

nuclear fission in the light of the cubic ellipsoid geometric model

Ronen Yavor

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Abstract

This paper examines the nuclear fission in the light of the cubic ellipsoid geometric model of the nucleus.

The main outputs of the research are:

- the explanation of the mechanism of the nuclear fission.
- the prediction of the most probable fission products (or fragments).

Both are based on the nuclear model and the nuclear instability as presented and discussed in the former papers in this series of the cubic ellipsoid model of the nucleus. [\[15\]](#)

These results provide additional reinforcement to the model.

Content

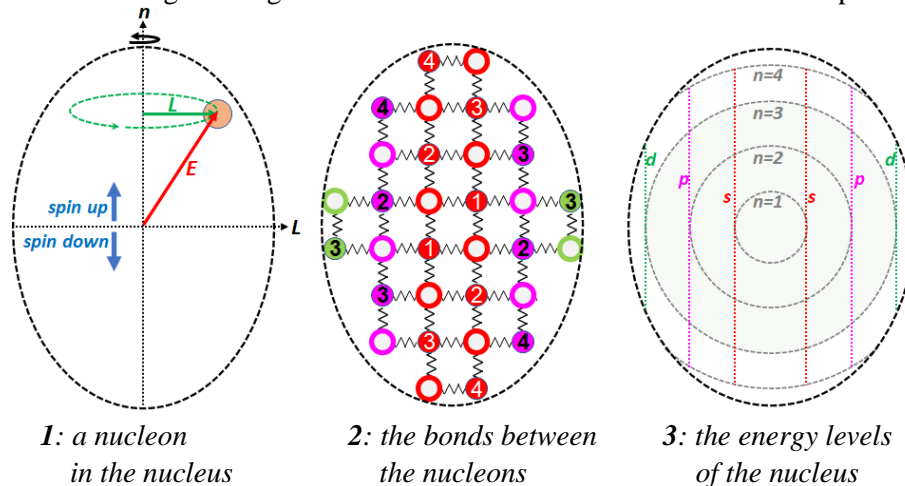
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The model at a glance

A brief description of the model [15]:

- The nucleus has an ellipsoid shape.
- The nucleons are connected in a cubic form.
- Protons are connected to neutrons (**p-n**).
- Neutrons are connected mainly to protons.
- The protons are populated and organized in shells in the nucleus in a full analogy to those of the electrons in the atom.
- The energy layers (principal quantum number **n**) grow along the **z**-axis of the nucleus in its both directions (more precisely **n** grows with its distance from the origin).
- The perpendicular distance from the **z**-axis in the **x-y**-plane reflects the angular momentum (**L**) and so the orbitals.
- The upper half of the ellipsoid is referred to as spin-up and the lower part as spin-down.
- The nucleus possibly rotates around its **z**-axis.

The following drawings describe the idea via cross sections in the **x-z**-plane of the nucleus.



1. One nucleon (orange circle) is observed inside the ellipsoid (dashed line) that encloses the nucleons and schematically defines the nucleus surface:
 - the distance from the origin represents its energy **E**.
 - the distance from the **z**-axis depicts its angular momentum **L**.
 - the nucleons in the upper half have spin up, and in the lower one spin down.
2. The bonds between the nucleons are shown for visibility as springs.
 - **Protons**: full circles of the **s**, **p** and **d** sub-orbitals. **Neutrons**: hollow circles.
3. The circles of equal energy states **n** in the ellipsoid.
 - the lines mark the development of the **s**, **p** and **d** sub-orbitals along the **z**-axis.
 - The **s** line crosses all **n** circles from 1 to 4 (**s1** to **s4**).
 - The **p** line begins by **n=2** and reaches till **n=4** (**p2** to **p4**).
 - The **d** line begins by **n=3** and reaches the ellipsoid border, before it reaches the **n=4** circle, and therefore there are no **d4** states at this stage (only **d3**).

Introduction: the fission hypotheses

We raise the following hypotheses regarding the fission mechanism and rules and then research and expand it through the next sections of the paper.

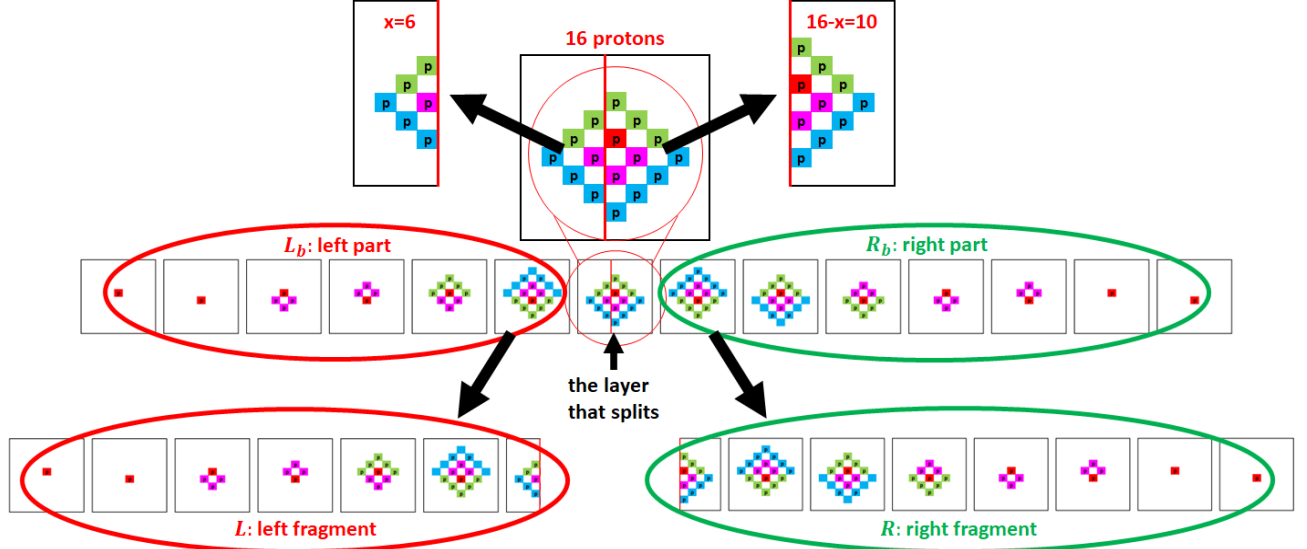
- A necessary but not sufficient condition for fission is that the nucleus is larger than Lead (Pb) and so has an unstable core. (see our former paper in our series on the source of instability by heavy nuclei [\[16\]](#)).
- The split of the nucleus occurs in one of the two nucleus center (most inner) layers. [\[16\]](#)
- The number of protons of the products (fragments) is the sum of the protons in the layers from both sides till the split point according to the nucleus description (or illustration) in cross sections along its z-axis.
- The number of neutrons must be a bit lower than the relatively more stable isotope of the nucleus. for example for Uranium the more stable isotope is U_{92}^{238} , so the unstable isotope is U_{92}^{236} ($U_{92}^{235} + n$). The assumption here is that this lack of several neutrons enables some movement of the nucleons in the nucleus and so after a radioactivity occurs in the center of the nucleus, a rearrangement of the inner parts enables the creation of the fragments.

In the following sections we describe the fission mechanism according to the model, explain how to calculate the size of the fragments and then demonstrate it for few examples.

The nuclear fission

The fission mechanism

The nuclear split occurs according to the model at one of the central layers (see illustration). For nuclei with even number of protons it doesn't matter if we select the right or the left centers as the one that splits, but for nuclei with odd number of protons, the two possibilities shall be considered separately.



The fission and the definition of the fragments

We define (see illustration):

- P : number of protons of the nucleus that undergoes fission.
- R_b : the number of protons of the right part of the nucleus till its center.
- L_b : as R_b , for the left part without its most inner layer (of 16 protons).
- x : the number of protons (out of 16) from the layer that splits.
- R : the number of protons of the right fragment.
- L : the number of protons of the left fragment.

with the values:

- $P := \begin{cases} 2m + 1 & P \text{ odd} \\ 2m & P \text{ even} \end{cases}$ with m integer
- $R_b := \frac{P}{2} = m$ (integer division)
- $L_b := \frac{P}{2} - 16 + \text{remainder} \left(\frac{P}{2} \right) = \begin{cases} m - 16 + 1 & P \text{ odd} \\ m - 16 & P \text{ even} \end{cases}$
- $R := R_b + 16 - x$
- $L := L_b + x$

and get that the sum of the fragments is equal, as required, to the total number of protons P :

$$\bullet \quad R + L = (m + 16 - x) + \begin{cases} m - 16 + 1 + x \\ m - 16 + x \end{cases} = \begin{cases} 2m + 1 \\ 2m \end{cases} = P$$

We take $x \in [6, 16 - 6] = [6, 10]$ and get the most probable fission product. We could possibly expand it to $x \in [1, 15]$ if we want to get additional potential fission products.

Fission products (fragments)

The following table shows the results of the above calculation for the nuclei from Thorium (Th_{90}) to Fermium (Fm_{100}) with the x values $x \in [6,10]$ (and near it the $16-x$ values).

nucleon	$x=6$	10	$x=7$	9	$x=8$	8	$x=9$	7	$x=10$	6
Th_{90}	Sb_{51}	Y_{39}	Te_{52}	Sr_{38}	I_{53}	Rb_{37}	Xe_{54}	Kr_{36}	Cs_{55}	Br_{35}
Pa_{91}	Sb_{51}	Zr_{40}	Te_{52}	Y_{39}	I_{53}	Sr_{38}	Xe_{54}	Rb_{37}	Cs_{55}	Kr_{36}
U_{92}	Te_{52}	Zr_{40}	I_{53}	Y_{39}	Xe_{54}	Sr_{38}	Cs_{55}	Rb_{37}	Ba_{56}	Kr_{36}
Np_{93}	Te_{52}	Nb_{41}	I_{53}	Zr_{40}	Xe_{54}	Y_{39}	Cs_{55}	Sr_{38}	Ba_{56}	Rb_{37}
Pu_{94}	I_{53}	Nb_{41}	Xe_{54}	Zr_{40}	Cs_{55}	Y_{39}	Ba_{56}	Sr_{38}	La_{57}	Rb_{37}
Am_{95}	I_{53}	Mo_{42}	Xe_{54}	Nb_{41}	Cs_{55}	Zr_{40}	Ba_{56}	Y_{39}	La_{57}	Sr_{38}
Cm_{96}	Xe_{54}	Mo_{42}	Cs_{55}	Nb_{41}	Ba_{56}	Zr_{40}	La_{57}	Y_{39}	Ce_{58}	Sr_{38}
Bk_{97}	Xe_{54}	Tc_{43}	Cs_{55}	Mo_{42}	Ba_{56}	Nb_{41}	La_{57}	Zr_{40}	Ce_{58}	Y_{39}
Cf_{98}	Cs_{55}	Tc_{43}	Ba_{56}	Mo_{42}	La_{57}	Nb_{41}	Ce_{58}	Zr_{40}	Pr_{59}	Y_{39}
Es_{99}	Cs_{55}	Ru_{44}	Ba_{56}	Tc_{43}	La_{57}	Mo_{42}	Ce_{58}	Nb_{41}	Pr_{59}	Zr_{40}
Fm_{100}	Ba_{56}	Ru_{44}	La_{57}	Tc_{43}	Ce_{58}	Mo_{42}	Pr_{59}	Nb_{41}	Nd_{60}	Zr_{40}

Table of the expected fission fragments from Th_{90} to Fm_{100} for $x \in [6,10]$

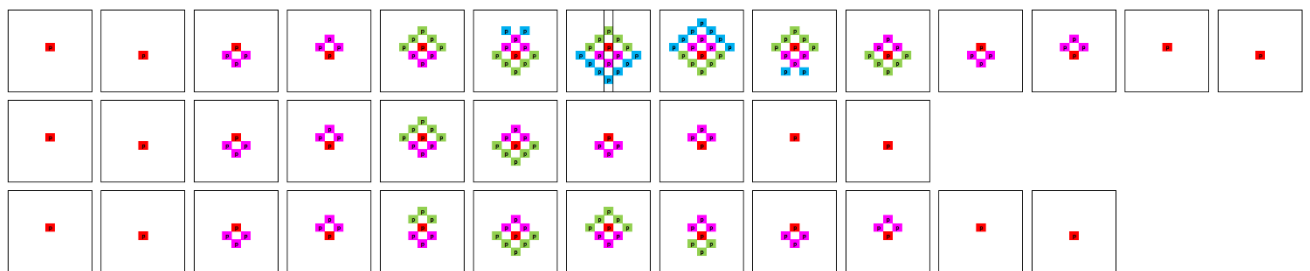
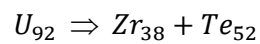
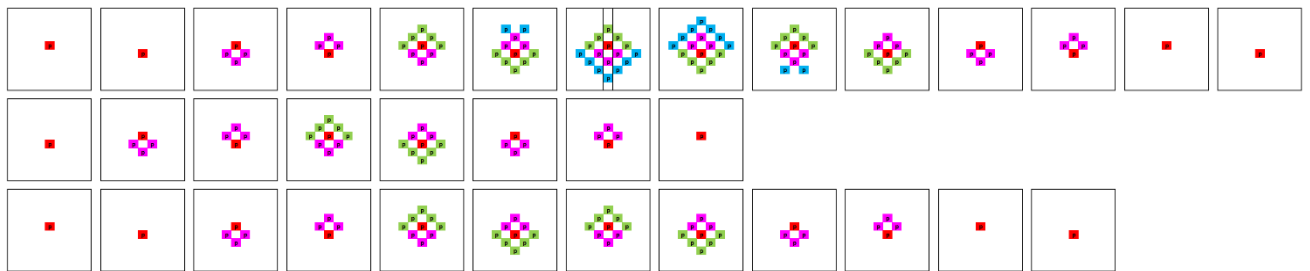
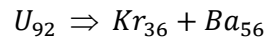
These results show the main fragments [13]; in order to get other fragments x could be taken from a wider range (e.g $x \in [3,13]$ or even $x \in [1,15]$).

Fission examples: observation of the protons solely

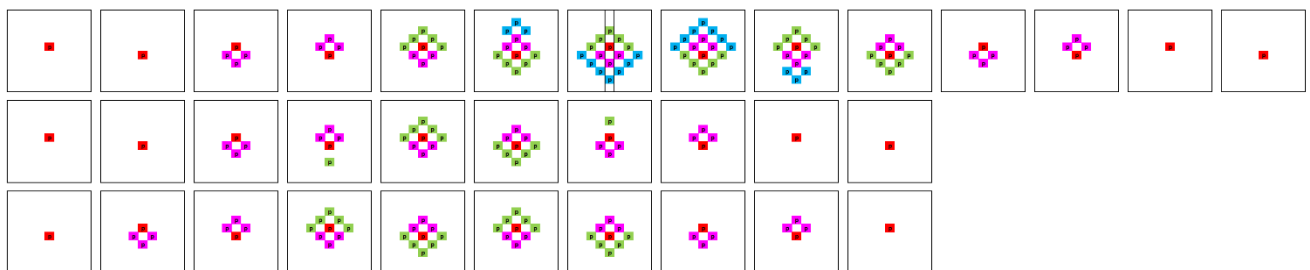
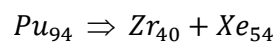
First we want to observe only the protons involved in the process; we choose for Uranium and Plutonium products [11], that have higher probability to appear and see first that according to the number of protons, the fission must occur at one of the two center layers as the model predicts.

The area of the split in one of the center layers is marked with two lines.

Uranium



Plutonium



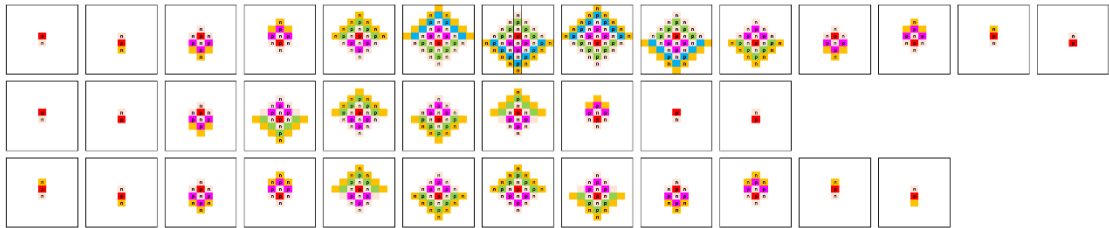
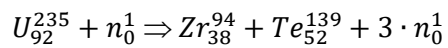
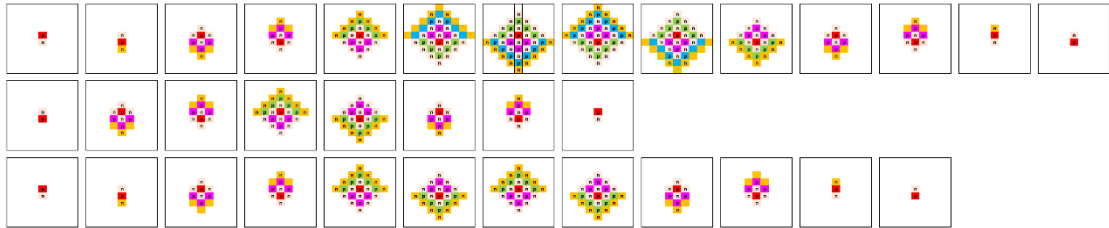
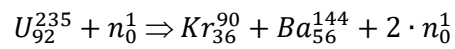
Fission examples - the full fragments

Now we consider the above nuclei as a whole [11].

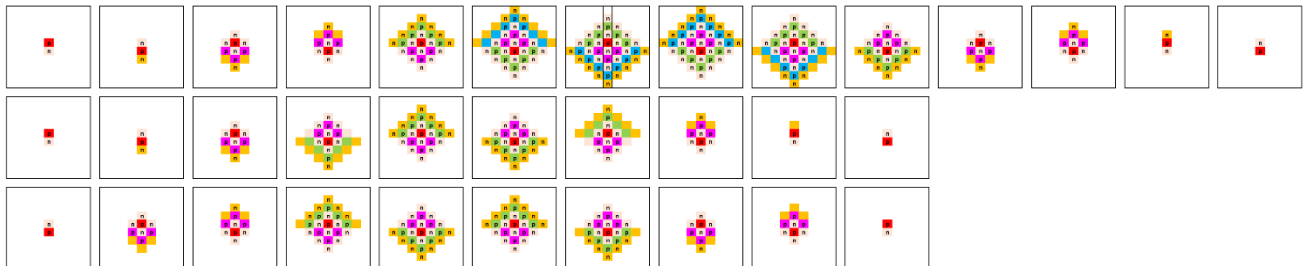
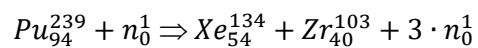
We see that for the nucleus that undergoes fission and also for the product nuclei almost all potential excess neutron positions are occupied.

The area of the split in one of the center layers is marked with two lines.

Uranium



Plutonium



Discussion of the results and conclusion

The main results of this research are:

- a tangible description of the fission mechanism.
- the prediction of the most probable products of the nuclear fission.

These are not a proof to the model, but strengthen its assumptions, just as the former papers of this series did.

We have got till this stage several results and all support the model on the one hand and none contradict the common nuclear theory or physics in general, so maybe it is worth continuing to study the model and deepen its understanding.

Sources and references

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