Proving the Goldbach Conjecture

Jim Rock

Abstract. In 1742 Christian Goldbach suggested that any even number four or greater is the sum of two primes. The Goldbach Conjecture remains unproven to the present day though it has been verified for all even numbers up to 4×10^{18}. This paper suggests an algorithm for checking the Goldbach conjecture for individual even numbers and a generalization that could be used to prove the Goldbach conjecture.

For any even number n there are $(n - 6) / 4 + 1$ pairs of odd integers between 3 and $n - 3$ that add up to n. We need every unique combination of odd numbers $3 \le m \le j$ and $9 \le (m)(j) \le n - 3$. We start with the largest prime $p_1 \le$ **sqroot(*n*)** and multiply it by itself and any larger primes such that $(p_1)(p_2)\ldots \le n - 3$. We choose the primes in descending order down to 3 and do the same thing.

The red superscripts highlight the matching composite pairs within this group.

Try it with ***200. (13)(13), (11)(11), (11)(13)[1], (11)(17), (7)(7), (7)(11)[2], (7)(13), (7)(17)[3], (7)(19), (7)(23)[4], (5)(5)[5], (5)(7)[6], (5)(11)[7], (5)(13)[8], (5)(17)[9], (5)(19)[10], (5)(23)[9], (5)(5)(5)[11], (5)(29)[7], (5)(31)[12], (5)(5)(7)[5], (5)(37)[13], (3)(3), (3)(5)[13], (3)(7), (3)(3)(3), (3)(11), (3)(13)[4], (3)(3)(5)[12], (3)(17), (3)(19)[1], (3)(3)(7), (3)(23), (3)(5)(5)[11], (3)(3)(3)(3)[3], (3)(29), (3)(31), (3)(3)(11), (3)(5)(7)[10], (3)(37), (3)(3)(13), (3)(41)[2], (3)(43), (3)(3)(3)(5)[8], (3)(47), (3)(7)(7), (3)(3)(17), (3)(53), (3)(5)(11)[6], (3)(3)(19), (3)(59), (3)(61), (3)(3)(3)(7), (3)(5)(13)***

There are a total of ***54*** composites with ***13*** matched pairs within this group. We have a total of ***41*** pairs containing composite integers. We subtract that number from $(200 - 6) / 4 + 1 = 49$ total pairs and are left with eight prime pairs. ***(3, 197), (7, 193), (19, 181), (37, 163), (43, 157), (61, 139), (73, 127), (97, 103)***

Try it with ***202. (13)(13)[1], (11)(11)[2], (11)(13), (11)(17)[3], (7)(7)[4], (7)(11)[5], (7)(13[6]), (7)(17), (7)(19)[7], (7)(23), (5)(5)[8], (5)(7), (5)(11)[9], (5)(13), (5)(17)[10], (5)(19), (5)(23)[11], (5)(5)(5)[5], (5)(29)[12], (5)(31), (5)(5)(7)[13], (5)(37), (3)(3), (3)(5)[3], (3)(7), (3)(3)(3)[13], (3)(11)[1], (3)(13), (3)(3)(5), (3)(17), (3)(19)[12], (3)(3)(7), (3)(23)[7], (3)(5)(5), (3)(3)(3)(3)[2], (3)(29)[11], (3)(31), (3)(3)(11), (3)(5)(7), (3)(37)[6], (3)(3)(13)[10], (3)(41), (3)(43), (3)(3)(3)(5), (3)(47), (3)(7)(7)[9], (3)(3)(17)[4], (3)(53), (3)(5)(11), (3)(3)(19), (3)(59)[8], (3)(61), (3)(3)(3)(7), (3)(5)(13)***

There are a total of ***54*** composites, but ***13*** are matched pairs within this group. We have a total of ***41*** pairs containing composite integers. We subtract that number from $(202 - 6) / 4 + 1 = 50$ total pairs and are left with nine prime pairs. ***(3, 199), (5, 197), (11, 191), (23, 179), (29, 173), (53, 149), (71, 131), (89, 113), (101, 101)***

Starting with the total number of composite integers between 3 and $n - 3$, we subtract the number of pairs with two composite integers that sum to n. This leave us with pairs with at least one composite integer that sum to n. If there are less than $(n - 6) / 4 + 1$, we must have prime pairs adding up to n.

The ratio of (prime pairs / total prime count) / (total prime count / odd numbers) remains close to 33% for large powers of two.

pwr2	tcomp	pripair	tprime	(pp/tp)/(tp/odd)
134217728	59505310	283746	7603552	32.9%
67108864	29596622	153850	3957808	33.0%
33554432	14713526	83467	2063688	32.9%
16777216	7310736	45746	1077870	33.0%
8388608	3630140	24928	564162	32.9%
4194304	1801204	13705	295946	32.8%
2097152	892964	7471	155610	32.4%
1048576	442262	4239	82024	33.0%
524288	218754	2367	43388	33.0%
262144	108071	1314	22999	32.6%

The ratio of (prime pairs / total prime count) / (total prime count / odd numbers) remains close to 33% for large numbers composed of powers of two multiplied by single prime numbers.

pwr2	prime	tcomp2	tcomp	pripair	tprime	(pp/tp) / (tp/odd)
16	8388593	26234306	59505201	283476	7603541	32.9%
16	8388617	26234972	59505373	284066	7603561	33.0%
32	4194301	26235027	59505264	284170	7603550	33.0%
64	2097143	26234226	59505052	283461	7603522	32.9%
64	2097169	26235158	59505796	284065	7603610	33.0%
128	1048573	26234314	59505138	283511	7603532	32.9%
256	524287	26234476	59505193	283650	7603541	32.9%
512	262139	26233960	59504169	283582	7603413	32.9%
512	262147	26234863	59505997	283681	7603633	32.9%
1024	131071	26234510	59504856	283829	7603494	32.9%
2048	65537	26234871	59506217	283597	7603669	32.9%
4096	32771	26237192	59510769	283926	7604237	33.0%
8192	16381	26229859	59494348	283798	7602226	32.9%
16384	8191	26231906	59498000	284241	7602670	33.0%
32768	4093	26215088	59461397	283546	7598313	32.9%
32768	4099	26253629	59549172	283464	7608842	32.9%
65536	2039	26117690	59241838	282827	7572112	33.0%
65536	2053	26299489	59651549	284291	7621153	32.9%
131072	1021	26156357	59329668	282816	7582586	32.9%
131072	1031	26417104	59914850	286061	7652764	33.0%
262144	509	26078990	59153994	282819	7561652	33.0%
524288	251	25715188	58334549	279710	7463593	33.0%
524288	257	26339548	59739380	285671	7631626	33.0%
1048576	127	26028729	59036877	284139	7547697	33.2%
2097152	61	24990915	56696127	276355	7267007	33.5%
2097152	67	27488235	62315329	300201	7939261	33.5%
4194304	31	25412242	57632297	285800	7379413	34.1%
8388608	16	26234625	59505310	283746	7603552	32.9%

Fewer prime factors in *n* makes the ratio of matched pairs within the composite group lower. This creates a lower percentage of matched prime pairs. The number of matched pairs in the composite group and matched prime pairs only gets larger as the number of prime factors in *n* increases.

There is a relationship for numbers between *3* and *n – 3, n ≥ 6* for total primes (tprime), total odd composites (tcomp), total composite pairs (tcomp2), and total prime pairs (pripair).

For *n = 4m*, *(2)(tcomp2 - pripair)(tcomp + tprime) – (tcomp)(tcomp) + (tprime)(tprime) = 0.*
For *n = 4m+2*, *(2)(tcomp2 - pripair)(tcomp + tprime+1) – (tcomp)(tcomp) + (tprime+1)(tprime+1) = 0.*

For n=200 $(2)(13-8)(54+44)-54^2+44^2=0$ *For n=202* $(2)(13-9)(54+46)-54^2+46^2=0$

Thanks for your interest in this paper. If you wish to make comments, send them to Jim Rock at collatz3106@gmail.com.