ-function, similar to that shown in the bottom left panel. The fit is normalised at the 3rd data point.

8 A linear relationship

The bottom left panels for the disk galaxies and galaxy clusters all show a strong linear relationship between the logarithms of our weighting function and distance. This is an observational result based on the measured baryonic and dynamical masses and our guess that the dynamical mass is a weighted sum of the baryonic mass. This linear relationship was entirely unexpected. The connection between the baryonic and dynamical masses, as given by equation (14) for disk galaxies and equation (15) for galaxy clusters, gave no hint that such a linear relationship for the ξ-function might exist.

The linear relationship only applies away from the centres of the disk galaxies and galaxy clusters. So, we are talking about the outer 80-90%. The inner 10-20% is the region where normal Newtonian gravity seems to apply and where dark matter is not required. The outer 80% is the region where the mass discrepancy shows itself and this is exactly the region where the linear relationship is a good explanation for what is happening.

The observed linear relationship means that the ξ-function is given by

$$\mathbf{log}(\boldsymbol{\xi}) \;=\; \boldsymbol{\alpha}\ \mathbf{log}(\boldsymbol{r}) \;+\; \mathbf{constant} \tag{22}$$

where α is the slope of the linear relationship.
Equation (22) can be rewritten, without the logarithms, as

$$\frac{\boldsymbol{\xi(r)}}{\boldsymbol{\xi_0}} \;=\; \left(\frac{\boldsymbol{r}}{\boldsymbol{r_0}}\right)^{\boldsymbol{\alpha}} \tag{23}$$

where the equation is normalised at the point (ξ_0, r_0), The observations show that the exponent α (the slope of the linear relationship) lies in the range

$$-\mathbf{0.5} \;>\; \boldsymbol{\alpha} \;>\; -\mathbf{1.5} \tag{24}$$

It is encouraging that the linear relationship, originally found for disk galaxies (JoKe, 2019), applies equally well to galaxy clusters. Of course, any hypothesis for explaining the mass discrepancies found in many astronomical scenarios, must be able to explain multiple scenarios, not just one. Nevertheless, it is a big step forward for our assumption, that a weighting function applied to the baryonic mass defines the dynamical mass. And that this assumption can explain both disk galaxies and galaxy clusters.

It is not surprising that the exponent, α, is not a fixed constant, but varies from galaxy to galaxy and from cluster to cluster. Just as disk galaxies and galaxy clusters come in different sizes and masses, so we would expect our weighting function to come in different shapes and sizes. However, it is surprising that the α exponent is the only parameter that is required. We can now explain the dynamical masses of both disk galaxies and galaxies clusters using an equation with just one free parameter, the ***α*** exponent; a different value of the ***α*** exponent is needed for each object.

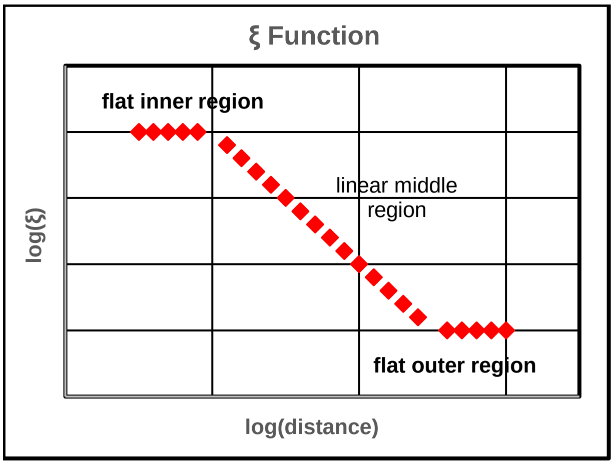


Figure 6. Expected behaviour of ξ-function. The logarithm of the ξ-function cannot be linear over its entire range. It must level off in the innermost regions and in the outermost regions where it merges into intergalactic space.

Although the data for both disk galaxies and galaxy clusters show a strong linear relationship, the logarithm of the ξ-function cannot stay linear over its entire range. Our expected behaviour is illustrated in Figure 6. The log function has an obvious discontinuity in the central regions as the distance shrinks to zero and the log function goes to infinity. Therefore we suggest the ξ-function must level off to a fixed value at the centre.

At large distances both galaxies and clusters end and we enter into intergalactic space where, again, we expect the ξ-function to level off to another fixed value. For disk galaxies this means we expect the rotation curve to return eventually to the standard Keplerian decline where the rotational velocity decreases as the inverse square root of the distance

$$v \propto \frac{1}{\sqrt{r}} \qquad (25)$$

9 Predicting the dynamical masses

Having established the observed linear behaviour of our weighting function, we are now in a position where we can predict the dynamical mass from the observed baryonic mass for both disk galaxies and galaxy clusters. We no longer need any dark matter to explain the discrepancy between the baryonic and dynamical masses.

The cumulative dynamical mass at $\boldsymbol{r}$ is given by equation (9), which can be written as

$$\boldsymbol{M}_{\mathbf{dyn}}(\boldsymbol{r}) = \frac{\mathbf{1}}{\boldsymbol{\xi}(\boldsymbol{r})} \sum_{\mathbf{0}}^{\boldsymbol{r}} \boldsymbol{\xi}(\boldsymbol{x})\, \boldsymbol{\Delta M}_{\mathbf{bar}}(\boldsymbol{x}) \tag{26}$$

where $\boldsymbol{M}_{\mathbf{dyn}}(\boldsymbol{r})$ is the cumulative dynamical mass from the centre to distance $\boldsymbol{r}$; $\boldsymbol{\xi}(\boldsymbol{r})$ is the weight at $\boldsymbol{r}$; $\boldsymbol{\Delta M}_{\mathbf{bar}}(\boldsymbol{x})$ is the increment of baryonic mass at $\boldsymbol{x}$; $\boldsymbol{\xi}(\boldsymbol{x})$ is the weight at $\boldsymbol{x}$. So if we know both the distribution of baryonic mass across the object and the weight $\boldsymbol{\xi}$, then we can predict the dynamical mass. We know the baryonic mass from observations, and we know the weight from equation (23), providing we know the value of the exponent $\boldsymbol{\alpha}$. Equation (26) enables us to start at the centre and work our way outwards, by adding in the incremental contributions of the baryonic mass.

It often turns out that the central regions of disk galaxies and galaxies clusters have large uncertainties. This means it is difficult to integrate from the centre outwards; we need a way of starting some way out. We can do this by normalising the data at a selected distance $\boldsymbol{R}$. For this point, equation (25) is

$$\boldsymbol{M}_{\mathbf{dyn}}(\boldsymbol{R}) = \frac{\mathbf{1}}{\boldsymbol{\xi}(\boldsymbol{R})} \sum_{\mathbf{0}}^{\boldsymbol{R}} \boldsymbol{\xi}(\boldsymbol{x})\, \boldsymbol{\Delta M}_{\mathbf{bar}}(\boldsymbol{x}) \tag{27}$$

where $\boldsymbol{M}_{\mathbf{dyn}}(\boldsymbol{R})$ is the cumulative dynamical mass from the centre to $\boldsymbol{R}$; $\boldsymbol{\xi}(\boldsymbol{R})$ is the weight at $\boldsymbol{R}$.

We can split the summation in equation (26) into two separate summations; (a) from the centre to our normalisation point $\boldsymbol{R}$, and (b) from $\boldsymbol{R}$ to a point $\boldsymbol{r}$ further out. Using equation (27), we can write equation (26) as

$$\boldsymbol{M}_{\mathbf{dyn}}(\boldsymbol{r}) = \frac{\mathbf{1}}{\boldsymbol{\xi}(\boldsymbol{r})} \left\{ \boldsymbol{\xi}(\boldsymbol{R}) \boldsymbol{M}_{\mathbf{dyn}}(\boldsymbol{R}) + \sum_{\boldsymbol{R}}^{\boldsymbol{r}} \boldsymbol{\xi}(\boldsymbol{x})\, \boldsymbol{\Delta M}_{\mathbf{bar}}(\boldsymbol{x}) \right\} \tag{28}$$

The predicted red lines in the bottom right panels of the diagrams for the disk galaxies and galaxy clusters were calculated using equation (28), normalising at the third data point out from the centre. You can judge for yourselves as to how well you think the predicted fit matches the observations.

We still have not fully predicted the dynamical masses, as we still need to know the exponent, $\boldsymbol{\alpha}$, of the weighting function in equation (23). One way to proceed is to normalise the fit at the 20% point, and to use the points from 20% to 40% to determine the value of $\boldsymbol{\alpha}$. Although we are not predicting the whole distribution of dynamical mass, we are predicting 60% of it, which is a pretty substantial amount.

The observed data show that the exponent, $\boldsymbol{\alpha}$, has a restricted range

$$-\mathbf{0.5} \ > \ \boldsymbol{\alpha} \ > \ -\mathbf{1.5} \tag{29}$$

and clusters around -1.0. So a good starting guess for the exponent would be $\boldsymbol{\alpha}$ = -1.0.

10 Gravitational acceleration

The dark matter problem is sometimes viewed as a problem with the gravitational acceleration. In all cases the predicted acceleration from the observed baryonic matter is too low and something needs to be done to make the acceleration larger. We take a brief look at the different solutions to the dark matter problem and how our weighting function conjecture fits in with these. We consider the simple situation of a central mass, $\boldsymbol{M}$, and a test particle a distance, $\boldsymbol{r}$, away. The masses and velocities involved with disk galaxies and galaxy clusters mean that we do not need to consider any relativistic effects. Simple Newtonian gravity should suffice.

Newtonian gravity. The basic Newtonian acceleration, $\boldsymbol{g_N}$, is given by

$$\boldsymbol{g_N(r)} = -\frac{\boldsymbol{G\,M}}{\boldsymbol{r^2}} \qquad (30)$$

Dark Matter. The dark matter problem is solved by the addition of large amounts of dark matter, which remains hypothetical and has never been detected in any experiment. The acceleration is given by

$$\boldsymbol{g(r)} = -\frac{\boldsymbol{G\,(M + M_{DM})}}{\boldsymbol{r^2}} = \boldsymbol{g_N(r)} - \frac{\boldsymbol{G\,M_{DM}}}{\boldsymbol{r^2}} \qquad (31)$$

where $\boldsymbol{M_{DM}}$ is the dark matter that is added to give the required acceleration. The basic form of Newtonian gravity is kept; we still have the inverse square law but the mass is increased by simply adding an the extra dark matter component.

Modified Gravity. The dark matter problem is changed by modifying the law of gravity. The best known example is MOND proposed by Milgrom (Sanders, 2010). In high acceleration regions normal Newtonian gravity applies, as given by equation (30). In low acceleration regions the acceleration is given by

$$\boldsymbol{g(r)} = -\sqrt{\frac{\boldsymbol{G\,M}}{\boldsymbol{r^2}}\;\boldsymbol{a_0}} = \sqrt{\boldsymbol{g_N(r)}}\;\sqrt{\boldsymbol{a_0}} \qquad (32)$$

where $\boldsymbol{a_0}$ is the limiting acceleration (~1.2×10^{-10} m.s^{-2}). Newton's law of gravity is modified. It is no longer an inverse square law but changes to an inverse linear law.

Weighting function. For our conjecture the acceleration is given by

$$\boldsymbol{g(r)} = -\frac{\boldsymbol{G}}{\boldsymbol{r^2}}\;\boldsymbol{M}\left\{\frac{\boldsymbol{\xi(0)}}{\boldsymbol{\xi(r)}}\right\} = \boldsymbol{g_N(r)}\;\frac{\boldsymbol{\xi(0)}}{\boldsymbol{\xi(r)}} \qquad (33)$$

where the baryonic mass is multiplied by the ratio of ξ values to give the dynamical mass. We retain the inverse square law but multiply the baryonic mass by a weighting function to give the dynamical mass.

The above examples hopefully clarify the differences between the various ways of solving the dark matter problem.

11 Gravitational lensing

Gravitational lensing is often used to support the existence of dark matter. The usual scenario is where the light from a remote galaxy is bent (lensed) by a galaxy cluster that is much closer to us.

General relativity gives the small angle, $\boldsymbol{\Delta\theta}$, that a light ray is bent through by a massive object as (d'Inverno, 1995)

$$\boldsymbol{\Delta\theta} = \frac{\boldsymbol{4\ G\ M}}{\boldsymbol{R\ \ c^2}} \tag{34}$$

where $\boldsymbol{M}$ is the mass of the object; $\boldsymbol{R}$ is the impact parameter or distance of closest approach by the light ray to the massive object.

For dark matter explanation the mass of the galaxy cluster is simply the sum of the baryonic mass and the dark matter mass, leading to

$$\boldsymbol{\Delta\theta} = \frac{\boldsymbol{4\ G\ (M_{\mathrm{bar}} + M_{DM})}}{\boldsymbol{R\ \ c^2}} \tag{35}$$

For galaxy clusters the estimate of the dark matter mass is in good agreement with the estimates from the virial theorem and the hot X-ray emitting gas.

For our alternative explanation, of a weighting function, the mass is simply the dynamical mass, leading to

$$\boldsymbol{\Delta\theta} = \frac{\boldsymbol{4\ G\ M_{\mathrm{dyn}}}}{\boldsymbol{R\ \ c^2}} \tag{36}$$

Using the dynamical mass in this way is fully consistent with the mass estimates determined by the virial theorem and X-ray emitting gas. So our alternative explanation is just as good as the dark matter explanation.

12 Discussion

In this paper we have approached the dark matter problem by assuming that there is no dark matter and that the observed dynamical masses are a weighted sum of the observed baryonic masses. We simply worked with the observed data and found that there is a simple linear relation for the weighting function. This is essentially an observational result. The linear relationship is in the data; it is not something that we have imposed on the data. The linear relationship seems to play a part in all disk galaxes and all galaxy clusters. This is a significant result and it cannot be ignored.

We postulate the existence of a weighting function with an approximate 1/r dependence. We add in the observed baryonic mass distribution and out pops the observed dynamical mass distribution. No dark matter, and no change to Newtonian gravitation. Just the introduction of a dimensionless scale field for our weighting function.

Our predicted dynamical masses are shown as the red curves in the bottom right panels of the figures for a few disk galaxies and galaxy clusters. These predicted red curves are calculated using the slope of the weighting function and the observed distribution of the baryonic masses. All the curves shown are excellent fits to the observed dynamical masses, all the way from the 3rd innermost data point to the outermost data point of each object.

Our predicted curves are clearly not perfect fits and we should not expect them to be. Our analysis is based on spherical symmetry being a good approximation for galaxy clusters and that for disk galaxies we can assume the mass acts as if concentrated at the centre. These are both reasonable assumptions but they are only first approximations. So, we should not expect perfect fits to the observations. There are also errors in the observations, which we have not taken into account. Nevertheless, we are impressed with the quality of the fits, which compare favourably with what dark matter and modified gravity can produce.

It is clear from equation (14) that the rotational velocity at any point of a disk galaxy is partly determined by the baryonic mass at that point. This has the potential to explain "Renzo's Rule" (Sancisi, 2003), where features in the matter distribution give rise to features in the rotation curve. This is something that it is difficult for a spherical dark matter halo to explain.

Any astronomer or scientist is free to repeat our analysis and check that our linear relationship is a real phenomenon. Any researcher with access to the observations of baryonic and dynamical masses can derive our weighting function for themselves by solving equation (15). The linear relationship we obtain is clearly telling us something and clearly needs more detailed scrutiny. It is important to note that there is no need for dark matter to exist and that the observed dynamical mass is determined solely by the observed baryonic mass.

Our weighting function is sufficient to explain away dark matter in disk galaxies, galaxy clusters, and gravitational lensing. However, dark matter is invoked in physical cosmology where it is needed to explain the acoustic peaks in the cosmic microwave background and the formation of structure. We have not considered these items here, but the introduction of our weighting function suggests that changes must be made to the Friedmann-Lemaitre-Robertson-Walker metric (FLRW metric), which takes us well outside the scope of this paper. One obvious aspect is the gravitational potential and potential theory; their basis on the baryonic mass must be changed to work with the dynamical mass instead. A separate paper is in preparation that shows how our weighting function can explain these additional topics of physical cosmology.

Much of the material in this paper has been published in previous papers that came to the problem from a different point of view (JoKe 2015 "On the variation of the energy scale: an alternative to dark matter"; JoKe 2019 "An analysis of the rotation curves of disk galaxies using the SPARC catalogue"; JoKe 2020 "Variation of the energy scale: an alternative to dark matter"; JoKe 2023 "A linear relationship between the baryonic and dynamical masses of disk galaxies and galaxy clusters"). These papers all started from the assumption that the energy scale can vary from location to location, and that this variation is described by a scalar field. In this paper our approach is more straightforward and simply suggests the existence of a weighting function that acts on the baryonic mass to give the dynamical mass. These two ways of looking at the dark matter problem are essentially the same.

Our weighting function clearly constitutes a scalar field, in that it has a single scalar value at every point of space. As such it should then be amenable to the physics of scalar fields, which then takes us into the area of potential theory including items such as Gauss's Theorem and Poisson's Equation. Again, going there is beyond the scope of this paper. One suggestion as to why the weighting function proposed here might actually exist is that the Higgs field (which determines the masses of the fundamental particles) is already known to be a scalar field. Another suggestion is that our scalar field may be related to the scalar field that is believed to be responsible for cosmic inflation. If that scalar field did not decay away completely, then perhaps its remnants give rise to our scalar field and weighting function.

A number of predictions can be made based on our suggestion that a weighting function exists that determines the dynamical mass from the baryonic mass. These are set out in viXra paper 2007.0017 ("Variation of the energy scale: an alternative to dark matter") and in "Predictions and Tests" (JoKe21, 2019). Some of these are not particularly helpful, such as the prediction that no dark matter particle will ever be detected. Others are testable. Perhaps the best testable prediction is that the motions of interacting galaxies are determined by the baryonic mass and not by the dynamical mass. This arises because we expect the weighting function to have similar values at the centres of all galaxies. This is the complete opposite of the prediction from both dark matter and modified gravity.
A second testable prediction is that we expect the rotation curves of disk galaxies to show the usual Keplerian decline at large distances, as given by equation (25). So, even those galaxies that appear to have a flat rotation curve will eventually move to a Keplerian decline.

So, at the end of the day, what does this all mean? If non-baryonic dark matter exists, independent of normal baryonic matter, then there should be no relationship between the observed baryonic and dynamical masses in disk galaxies and galaxy clusters. But we have shown that there is a simple linear relationship that enables us to calculate the dynamical mass from the baryonic mass. We have a reductio ad absurdum argument that forces us to conclude that dark matter does not exist, at least neither in disk galaxies nor in galaxy clusters. In short, there is no dark matter. It also means that Newtonian gravity is intact; it is what should apply in the regimes of disk galaxies and galaxy clusters. We can sigh with relief that we do not have to get involved with modifications of the General Theory of Relativity. In short, Newtonian gravity is safe.

In summary, we started with a guess, uncovered a linear relationship that accounts for the baryonic and dynamical masses in disk galaxies and galaxy clusters, and enables us to predict the dynamical masses from the baryonic masses. We do not need to add any extra mass in the form of dark matter and we do not need to modify the law of gravity. Clearly, we have made some progress but more work is still to be done.

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