Solving cosmological constant problem

Felix M Lev

Artwork Conversion Software Inc.
509 N. Sepulveda Blvd Manhattan Beach CA 90266 USA

Email: felixlev314@gmail.com

Abstract

Physicists usually believe that physics cannot (and should not) derive the values of c and $\hbar$ but should derive the value of the cosmological constant Λ. This problem is considered fundamental after the phenomenon of cosmological acceleration (PCA) was discovered in 1998. This phenomenon is usually considered in the framework of General Relativity (GR) and here the main uncertainty is how the background space is treated. If it is flat, PCA is usually treated as a manifestation of dark energy and (as acknowledged in the literature) currently its nature is a mystery. On the other hand, if it is curved then a problem arises why the value of Λ is as is. However, in our approach based only on universally recognized results of physics, the solution of the problem does not contain uncertainties because PCA is an inevitable kinematical consequence of quantum theory in semiclassical approximation. Since the de Sitter (dS) algebra is semisimple, it is the most general ten-dimensional Lie algebra because it cannot be obtained by contraction from other ten-dimensional Lie algebras. Let R be the parameter of contraction from the dS algebra to the Poincare one. Then the problem why the quantities $(c, \hbar, R)$ are as are does not arise because they are contraction parameters for transitions from more general Lie algebras to less general ones. In our approach, background space and its geometry (metric and connection) are not used but, in semiclassical approximation, the result for PCA is the same as in GR if $\Lambda = 3/R^2$.

Keywords: quantum de Sitter symmetry; cosmological acceleration; irreducible representations

1 Introduction

In the phenomenon of cosmological acceleration (PCA), only nonrelativistic macroscopic bodies are involved, and one might think that here there is no need to involve quantum theory. However, ideally, the results for every classical (i.e., non-quantum) problem should be obtained from quantum theory in semiclassical approximation. We will see that, considering PCA from the point of view of quantum theory sheds a new light on understanding this problem.

In PCA, it is assumed that the bodies are located at large (cosmological) distances from each other and sizes of the bodies are much less than distances between them. Therefore, interactions between the bodies can be neglected and, from the formal point of view, the description of our system is the same as the description of N free spinless elementary particles.

However, in the literature, in view of mainly historical reasons, PCA is usually considered in the framework of dark energy and other exotic concepts. In Sec. 2 we argue that such considerations are not based on rigorous physical principles. In Sec. 3 we explain how symmetry should be defined at the quantum level. In Sec. 4 we describe PCA in the framework of our approach. Finally, Sec. 5 is conclusion.

2 History of dark energy

This history is well-known. First Einstein introduced the cosmological constant Λ because he believed that the universe was stationary and his equations can ensure this only if $\Lambda \neq 0$. But when Friedman found his solutions of equations of General Relativity (GR) with $\Lambda = 0$ and Hubble found that the universe was expanding, Einstein said (according to Gamow's memories) that introducing $\Lambda \neq 0$ was the biggest blunder of his life. After that, the statement that Λ must be zero was advocated even in textbooks.

The explanation was that, according to the philosophy of GR, matter creates a curvature of space-time, so when matter is absent, there should be no curvature, i.e., space-time should be the flat Minkowski space. That is why when in 1998 it was realized that the data on supernovae could be described only with $\Lambda \neq 0$, the impression was that it was a shock of something fundamental. However, the terms with Λ

in the Einstein equations have been moved from the left-hand side to the right-hand one, it was declared that in fact $\Lambda = 0$, but the impression that $\Lambda \neq 0$ was the manifestation of a hypothetical field which, depending on the model, was called dark energy or quintessence. In spite of the fact that, as noted in wide publications (see e.g., [1] and references therein), their physical nature remains a mystery, the most publications on PCA involve those concepts.

Several authors criticized this approach from the following considerations. GR without the contribution of Λ has been confirmed with a high accuracy in experiments in the Solar System. If Λ is as small as it has been observed, it can have a significant effect only at cosmological distances while for experiments in the Solar System the role of such a small value is negligible. The authors of [2] titled "Why All These Prejudices Against a Constant?" note that it is not clear why we should think that only a special case $\Lambda = 0$ is allowed. If we accept the theory containing the gravitational constant G which is taken from outside, then why can't we accept a theory containing two independent constants?

Let us note that currently there is no physical theory which works under all conditions. For example, it is not correct to extrapolate nonrelativistic theory to cases when speeds are comparable to c, and it is not correct to extrapolate classical physics for describing energy levels of the hydrogen atom. GR is a successful classical (i.e., non-quantum) theory for describing macroscopic phenomena where large masses are present, but extrapolation of GR to the case when matter disappears is not physical. One of the principles of physics is that a definition of a physical quantity is a description of how this quantity should be measured. The concepts of space and its curvature are pure mathematical. Their aim is to describe the motion of real bodies. But the concepts of empty space and its curvature should not be used in physics because nothing can be measured in a space which exists only in our imagination. Indeed, in the limit of GR when matter disappears, space remains and has a curvature (zero curvature when $\Lambda = 0$, positive curvature when $\Lambda > 0$ and negative curvature when $\Lambda < 0$) while, since space is only a mathematical concept for describing matter, a reasonable approach should be such that in this limit space should disappear too.

A common principle of physics is that when a new phenomenon is discovered, physicists should try to first explain it proceeding from the existing science. Only if all such efforts fail, something exotic can be involved. But for PCA, an opposite approach was adopted: exotic explanations with dark energy or quintessence were accepted without serious efforts to explain the data in the framework of existing science.

Although the physical nature of dark energy remains a mystery, there exists a wide literature where the authors propose quantum field theory (QFT) models of dark energy. While in most publications, only proposals about future discovery of dark energy are considered, the authors of [1] argue that dark energy has already been discovered by the XENON1T collaboration. In June 2020, this collaboration reported an excess of electron recoils: 285 events, 53 more than expected 232 with a statistical significance of 3.5σ. However, in July 2022, a new analysis by the XENONnT collaboration discarded the excess [3].

As shown in our publications and in the present paper, PCA can be explained without uncertainties proceeding from universally recognized results of physics and without involving models and/or assumptions the validity of which has not been unambiguously proved yet.

3 Symmetry at quantum level

In the literature, symmetry in QFT is usually explained as follows. Since Poincare group is the group of motions of Minkowski space, the system under consideration should be described by unitary representations of this group. This approach is in the spirit of the Erlangen Program proposed by Felix Klein in 1872 when quantum theory did not yet exist.

However, background space is only a mathematical concept: in quantum theory, each physical quantity should be described by an operator but there are no operators for the coordinates of background space. There is no law that every physical theory must contain a background space. For example, it is not used in nonrelativistic quantum mechanics and in irreducible representations (IRs) describing elementary particles. In particle theory, transformations from the Poincare group are not used because, according to the Heisenberg S-matrix program, it is possible to describe only transitions of states from the infinite past when $t \to -\infty$ to the distant future when $t \to +\infty$. In this theory, systems are described by observable physical quantities —

momenta and angular momenta. So, symmetry at the quantum level is defined not by a background space and its group of motions but by a representation of a Lie algebra A by self-adjoint operators (see [4, 5] for more details).

Then each elementary particle is described by an IR of A and a system of N noninteracting particles is described by the tensor product of the corresponding IRs. This implies that, for the system as a whole, each momentum operator is a sum of the corresponding single-particle momenta, each angular momentum operator is a sum of the corresponding single-particle angular momenta, and *this is the most complete possible description of this system*. In particular, nonrelativistic symmetry implies that A is the Galilei algebra, relativistic symmetry implies that A is the Poincare algebra, de Sitter (dS) symmetry implies that A is the dS algebra so(1,4) and anti-de Sitter (AdS) symmetry implies that A is the AdS algebra so(2,3).

In his famous paper "Missed Opportunities" [6] Dyson notes that:

- a) Relativistic quantum theories are more general (fundamental) than nonrelativistic quantum theories even from pure mathematical considerations because Poincare group is more symmetric than Galilei one: the latter can be obtained from the former by contraction $c \to \infty$.

- b) dS and AdS quantum theories are more general (fundamental) than relativistic quantum theories even from pure mathematical considerations because dS and AdS groups are more symmetric than Poincare one: the latter can be obtained from the former by contraction $R \to \infty$ where R is a parameter with the dimension *length*, and the meaning of this parameter will be explained below.

- c) At the same time, since dS and AdS groups are semisimple, they have a maximum possible symmetry and cannot be obtained from more symmetric groups by contraction.

As noted above, symmetry at the quantum level should be defined by a Lie algebra, and in [5], the statements a)-c) have been reformulated in terms of the corresponding Lie algebras. It has also been shown that the fact that quantum theory is more general (fundamental) than classical theory follows even from pure mathematical considerations because formally the classical symmetry algebra can be obtained from the symmetry algebra in quantum theory by contraction $\hbar \to 0$. *For these reasons, the most general consideration of PCA should be carried out in terms of quantum dS or AdS symmetry.*

The definition of those symmetries is as follows. If M^{ab} $(a, b = 0, 1, 2, 3, 4,\ M^{ab} = -M^{ba})$ are the angular momentum operators for the system under consideration, they should satisfy the commutation relations:

$$[M^{ab}, M^{cd}] = -i(\eta^{ac}M^{bd} + \eta^{bd}M^{ac} - \eta^{ad}M^{bc} - \eta^{bc}M^{ad}) \qquad (1)$$

where $\eta^{ab} = 0$ if $a \neq b$, $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = 1$ and $\eta^{44} = \mp 1$ for the dS and AdS symmetries, respectively.

Although the dS and AdS groups are the groups of motions of dS and AdS spaces, respectively, the description in terms of relations (1) does not involve those groups and spaces at all, and *those relations can be treated as a definition of dS and AdS symmetries at the quantum level* (see the discussion in [4, 5]). In QFT, interacting particles are described by field functions defined on Minkowski, dS and AdS spaces. However, since we consider only noninteracting bodies and describe them in terms of IRs, at this level we don't need these fields and spaces.

The procedure of contraction from dS or AdS symmetry to Poincare one is defined as follows. If we *define* the momentum operators P^μ as $P^\mu = M^{4\mu}/R$ $(\mu = 0, 1, 2, 3)$ then in the formal limit when $R \to \infty$, $M^{4\mu} \to \infty$ but the quantities P^μ are finite, Eqs. (1) become the commutation relations for the Poincare algebra (see e.g., [4, 5]). Here R *is a parameter which has nothing to do with the dS and AdS spaces.* As seen from Eqs. (1), quantum dS and AdS theories do not involve the dimensionful parameters $(c, \hbar, R)$ at all because (kg, m, s) are meaningful only at the macroscopic level.

In particle theories, the quantities c and $\hbar$ typically are not involved and it is said that the units $c = \hbar = 1$ are used. Physicists usually understand that physics cannot (and should not) derive that $c \approx 3 \cdot 10^8 m/s$ and $\hbar \approx 1.054 \cdot 10^{-34} kg \cdot m^2/s$ and those values are as are simply because, mainly due to historical reasons, people want to describe velocities in m/s and angular momenta in $kg \cdot m^2/s$. At the same time, physicists usually believe that physics should derive the value of Λ and that the solution to the dark energy problem depends on this value.

At the classical level, Λ is the curvature of the background space and equals $\pm 3/R^2$ for the dS and AdS spaces, respectively, where R is the radius of those spaces. As noted below, in semiclassical approximation, R is the same as the parameter R in quantum theory where this parameter is only the coefficient of proportionality between $M^{4\mu}$ and P^μ. As follows from the above discussion, at the level of contraction parameters, the quantity R is fundamental to the same extents as c and $\hbar$. Here the question why R is as is does not arise simply because the answer is: because people want to describe distances in meters. There is no guaranty that the values of $(c, \hbar, R)$ in (kg, m, s) will be the same during the whole history of the universe.

4 Explanation of cosmological acceleration

Standard particle theories involve IRs of the Poincare algebra by self-adjoint operators. They are described even in textbooks and do not involve Minkowski space. Therefore, when Poincare symmetry is replaced by more general dS or AdS one, dS and AdS particle theories should be based on IRs of the dS or AdS algebras by self-adjoint operators, respectively. However, physicists usually are not familiar with such IRs because they believe that dS and AdS quantum theories necessarily involve quantum fields on dS or AdS spaces, respectively.

The mathematical literature on unitary IRs of the dS group is wide but there are only a few papers where such IRs are described for physicists. For example, the excellent Mensky's book [7] exists only in Russian. At the same time, to the best of our knowledge, IRs of the dS algebras by self-adjoint operators have been described from different considerations only in [5, 8, 9, 10].

In the framework of our approach, the explanation of cosmological acceleration consists of the following steps. First, instead of the angular momentum operators $M^{4\mu}$ we work with the momentum operators $P^\mu = M^{4\mu}/R$, and, in the approximation when R is very large, different components of P^μ commute with each other. Then we use the explicit expressions for the operators M^{ab} of IRs of the dS algebra — see e.g., Eqs. (3.16) in [5], Eqs. (17) in [9] or Eqs. (3) in [10]. Those operators act in momentum representation and *at this stage, we have no spatial coordinates yet*. For describing the motion of particles in terms of spatial coordinates, we must define the position operator. A question: is there a law defining this operator?

The postulate that the coordinate and momentum representations are related by the Fourier transform was taken at the dawn of quantum theory by analogy with classical electrodynamics, where the coordinate and wave vector representations are related by this transform. But the postulate has not been derived from anywhere, and there is no experimental confirmation of the postulate beyond the nonrelativistic semiclassical approximation. Heisenberg, Dirac, and others argued in favor of this postulate but, for example, in the problem of describing photons from distant stars, the connection between the coordinate and momentum representations should be not through the Fourier transform, but as shown in [5]. However, since, PAC involves only nonrelativistic bodies then, as follows from the above remarks, the position operator in momentum representation can be defined as usual, i.e., as $\mathbf{r} = i\hbar\partial/\partial\mathbf{p}$ where $\mathbf{p}$ is the momentum. Then in semiclassical approximation, we can treat $\mathbf{p}$ and $\mathbf{r}$ as usual vectors.

The next step is to take into account that the representation describing a free N-body system is the tensor product of the corresponding single-particle IRs. It means that every N-body operator M^{ab} is a sum of the corresponding single-particle operators. Then one can calculate the internal mass operator for any two-body subsystem of the N-body system, and the result is given by Eq. (3.68) in [5], Eq. (61) in [9] or Eq. (17) in [10]. Now, as follows from the Hamilton equations, in any two-body subsystem of the N-body system, the relative acceleration in semiclassical approximation is given by

$$\mathbf{a} = \mathbf{r}c^2/R^2 = \frac{1}{3}c^2\Lambda\mathbf{r} \tag{2}$$

where $\mathbf{a}$ and $\mathbf{r}$ are the relative acceleration and relative radius vector of the bodies, respectively, and $\Lambda = 3/R^2$.

Let us note the following. Since c is the contraction parameter for the transition from Poincare invariant theory to Galilei invariant one, the results of the latter can be obtained from the former in the formal limit $c \to \infty$, and Galilei invariant theories do not contain c. Then one might ask why Eq. (2) contains c although we assume that the bodies in PCA are nonrelativistic. The matter is that Poincare invariant theories do not contain R but we work in dS invariant theory and assume that, although c and R are very large, they are not infinitely large, and the quantity c^2/R^2 in Eq. (2) is finite.

An analogous calculation using the results of Chap. 8 of [5] on IRs of the AdS algebra gives that, in the AdS case, $\mathbf{a} = -\mathbf{r}c^2/R^2$, i.e., we have attraction instead of repulsion. The experimental facts that the bodies repel each other show that in PCA, dS symmetry is more relevant than AdS one. The fact that the relative acceleration of noninteracting bodies is not zero does not contradict the law of inertia, because this law is valid only in the case of Galilei and Poincare symmetries, and in the formal limit $R \to \infty$, $\mathbf{a}$ becomes zero as it should be.

The relative accelerations given by Eq. (2) are the same as those derived from GR if the curvature of dS space equals $\Lambda = 3/R^2$, where R is the radius of this space. *However, the crucial difference between our results and the results of GR is as follows. While in GR, R is the radius of the dS space and can be arbitrary, in quantum theory, R is the coefficient of proportionality between $M^{4\mu}$ and P^μ, this coefficient is fundamental to the same extent as c and $\hbar$, and a question why R is as is does not arise.* **Therefore, our approach gives a clear explanation why Λ is as is.**

In GR, the result (2) does not depend on how Λ is interpreted, as the curvature of empty space or as the manifestation of dark energy. However, in quantum theory, there is no freedom of interpretation. Here R is the parameter of contraction from the dS Lie algebra to the Poincare one, it has nothing to do with the radius of the background space and with dark energy and it must be finite because dS symmetry is more general than Poincare one.

5 Conclusion

We have shown that the phenomenon of cosmological acceleration is simply a consequence of quantum theory in semiclassical approximation, and this conclusion has been made without involving models and/or assumptions the validity of which has not been unambiguously proved yet. From our consideration, it is clear that the cosmological constant Λ has a physical meaning only in semiclassical approximation.

In the literature, the cosmological constant problem is usually described in the framework of Poincare invariant QFT of gravity on Minkowski space. This theory contains only one phenomenological parameter — the gravitational constant G, and Λ is defined by the vacuum expectation value of the energy-momentum tensor. The theory contains strong divergencies which cannot be eliminated because the theory is not renormalizable. The results can be made finite only with a choice of the cutoff parameter. Since G is the only parameter in the theory, the usual choice of the cutoff parameter in momentum space is $\hbar/l_P$ where l_P is the Plank length. Then, if $\hbar = c = 1$, G has the dimension $length^2$ and Λ is of the order of $1/G$. However, this value is more than 120 orders of magnitude greater than the experimental one.

As explained above, in quantum theory, Poincare symmetry is a special degenerate case of dS symmetry in the formal limit $R \to \infty$ where R is a parameter of contraction from the dS algebra to the Poincare one. This parameter is fundamental to the same extent as c and $\hbar$, it has nothing to do with the relation between Minkowski and dS spaces and the problem why R is as is does not arise by analogy with the problem why c and $\hbar$ are as are. As noted in Sec. 4, the result for cosmological acceleration in our approach and in GR is given by the same expression (2) but the crucial difference between our approach and GR is as follows. While in GR, R is the radius of the dS space and can be arbitrary, in our approach, R is defined uniquely because it is a parameter of contraction from the dS algebra to the Poincare one. Therefore, in our approach, the problem why the cosmological constant is as is does not arise.

Therefore, the phenomenon of cosmological acceleration has nothing to do with dark energy or other artificial reasons. This phenomenon is an inevitable kinematical consequence of quantum theory in semiclassical approximation and the problem of cosmological constant does not arise.

Since 1998, the fact that $\Lambda > 0$ has been confirmed in several experiments, and it is now accepted [11] that $\Lambda = 1.3 \cdot 10^{-52}/m^2$ with the accuracy 5%. Since Λ is very small and the evolution of the universe is the complex process, cosmological repulsion does not appear to be the main effect determining this process, and other effects (e.g., gravity, microwave background and cosmological nucleosynthesis) may play a much larger role.

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