Why Quantum Mechanics Must Be a Probability Theory

Joseph Palazzo

It’s not a question of nature being inherently chaotic or random, an issue like a dead horse that has been beaten multiple times. It’s a question of reality versus fantasy. As curious beings, we have a need to know, we want to understand, and the data required to validate our theories compels us to observation. By re-examining what the Heisenberg Uncertainty Principle entails at quantum scale, we are faced with the brute fact that the very nature of the real world demands that any theory describing it faithfully is a probability theory. And Quantum Mechanics is such a theory. Einstein once famously said that God doesn’t play dice with the universe. We agree with this view, however for different reasons. In this paper a deity will play, surprisingly, a different role.

Part 1: Tossing a fair coin

Some would object that quantum mechanics is anything like tossing a fair coin. We beg to differ. That are many concepts that overlap. But there are also differences, which we shall not neglect.

Technically if one knew exactly the strength of the force applied to the fair coin, the exact position where that force was applied, and every other forces acting on the coin (air friction, atmospheric pressure, wind factor, etc.) one could in theory calculate on which side of the coin – heads or tails – it would land. However, no one would discuss in which state the coin resides while twirling in space. What counts is on which side it lands. Call the latter the observable. In a world in which the coin would twirl forever there would be no result. The coin must land. In a similar case, finding the spin of an electron requires that we must pass it through a magnetic field. It's the equivalent of the coin landing. Also the coin's itinerary into space could loosely constitute a set of states. But those have very little in common with quantum states, which form a complete set of orthonormal set in a Hilbert space.

Contrary to Classical physics, the fundamentally different approach of Quantum Mechanics (QM) requires that we separate the notion of a state from its observables So that observables like position, momentum and energy are now represented as operators in the theory, which often do not necessarily commute as mathematical objects. The quantum states are themselves found in the wave function which in essence is used to calculate probabilities.

Nevertheless we must examine how we go about to measure the observables that are of fundamental importance

Part 2: An electron moving in a straight line

How do we measure the velocity of a car? We see the car because an enormous number of photons are hitting the car in all directions, and some of them will reach our eyes. We can then note where it is at a given time, call that x_1 and t_1. At a later time t_2, we observe the car again at some other position x_2. Proceeding in that way, we get a whole set of these points, plot it on a graph, get the velocity, and determine if it is in uniform motion or if it is accelerating or decelerating, etc. Consider the case of an electron moving from left to right (FIG 1).

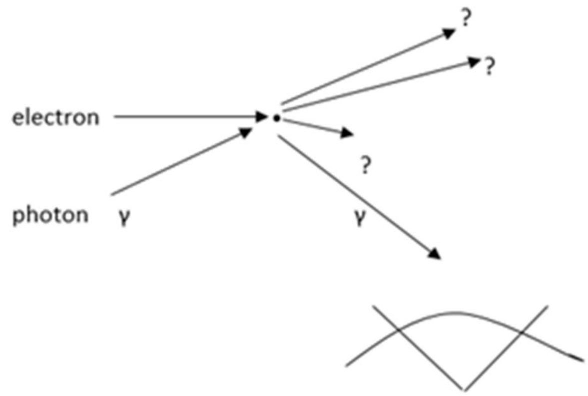

FIG. 1

So it goes for an electron if we want to find out anything about its velocity and the trajectory it might undergo - the idea is to shoot a whole bunch of photons, but in this case short-wave or high-energy photons due to the smallness of the electron. We get lucky if one of those photons hits the electron, and with very much luck, bounces in the right way to reach our eyes.

This is what the photon would be telling us if it could speak,

“Sir, that electron is right there,” call that position X, even though X is really a smeared area as our electron was jiggling around when it was hit, “but then guess what Sir, I’ve also thrown it off its position, and I haven’t a clue in what direction it’s going.”

Hitting the electron with a photon would be like hitting the car in the previous case with a missile. It would be unlikely that the car would have continued along its path. It would follow some undetermined path as in FIG 1. Hitting the electron with a second photon to get another position and time, x_2 and t_2, would be a nearly impossible task. In other words, the electron’s path after the collision with the photon becomes unpredictable. It can still be dealt with a probability theory, which is what QM is, by placing a series of detectors surrounding the location of impact.

Part 3: An electron passing through a magnetic field

Suppose we have a beam of electrons flowing from right to left, FIG 2.

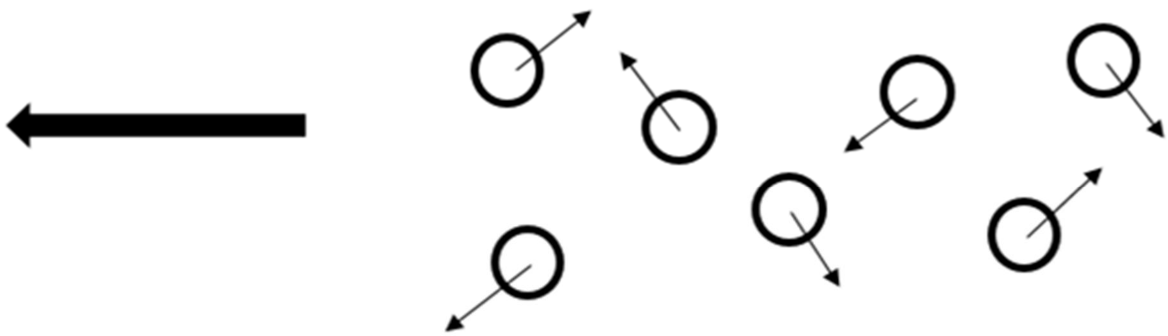

FIG 2

Notice this is a thought experiment as we really don't know in what direction the spin of each individual electron is pointing. We can safely say that these directions are at random. Yes, Einstein was correct in this particular case: "… there exists a physical reality independent of substantiation and perception". But we need to make observations if we want to understand the underlying principles that govern the universe. And in this case, we can't help ourselves but to interfere when we do that.

Now physicists are interested in measuring these spins.

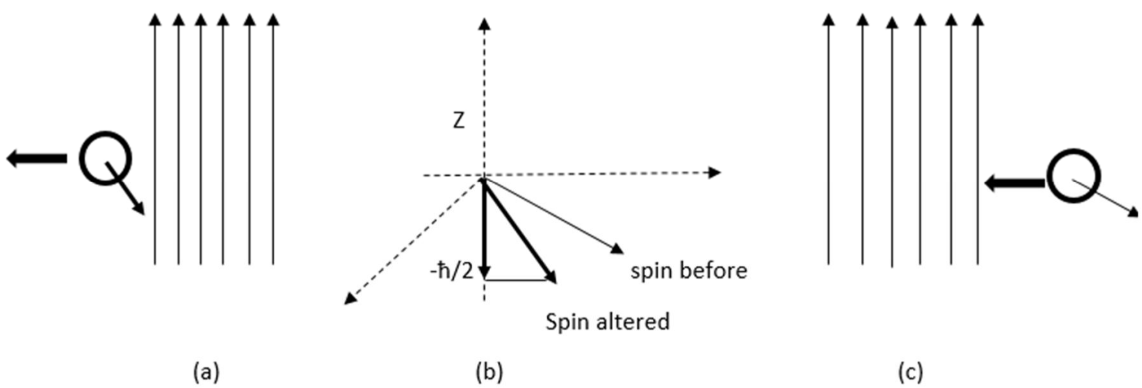

FIG 3

So what is needed is some kind of apparatus, and the good news is that there exists one – a magnetic field. Trouble is that these electrons, with their spin, are also tiny magnets, and we know that magnets placed in a magnetic field will align (or anti-align) with the magnetic field. Suppose a magnetic field is placed along a certain direction, say the z-axis. Now let's look at one specific electron as it approaches the magnetic field (FIG 3c).

When that electron penetrates the magnetic field, it will align its spin such that its z-component will yield the value of $-\hbar/2$, in this case along the z-axis, a spin down, which can be represented as in FIG 3b. Note that after passing the magnetic field, the electron's total spin has been altered. It leaves the magnetic fields with a different total spin, with nevertheless a spin of -$\hbar/2$ in the z-direction (FIG 3a).

On the whole, 50% of all the electrons will align with the magnetic field (spin =+$\hbar/2$, or up), and 50% will anti-align (spin = -$\hbar/2$, or down).

Comments

(i) Before the measurement, the spin of an electron can be in any direction. Passing the electron through the magnetic field forces the electron to change its spin orientation such that it either aligns or anti-aligns with its z-component to be ± $\hbar/2$. This is what distinguishes quantum physics from classical physics: the act of measuring a quantity will disturb the system. In the case of the car previously mentioned, that doesn't happen.

(ii) The other components of the spin are indeterminate: if we were to pass these electrons into another magnetic field, say aligned with the x-axis, again it will be found that 50% of the electrons will align with the magnetic field (spin = +$\hbar/2$), and 50% will anti-align (spin = -$\hbar/2$), this time along the x-axis. On the other hand the spin along the z-axis is no longer known for these particles.

(iii) One way to mathematically represent this quantum system (read, the wave function) is the following equation:

$$|\psi\rangle = (1/2)^{1/2}\,(|\uparrow\rangle - |\downarrow\rangle) \qquad 1$$

This is called a superposition of two quantum states, the up and down states. Note that if we want to calculate the probability that the electron has a spin up, we take the product of the vector $|\uparrow\rangle$ with the wave function | ψ>, and square that.

$$\begin{aligned} P &= \left|\langle\uparrow|\psi\rangle\right|^2 \qquad 2 \\ &= 1/2\,[\langle\uparrow|\,(|\uparrow\rangle - |\downarrow\rangle)]^2 \\ &= 1/2\,[\langle\uparrow|\uparrow\rangle - \langle\uparrow|\downarrow\rangle]^2 \end{aligned}$$

Using the orthogonality condition, which is a fundamental property of a Hilbert space,

$$\langle\uparrow|\uparrow\rangle = 1 \; and \; \langle\uparrow|\downarrow\rangle = 0$$

We get,

$$P = 1/2, or\ 50\%, \qquad 3$$

Which is what is observed in the lab.

(4) After the electron has passed through the magnetic field, if passing again through the same or similar magnetic field, the result will be the same. This is the notion of preparing a particle in a given quantum state.

(5) Now here comes the real crunch. Writing | ψ > = $(1/2)^{½}$ (|↑ > – | ↓ >) in equation 1 is called a superposition but it's not meant to mean that the electron "lives" simultaneously in two states and can't make up its "mind" in which one it wants to live. It does not mean that the electrons live in two different states, but rather that it indicates the possible outcomes should a measurement be taken. The superposition of these states is not about how weird QM is but about the mathematical structure of a Hilbert space. And it's not that an act of observation is needed in order for a particle to acquire a well-defined position or momentum. It already has. But trying to get any info (passing through a magnetic field is that attempt) changes its prior state.

To be kept in mind is that those states, in their mathematical sense, do not represent ordinary vectors of real objects, like velocities, acceleration, forces. If that were the case, then since these two vectors are equal in magnitude and opposite in direction we would be able to claim that, | ↑ > = (-1) | ↓ >. And the orthogonal condition would no longer hold, and P would not equal to 50%. What needs to be reminded is that the two vectors, | ↑ > and | ↓ > mathematically represent possible states in some abstract space called the Hilbert space. And the beauty of it all is that they form a complete set of orthogonal unit vectors, which provides a powerful method of calculating probabilities.

Part 4: A simple electron

Here's a thought experiment. Suppose you were God and you could grab an electron and deposit it at a certain fixed position. As God, you've just violated the Heisenberg Uncertainty Principle, which states that we cannot know both the position and speed of a particle, such as a photon or electron, with perfect accuracy. But that's okay, an all-powerful God can do that. We could depict this as in FIG 4.

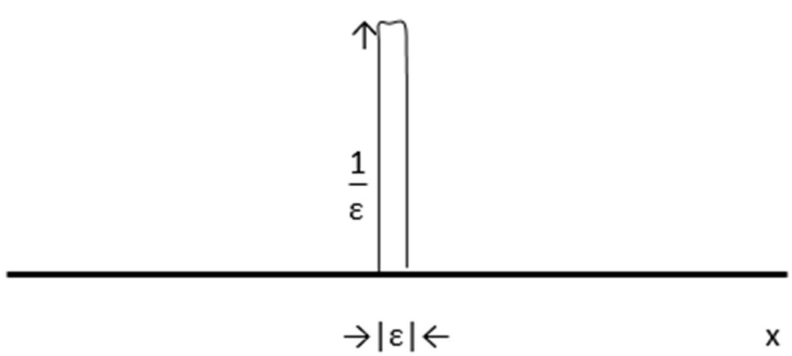

FIG 4

But as soon as you as God were to release the electron, it would look like FIG 5.

FIG 5

That is, after a certain time, the position of the electron has spread out. The question is: what does that tell us? It looks like the particle is doing some kind of motion, some jiggling or wobbling. It also means that for microscopic particles, they are never at rest. In classical physics, you can choose an inertial frame in which an object is at rest. The walls in your room are at rest with you. But in QM, no object is at rest. And that's a fundamental difference with classical physics. In other words, there is no location in all the corners of the universe where an electron can be placed at rest. It also means that these objects can be described with wave-like features.

This brings up an important point: if an observer could interact with a particle at quantum scale *without* altering its state, or know simultaneously both position and velocity, then he or she would need to have magical powers. In a fairy tale, that is always possible. In the real world, not so much. Now if the real world behaved like a fairy tale, it would be weird! The conclusion is that observing objects at quantum scale alters the state of those particles, which is unavoidable. And so to deal with the very basic nature of the real world in our quest to observe it, we can only do that with a probability theory, which QM is, and has proven to be right in its multiple predictions.

PS: There is a story circulating on the Internet, which is most likely untrue, but brings up the point of this paper, and it goes as follows: Heisenberg gets pulled over for speeding. The cop asks Heisenberg, "Do you know how fast you were going?" Heisenberg replies, "No, but I know exactly where I am!" The officer looks at him confused and says "you were going at 110 kilometers per hour!" Heisenberg throws his arms up and cries out, "Great! Now I'm lost!"

Now we know as a classical object, Heisenberg would be wrong. On the other hand, as an electron, he would be definitely right.