On V-Categories

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Abstract

Lawvere introduced a deceptively simple category, V, which is complete, symmetric, and monoidal closed. Here, we extend this construction to describe a rather general notion of localization called Γ-truncation. We show that this procedure produces tame, realizable n-cells in a standard Grothendieck universe, $\mathfrak{U}$. Finally, we clarify our notion of smallness for objects of stable rings in $\mathfrak{U}$.

I. Background

Definition 1.0.1 A *V-category* satisfies the following properties:

- The closed interval, $[0,\infty]\sim|\mathbb{Z}^+|$ (i.e., the upper half plane) is the domain, dom(V).
- The antisymmetric relationship, R:=(≥) provides the maps
- + as a tensor symbol
- Truncated subtraction is the adjoint "hom"

These properties were delineated by the classical 1973 paper of [**Lawv**], which was an early foray into the study of enriched categories.. By 2005, [**Kelley**] had established a canonical forgetful 2-functor:

$$(-)_0\colon \mathcal{V}\textbf{-Cat} \to \textbf{Cat}$$

in order to provide a *minimal* model over every category which trivially enriches it with a V-categorical structure. Alongside V, there is a shadow category, V_0, and the correspondence:

$$V_{COR}(V_0)$$

produces a valid binary logic. There is a projective morphism

$$V_0 \to (\tau = d(V,V_0)),$$

Where:

- $\tau=\mathbf{T}$ iff $d(V,V_0) = 0$
- $\tau=\mathbf{F}$ if V is not comparable with V_0 via a binary relationship ϑ.

Lawvere interpreted the second case as an *infinite distance*, thus fulfilling the supremum of the interval containing all the members of V. In this way, falsity completes the span of truth values. By assigning maximal truth to the distance zero, we obtain $\mathbf{0}_{\mathbf{id}}$, the numerical zero quantity, as our unit object. Thus, the quantities of all other elements *k* of V are inherently *relationally defined*. In this way, we are able to write

$$a\ R_k\ b,$$

where k is a fixed numerical invariant, and obtain

$$d(a,b) \equiv_k \frac{d(a,b)}{k}.$$

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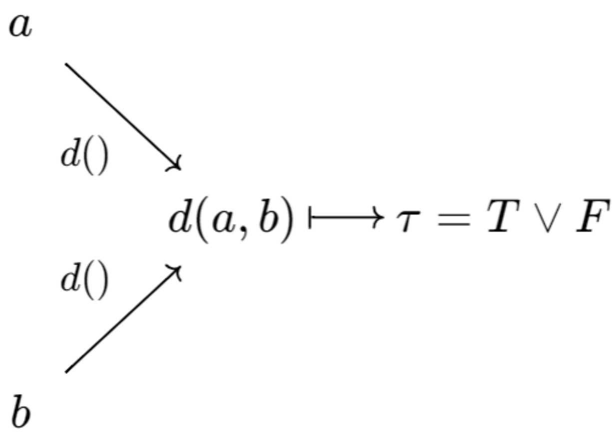

More generally, we may be interested in constructing a ring $\mathcal{R}_\tau$ of truth values which has as its supremum a top object in the category of frames, $\boldsymbol{t}$, which corresponds to a *truncation* of the standard infinity symbol. Locally, we define an accessibility relation, $a \to\!|_k b$ to consist of a *class of keys*, k, alongside a pullback to a "secret," a. Then, we have $a \to\!|_k \mathbf{t} = \mathbf{F}$ for all $a{:}\boldsymbol{\mathcal{R}}_{\tau_{FIN}}$. Then, the judgment, $a{:}_k\mathbf{t}$ is contextually equivalent to the judgment $a{:}_\gamma\infty$ for all γ.

II. Localization

Let, for every $\gamma_i \in \Gamma$, there is a γ_j some distance ε away, so that $d(\gamma_i,\gamma_j)$ is non-trivial and non-unital. Let k be a real number for every true proposition $\gamma_i \to_k \gamma_j$. Then,

Proposition 2.0.1 for some set Σ whose closure, λ, lies in γ_i, there is an extension, $\Sigma_{EXT} = \overline{\Sigma}$ whose closure lies in γ_j.

Proof Since $\gamma_i \to_k \gamma_j$ holds, and since $\gamma_i \neq \gamma_j$, then $d(\gamma_i,\gamma_j) = nk$ for n some natural number. Therefore, $\sup(\gamma_i)\mathbf{R}\sup(\gamma_j)$ for some binary transitive relationship $\mathbf{R}$, and thus the interval $[\gamma_i,\gamma_j]$ is a poset, and the cardinality of γ_j strictly exceeds the cardinality of γ_i, such that we may write:

$$\mathscr{C}(\gamma_j) >_{nk} \mathscr{C}(\gamma_i)$$

Definition 2.0.2 If $\Sigma \cup \lambda^+ = \overline{\Sigma}$ holds, then the ring of Σ is *k-local* with respect to the ring of $\overline{\Sigma}$.

Remark We may be especially interested in the case when k represents a particular value place; in this case, we say that Σ is *k-digital*, and symbolize by k the number of digits truncated to.

We are now ready to define Γ-truncation.

Definition 2.1.0 Let $\mathscr{V}$ be an ∞-category, and $\mathscr{V}_0$ a small category with finite colimits. The map, $\mathscr{V} \to \mathscr{V}_0$, which has as its left adjoint $\mathscr{V}_0 \to\!|_\tau \mathscr{V}$, is said to be Γ-truncated if, for every $d(\gamma_i,\gamma_j) \in \Gamma$, there is a set of keys, Γ_k, which allows $\sup(\lambda^+)$ to be *externally accessible* to every extension of Γ.

Our definition says nothing of internal accessibility; we assume that every $\kappa \gtrless \lambda$ is not internally accessible, and as a result the cardinal κ locally models infinity. So, we shall call κ an "∞-ideal" so long as there is no internal system for topologizing or geometrically realizing a space of arity κ. In other words, for a bounded set $\mathscr{U}(\mathbf{S})$, there is no set of internal keys such that

$$(\mathscr{U}(\mathbf{S}) \to_k \mathfrak{p}; ((\mathfrak{p} \notin \mathscr{U}(\mathbf{S})) \wedge (k \in \mathscr{U}(\mathbf{S})))) = \mathbf{T}$$

holds.

III. Universes

Let $\mathfrak{S}$ be a bounded set, and λ=[inf($\mathfrak{S}$),sup($\mathfrak{S}$)] be its primary key. Then, the interval which is "unlocked" by λ is *well-founded* within the universe $\mathfrak{U}|_{\lambda}$. Equivalently, all of the idempotents of $\mathfrak{S}$ are finitely contained within $\mathfrak{U}|_{\lambda}$, and the span of $\mathfrak{U}|_{\lambda}$ is

$$\widehat{\lambda} = |\sup(\mathfrak{S}) - \inf(\mathfrak{S})|$$

As a dynamical system, we have

$$\hom(\mathfrak{S},\mathfrak{S}) = 2^{\sup(\lambda)}(\dot{a}^{\dagger}),$$

where $\dot{a}^{\dagger}$ is a conscious agent.

Suppose $\mathfrak{S} \vDash \mathcal{LX}$, and let $\mathcal{LX}$ be a looped space, with a barycenter $\mathfrak{b}_{\mathcal{LX}}$=$\mathfrak{b}(\mathfrak{S})$. We make the appropriate translation

$$0\sim(\mathfrak{b}(\mathbb{R}^{+})) \rightarrow 0\sim(\mathfrak{b}(\mathfrak{S}))$$

and define the infinity ideal as

$$0 + \varepsilon = -\infty$$

$$0 - \varepsilon = \infty$$

so that a minimum distance in one direction is infinitely extended in the other direction of a Mobius band.

Definition 3.1.0 A *stable ring*, $\mathcal{R}_{\text{STAB}}$, is a looped space $\mathcal{LX}$ along with a defined positive and negative infinity ideal.

We may proceed to define *smallness* as follows. In the ordinary sense, we use the word "smaller than" to mean "less than," as in the arithmetic case. For the stable ring, however, there is a special property; for there to exist a *smaller stable ring* means that all of the values are inside, and completed by, the larger ring. Thus, for Γ-truncation to occur over a stable ring is to uniquely determine a class of floating points to which every integer and logical operation refers.

Write δ_i(x) for a δ_i-small object x. Here, we define δ_i-smallness as a *presentable slice* of a category which is *small* within a Grothendieck universe whose representation is a stable ring. Thus, a δ_i-small object, x, exhibits a rank-one isomorphism with its algebraic identity

$$x_{id} \in \mathfrak{U}|_i \leftrightarrow \delta_i(x)$$

which evaluates the distance function d(x,x_{id}) as a first-order truth.

One may begin to feel frustrated at the futility of this assignment; at first, it does not seem promising. Yet, the notion of such a bijection-on-object-identity leads immediately to the contrapuntal case, in which no such bijection exists. We may, for instance, consider superposition as the failure of an object's "classical" identity (with respect to a given universe) to biject to its algebraic smallness. As a result, the object(s) in superposition are not representable classically.

References

F.W. Lawvere *Metric Spaces, Generalized Logic, and Closed Categories* **(1973; reprinted 2002)**
G.M. Kelley *Basic Concepts of Enriched Category Theory* (**2005**)