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The link between the de-Vries-formula for the fine structure constant and the power-of-two value of 128

Content

Abstract

Almost 20 years ago, de Vries proposed a recursive equation for the fine structure constant α that depends only on the mathematical constants e and π [1]. It gives a value for 1/α = 137.0359990958.

$$\alpha = \left((1 + \frac{\alpha}{(2\pi)^0} \cdot (1 + \frac{\alpha}{(2\pi)^1} \cdot (1 + \frac{\alpha}{(2\pi)^2}(1 + \ldots)))) \right)^2 \cdot e^{-\frac{\pi^2}{2}} \qquad (1)$$

By applying the term α/(2π) in a geometric series, this equation has the flavour of mainstream physics (see Schwinger-Dyson series on calculating the electron's g-factor), which in combination with the agreement with the experimentally found value is probably the reason why this formula, unlike many other numerology attempts, is not forgotten.

The starting point and core element of this equation is the term 1/e^(π^2/2). Despite interesting approaches [2], it has not yet been possible to give this term an unequivocal physical meaning. So I thought it worthwhile to investigate the proximity of this value to the closest power-of-two value, i.e. 128. This resulted from the considerations I made in the previous works, where I (among many others [3][4][5]) became aware of the importance of the number 2^128 as a link between the order of magnitude of the elementary particles and the universe [6].

Finally, another equation for α was obtained, which is roughly equivalent in complexity to the de Vries equation, depends only on π and 128, and whose result differs from the de Vries result by only about 2·10^-10.

It was during my playing around with the numerical values that I found the literature that told me that at extremely high energies the fine structure „constant“ increases to the value of about 1/128 [7]. So I saw another reason to publish these number juggleries. Here they are.

TL;DR

I will include side calculations and obvious dead ends when presenting the numerical matches. If you only want to get a rough overview, I recommend that you first look at the framed equations, which I present here in advance, removed from the context.

`Moreover, the equations 3,7 and 8, which are particularly noteworthy, are programmed in Python in the appendix.`

The value that will run like a thread through this work I call q and it is defined as follows:

$$q = \ln(2^{128}) \cdot \frac{\pi}{2} = \ln 2 \cdot 64\pi = 139.36550977943054 \tag{2}$$

There are interesting numerical connections to q with the term e^(π^2/2), which is the core term of the de Vries equation for the fine structure constant α:

$$\frac{q}{1 + \frac{1}{(q-1)\cdot\pi}} = \frac{e^{\frac{\pi^2}{2}}}{1.00000001004} \tag{3}$$

$$\frac{q}{1 + \frac{(\alpha\cdot q)-1}{e^2}} = \frac{e^{\frac{\pi^2}{2}}}{1.00000010936} \tag{4}$$

$$e^{\frac{\pi^2}{2}} - 2\pi \cdot \left(q - e^{\frac{\pi^2}{2}}\right) = \frac{\alpha^{-1}}{1.00000134648} \tag{5}$$

α: value according the de-Vries-equation (1) = 137.0359990958

From q and its interaction with e^(π^2/2) and 1/*α*, another interesting value crystallises (see chapter „*Let's close the gap*“), which is called z in the following and is defined as follows:

$$\begin{aligned} z &= 128 \cdot \ln 128 \cdot \ln 2 \quad = 7 \cdot 2^7 \cdot \ln^2 2 \\ &= 430.48590047 \end{aligned} \tag{6}$$

With z an equation can be formulated. which is roughly equivalent in complexity to the de Vries equation, depends only on π and 128, and whose result differs from the de Vries result by only about 2·10^-10:

$$\begin{aligned} \alpha^{-1} &= \frac{z}{\pi} \cdot \prod_{n=1}^{\infty} 1 + (-1)^{n+1} \cdot \left(\frac{1 + \frac{1}{z}}{4\pi^2 \cdot z} \right)^n \\ &= 137.03599911925744 \end{aligned} \tag{7}$$

or simplified without infinite product:

$$\begin{aligned} \alpha^{-1} &= \frac{z}{\pi} \cdot \left(1 + \frac{1 + \frac{1}{z}}{4\pi^2 \cdot z + 1 + \frac{1}{z}} \right) \\ &= 137.03599911928552 \end{aligned} \tag{8}$$

It's all just numerology - so what?

"Numerology can be used to prove any nonsense, and yet it is not meaningless" begins an article worth reading in the Swiss daily newspaper NZZ - published in 2004 [8]. As a classic example, Kepler's Third Law is cited there, which remained a numerical curiosity for decades until Newton was able to put it on a theoretical basis. And Consa, in his critical examination of QED [9], shows that the fundamental expression for calculating the g-factor of electrons, α/2π, which is engraved on the tombstone of its discoverer Julian Schwinger, must also have been pure numerology when it was first published in December 1947 [10].

When I started to write this paper, I was surprised at how many ideas and possibilities there already are for expressing a value of approx. 137.035999. The archive of the talk page on the English Wikipedia article on the fine structure constant is a veritable treasure trove in this respect.

My favourite here is the woof-woof formula [11], which an anonymous dog posted in there:

$$137 + \frac{ln137}{137} + \frac{lnln137}{137^2} + \frac{ln137}{137^3} + \frac{lnln137}{137^4} + \frac{ln137}{137^5} + \ldots = 137.0359990777649 \qquad (9)$$

And here the question arises: is a numerology attempt to express the number 137.035999 more promising if it builds up with 137 or with 128?

Honestly, I can't say for sure. I can only say from a gut feeling that a power-of-two number might be more promising to derive once from a large context than the nearest integer - even if it is a prime number.

And I had already mentioned the vague possibilities for this at the beginning:

- The strangest coincidence in physics is the fact that the order of magnitude of the elementary particles and that of the entire universe is about 2^128, which also corresponds to the ratio between electromagnetic and gravitational forces.
- The assumption made by Eddington that the universe has 1/α * 2^256 protons (dto electrons) is still considered a good estimate today, see https://en.wikipedia.org/wiki/Eddington_number.
- The fine structure constant is no longer considered to be a correct constant, but it is thought that the coupling increases to about 1/128 at high energies.

First of all a dead end

So, what numerical relationship could there be between e^(π^2/2) and 128?

If you divide the first term by the second, you get the quotient **1.086294**. This is in approximate order of ***1+√α=1.08542***, but I could not find anything meaningful in the gap between these values until now. It is still interesting that ***2*e/0,086294 = 63,000483***, thus approximately ***128/2-1***. But also here I find no further connecting points.

So let's see if we can find more if we consider only the exponent from the De-Vries term, i.e. only π^2/2 and compare it with the corresponding exponent for 128, i.e. for the x in e^x=128, i.e. ln(128). Then we have:

$$\frac{\pi^2}{2 \cdot ln128} = 1.0170592375 \qquad (10)$$

This value also looks like a dead end at first. But let's remember it anyway, I will come back to it again.

The interaction of e^2, 2π and α

So, let's turn to the role of 128, where this power-of-two value is itself an exponent of a power-of-two, i.e., 2^128. Here, it's interesting to know what the equivalent value is in the power of e, i.e., 2^128 = e^x. This gives x = ln(2)*128 = 88.7228391. If we now compare this value with e^(π^2/2), we get:

$$\frac{e^{\frac{\pi^2}{2}}}{\ln 2 \cdot 128} = 1.567191 \qquad (11)$$

This value is close to ***π/2 = 1.570796*** . We already had something like this in the last section with *1+√α*≈1.086294, but now the gap value turns out to be interesting:

$$\frac{\ln 2 \cdot 128\pi}{2 \cdot e^{\frac{\pi^2}{2}}} = \frac{\ln 2 \cdot 64\pi}{e^{\frac{\pi^2}{2}}} = 1.00230049 \qquad (12)$$

So here it is the term I already presented under TL;DR:

$$\boxed{\begin{aligned} q &= \ln(2^{128}) \cdot \frac{\pi}{2} = \ln 2 \cdot 64\pi \\ &= 139.36550977943054 \end{aligned} \qquad (2)}$$

The gap, i.e. the quotient **1.00230049** is now of particular interest. Because it results that:

$$\frac{1}{(1.00230049 - 1) \cdot \pi} = 138.36611266 \approx q - 1 \qquad (13)$$

This then also results in the interesting numerical relationship that I have already presented in equation (3):

$$\frac{q}{1+\frac{1}{(q-1)\cdot\pi}} = \frac{e^{\frac{\pi^2}{2}}}{1.00000001004} \qquad (14)$$

Further, it is now interesting to note that the distance between 1/α and q is approximately e^2 times the distance between q and e^(π^2/2) - and likewise that the distance between 1/α and e^(π^2/2) is approximately 2π times the distance between q and e^π^2/2.
First, let's look at the distance between q and 1/α. In absolute values this described ratio is a bit inaccurate:

$$q - e^2 \cdot \left(q - e^{\frac{\pi^2}{2}}\right) = 137.00194936 = \frac{\alpha^{-1}}{1.0002485} \qquad (15)$$

But it becomes more precise if we look at the by 1 subtracted quotient :

$$q \cdot \frac{1}{1+\left(\frac{q}{e^{\frac{\pi^2}{2}}}-1\right)\cdot e^2} = 137.03610823 \qquad (16)$$
$$= \alpha^{-1} \cdot 1.0000007964$$

or as already formulated in the TL;DR:

$$\frac{q}{1+\frac{(\alpha\cdot q)-1}{e^2}} = \frac{e^{\frac{\pi^2}{2}}}{1.00000010936} \qquad (4)$$

Unfortunately, so far I have not been able to see a straight-line pattern to fill this gap of about 10^-7.Therefore, let's look at the other distance between e^(π^2/2) and 1/α. As already presented in TL;DR, the gap in absolute values is already much smaller here:

$$e^{\frac{\pi^2}{2}} - 2\pi \cdot \left(q - e^{\frac{\pi^2}{2}}\right) = \frac{\alpha^{-1}}{1.00000134648} = 137.03581458 \tag{5}$$

This value can be refined in a further step with 4π^2, a value that certainly would not need to be called an arbitrary "fudge factor" in this context:

$$137.03581458 + \frac{1}{137.03581458 \cdot 4\pi^2} = 137.0359994249 \tag{17}$$

Unfortunately also here, this pattern cannot be continued in this way to further approach the desired value of 137.0359990958 (at least from my point of view). The best I have found yet would be:

$$137.03581458 + \frac{1}{137.03581458 \cdot 4\pi^2} - \frac{1}{137.03581458^2 \cdot 16\pi^2} = 137.0359990877 \tag{18}$$

But that would be an inconsistent continuation of the use of the 4π^2 factor, since 16π^4 and not 16π^2 would be the consistent continuation.

In summary, one can say that with q = ln(2)*64π one can get interesting approximations to the „De-Vries-value“ of 137.0359990958 with both the e^2 approach and the 2π approach. However, the bull's eye is not hit. But maybe I missed something...

Let's close the gap

So let's go back to equation (10):

$$\frac{\pi^2}{2 \cdot ln128} = 1.0170592375 \qquad (10)$$

After a lot of experimentation with the value q in the last section without hitting the bull's eye, perhaps it can still provide an approach here. And you can see that

$$q \cdot \alpha = 1.01699926 \qquad (19)$$

is pretty close to the equation (10) value. This means that if q is not multiplied by α, but by the reciprocal value from equation (10), you should also get a value that comes pretty close to α. I call this value z* and with q = ln2*64π and the reciprocal from equation (10) can thus be defined:

$$\begin{aligned} z* &= q \cdot \frac{2 \cdot ln128}{\pi^2} = \ln 2 \cdot 64\pi \cdot \frac{2 \cdot ln128}{\pi^2} \\ &= \frac{\ln 128 \cdot 128 \cdot \ln 2}{\pi} \\ &= 137.02791798 \end{aligned} \qquad (20)$$

So, and this value z* has the wonderful feature that the values found near 1/α in the previous section did not have: a pattern is found to close the small gap to the 1/α value:

$$137.02791798 \cdot \left(1 + \frac{1}{1 + \left(137.02791798 \cdot 4\pi^3 \cdot \left(1 - \frac{1}{137.02791798 \cdot \pi}\right)\right)}\right) \qquad (20)$$

$$= 137{,}03599916289$$

The correction factor 4π^3 used there is actually the nice looking 4π^2 factor, because there is an 1/π in our z* term, so with the definition

$$\begin{aligned} z &= z* \cdot \pi \\ &= \ln 128 \cdot 128 \cdot \ln 2 = 7 \cdot 2^7 \cdot \ln^2 2 \\ &= 430.48590047 \end{aligned} \tag{6}$$

we can formulate this gap filling very straightforwardly without magic fudge factors. As already presented in TL;DR, we get:

$$\boxed{\begin{aligned} \alpha^{-1} &= \frac{z}{\pi} \cdot \prod_{n=1}^{\infty} 1 + (-1)^{n+1} \cdot \left(\frac{1 + \frac{1}{z}}{4\pi^2 \cdot z} \right)^n \\ &= 137.03599911925744 \end{aligned}} \tag{7}$$

or simplified without infinite product:

$$\boxed{\begin{aligned} \alpha^{-1} &= \frac{z}{\pi} \cdot \left(1 + \frac{1 + \frac{1}{z}}{4\pi^2 \cdot z + 1 + \frac{1}{z}} \right) \\ &= 137.03599911928552 \end{aligned}} \tag{8}$$

We have thus found an equation for 1/α that, just like the de Vries formula, yields a value that is absolutely within the range of the results of the high-precision experiments for determining the fine structure constant. And this will probably not change in the next decades, because already at the beginning of this century a value slightly above 137.035999 has manifested itself in these experiments.

But here, too, I missed my actual goal - to formulate the exact value of the De Vries formula by other means - albeit extremely narrowly.

Summary and outlook

I have presented here an intermediate state of numerical trial and error to pursue the question whether and if so how the fine structure constant is related to the powers of two and here in particular to 128 and 2^128. If I have animated some readers to participate in answering this question, then this work has served its purpose.

Even if the original goal of the trial and error, namely to find a straight line alternative formulation of the De-Vries equation with the indentical result value, has not been achieved (so far), I have personally come to the conclusion that the matches found are not mere coincidence - as long as the opposite is proven. To be continued...

Appendix: Python code

Please note the comment above the deVries()-function.

```
#!/usr/bin/env python2


from math import pi, e, log

iterations = 25

def dv_core():
   return pow(e, (pow(pi, 2)/2))

# q = ln2*64pi
def q_value():
   return 64 * log(2) * pi

# z = 128*ln128*ln2
def z_value():
   return 128 * log(128) * log(2)


def q_value_minus_1_relation():
   return q_value() / (1 + (1/((q_value()-1)*pi) ))


def z_func():
    previous = 1
    for i in range(1, iterations):
       q = (1 + 1/z_value())/(z_value()*4*pi**2)
       previous = previous * (1 + (-1)**(i+1) * q**i)
    return z_value()/pi * previous


def z_func_simple():
    q = 1 + (1 + 1/z_value())/(z_value()*4*pi**2 + 1 +
1/z_value())
    return z_value()/pi * q
```

```
#This function is a simplification of the De-Vries formula
#implementation of Luke Kenneth Casson Leighton, see
#http://lkcl.net/reports/fine_structure_constant/alpha.py
#It is recommended to also look at this code and also to look at
#his explanations in [12]
#For the sake of form, it should therefore be mentioned that the
#Python code presented in this paper falls under the Gnu General
#Public License.
# http://www.gnu.org/licenses/gpl-3.0.html
def deVries():
    result = 1
    for x in range(1, iterations):
        gamma = 0.0
        t = 0.0
        for i in range(iterations):
            gamma = gamma + pow(result, i)/pow(2*pi, t)
            t = t + i
        result = pow(gamma, 2)/(pow(e, (pow(pi, 2)/2)))
    return result


alpha = deVries()
print("De-Vries 1/alpha:",  1/alpha)

print("Equation (3) - dv_core / q_value_minus_1_relation:",
dv_core() / q_value_minus_1_relation() )

print("Equation (7) (infinite product):", z_func())
print("Equation (8) (simplified):", z_func_simple())
```

```
c:\Space>python alpha128.py
De-Vries 1/alpha: 137.0359990958296
Equation (3) - dv_core / q_value_minus_1_relation: 1.000000010041539
Equation (7) (infinite product): 137.03599911925744
Equation (8) (simplified): 137.03599911928552
```

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