Title: The Correct Way to Calculate the Rapidity in Special Relativity

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Abstract:

Taylor and Wheeler, in their SPACETIME PHYSICS book (1966 edition), give the “rapidity” (or “velocity parameter”) as theta = A * tau, where “A” is the acceleration in ly/y/y, and “tau” is the age in years of the person on the trailing rocket (assuming that the rocket fires when that person has just been born, and is at the origin of the chart at birth). The chart’s vertical axis gives the distance “X” from the starting point, and the horizontal axis gives the age “tau” of the person on the trailing rocket. The velocity, according to that person, is then v = tanh(theta) ly/y, and the distance traveled from X = 0 by that person is just the integral of “v” with respect to “tau”. If desired, OTHER rockets can start from positions farther up the “X” axis, at positions D = D1, D = 2*D1, etc. The curve of the second-lowest curve, starting at X = D1 and tau = 0, is exactly the same shape as the lowest curve, just shifted upward by the amount D1. It is possible to have any number of such rockets, each with a curve shifted upward by some distance “D” above the immediately lower curve, and having exactly the same shape as the lowest curve. The result is the view according to the people in the lowest rocket.

NOW, one can use the length contraction equation (LCE) to get the corresponding chart according to the initial inertial observer (IIO), with time variable “t” and position “x”: one just divides each point of each curve by the factor gamma, where

gamma = 1 / { 1 - sqrt [(v * v)] }.

But when one does that, the results are ABSURD. If there is just a single rocket, starting from

x = 0 at t = 0,

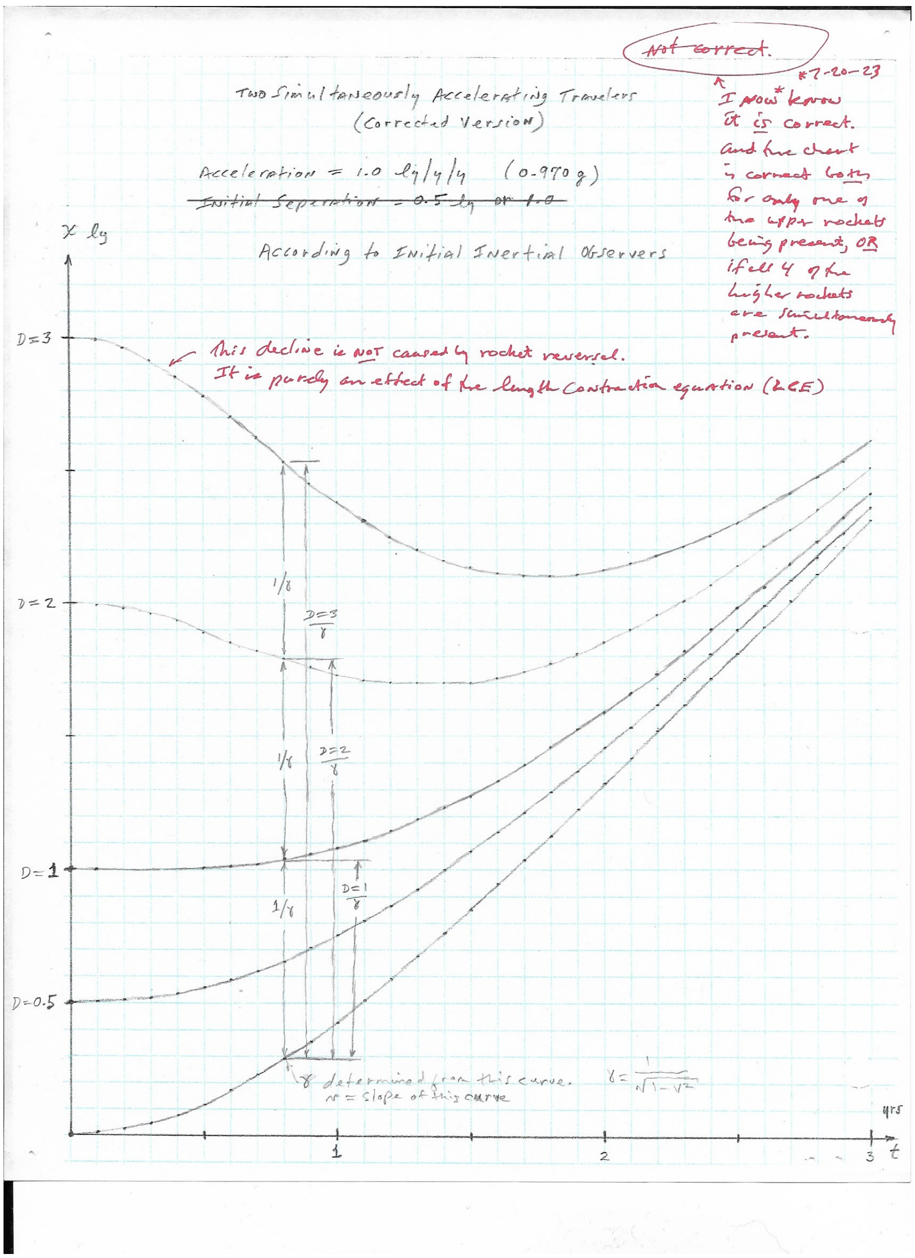
then according to the IIO, that rocket will first move a large distance away from x = 0, but will eventually start to move back toward the IIO (even though its rocket is always pointing away from x = 0, and is still producing the same thrust). Ultimately, that rocket gets arbitrarily close to the IIO, as “t” goes to infinity. Clearly, that is absurd, and it can’t be true. So we must reject Taylor and Wheeler’s scenario and their equation for the rapidity.

IS there an alternative scenario and rapidity equation which DOESN’T have an absurd outcome? YES! We use the viewpoint of the IIO who is present (and stationary) at the launch. According to that IIO, at time “t” in the IIO’s life, the trailing rocket is moving at the speed

v = tanh(theta),

where theta is NOW given by theta = A * t, not theta = A * tau. And a rocket starting at distance D1 above the origin is vertically separated from the lowest curve by the amount D1 / gamma. Likewise, another rocket starting at a distance D2 will be vertically separated from the lowest curve by amount D2 / gamma, etc. For this alternative scenario, there are no absurdities like there were in the first scenario.

The above abstract says almost everything that needs saying. What I need to do now is provide a chart that matches the portion of the wording above for the viewpoint of the initial inertial observers (IIO's). Here is that chart, together with some additional important observations on the next page.



In contrast to the above chart for the viewpoint of the initial inertial observer, the chart for the viewpoint of the people on the trailing rocket, plotted versus their age “tau”, would have the same lower curve as the above chart, and each of the upper curves would have EXACTLY the same shape as the lowest curve, just shifted up by 0.5 ly, 1.0 ly, 2.0 ly, or 3.0 ly. But that viewpoint, when converted (using the length contraction equation) to the viewpoint of the initial inertial observers (the IIO’s), gives the absurd result described earlier, and so that scenario must be rejected.

Some additional comments are important about the above chart. Looking at the top curve (that starts from D = 3 ly), note that the curve decreases initially (that rocket moves toward x = 0 for a while). That is NOT due to the rocket being turned around … the rocket always is pointing away from the starting point, and still thrusting. And more startling is the fact that the negative slope of that line at some points is greater in magnitude than 1.0, which corresponds to a speed greater than the speed of light. But that is NOT a problem! Such greater than light-speed motion occurs in special relativity in LOTS of situations. For example, in the twin paradox scenario, if at the instantaneous turnaround, the traveling twin (he) suddenly changes his speed relative to the home twin (her) to ZERO, he will say that the distance between them instantaneously gets much larger. So he is effectively saying that she instantaneously moves away from him, at an infinite speed! Another example of greater than light-speed motion is when a yardstick suddenly increases its speed with respect to an inertial observer (her): in that case, the length of the yardstick suddenly gets shorter, so according to her, the ends of that yardstick move essentially infinitely fast toward each other!