Nuclear stability depending on the number of excess neutrons

research focus: nuclei with closed sub-orbitals

A continuation and further development of the cubic ellipsoid model of the nucleus [5]

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Abstract

This paper analyzes the sources of nuclear stability according to the cubic ellipsoid model [5] in dependency on the number of excess neutrons (neutrons beyond the number equal to that of the protons).
Although the research included all nuclei up to Lead (Pb), the focus here is on nuclei with closed sub-orbitals, because this simplifies the analysis; further works will expand the discussion to nuclei with even number of protons and then to all nuclei.

According to the model the nuclei are built up of layers and each of them has a population range of excess neutrons, in which the nucleus is stable.

If the number of these excess neutrons is below the minimum for this range then there is lack of neutrons that stabilizes the proton bonds and a proton emission occurs. [4]
Above the maximum number of neutrons for the range (neutrons excess) a neutron emission occurs due to lack of protons that stabilize the neutrons. [3]

Remark: these statements regarding the mechanisms that cause proton or neutron instability and their emission are not part of this work.

The population range of the excess neutrons depends on the number of layers of the nucleus and the protons population of each specific layer.
This is the focus of this work.

The results of this research are guidelines for the population of stable nuclei with excess neutrons in a similar manner to Hund's rules in atomic physics (for the electron population of light atoms).

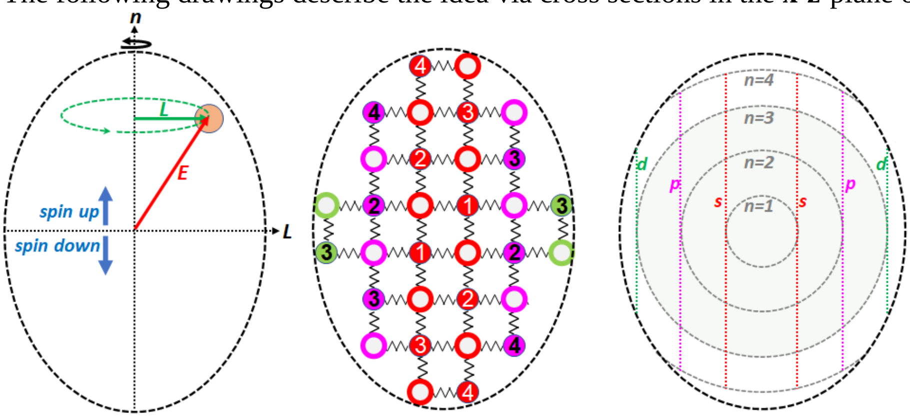
Content

The model at a glance

A brief description of the model [5]:

- The nucleus has an ellipsoid shape.
- The nucleons are connected in a cubic form.
- Protons are connected to neutrons (**p**-**n**).
- Neutrons are connected mainly to protons.
- The protons are populated and organized in shells in the nucleus in a full analogy to those of the electrons in the atom.
- The energy layers (principal quantum number **n**) grow along the **z**-axis of the nucleus in its both directions (more precisely **n** grows with its distance from the origin).
- The perpendicular distance from the **z**-axis in the **x**-**y**-plane reflects the angular momentum (**L**) and so the orbitals.
- The upper half of the ellipsoid is referred to as spin-up and the lower part as spin-down.
- The nucleus possibly rotates around its **z**-axis.

The following drawings describe the idea via cross sections in the **x**-**z**-plane of the nucleus.



***1**: a nucleon in the nucleus* *2: the bonds between the nucleons* *3: the energy levels of the nucleus*

1. One nucleon (**circle**) is observed inside the ellipsoid (dashed line) that encloses the nucleons and schematically defines the nucleus surface:
   - the distance from the origin represents its energy **E**.
   - the distance from the **z**-axis depicts it angular momentum **L**.
   - the nucleons in the upper half have spin up, and in the lower one spin down.
2. The bonds between the nucleons are shown for visibility as springs.
   - **Protons**: full circles of the **s**, **p** and **d** sub-orbitals. **Neutrons**: hollow circles.
3. The circles of equal energy states **n** in the ellipsoid.
   - the lines mark the development of the **s**, **p** and **d** sub-orbitals along the **z**-axis.
   - the **s** line crosses all **n** circles from 1 to 4 (**s1** to **s4**).
   - the **p** line begins by **n**=2 and reaches till **n**=4 (**p2** to **p4**).
   - the **d** line begins by **n**=3 and reaches the ellipsoid border, before it reaches the **n**=4 circle, and therefore there are no **d4** states at this stage (only **d3**).

Introduction

The cubic ellipsoid model of the nucleus [5] offers a tangible description for the nucleus.
The first paper dealt with the model itself and its mass formula.
In this research we want to deepen the understanding of the model and expand it by investigating the nucleus stability in dependency on the number of excess neutrons in it.
The number of neutrons has two limits:

- lower limit: below it a lack of neutrons leads to proton emission. [4]
- upper limit: beyond which the excess of neutrons leads to neutron emission. [3]

Elements larger than Lead (Pb) don't have stable nuclei at all and therefore other mechanisms shall be considered in addition to the lack or excess of neutrons; these will be discussed in a following research.
We observe here mainly nuclei of full sub-orbitals (s, p, d, f) in order to simplify the discussion and make the idea clearer. Further works shall expand the discussion to nuclei with even number of protons and then to all nuclei.
According to the model the core of the nucleus is built from an equal number of protons and neutron that are connected in cubic p-n bonds.
Till approximately the end of the third row of the periodic system, meaning around Z=18 or 20 protons, additional neutrons are not crucial for the stabilization of the nucleus; the excess neutrons help by the stabilization of nuclei with odd number of protons or appear also in several isotopes, but there are also stable nuclei with no excess neutrons.
From the fourth row excess neutrons are required for stability in general and we assume here that this is due to the electric forces on the surface protons.
The electric force is analyzed in this work and we can see a correlation between the number of protons on which a force above certain level acts and the number of neutrons required for the nucleus stability.
How the excess neutrons stabilize the protons we don't try to solve at this stage, but detailed table are offered here for the number of excess neutrons along the filling process of every sub-orbital.

Population rules for the excess neutrons of stable nuclei

The force on the surface protons of the nucleus ellipsoid

According to the model the excess neutrons are located in the envelope of the nucleus.
We first calculate the relative electric force for each proton in the envelope.
The electric force on the proton $\boldsymbol{j}$ in the $\boldsymbol{x}$ direction:

$$F_{j_x} = \frac{e^2}{4\pi\epsilon_0}\frac{1}{d_0^3}\left\{\sum_{i\neq j}^{Z}\frac{(x_i - x_j)}{d_{i,j}{}^3}\right\} = \frac{e^2}{4\pi\epsilon_0}\frac{1}{d_0^3} f_{j_x}$$

the unitless relative electric force in the $\boldsymbol{x}$ direction is defined as: $f_{j_x} := \sum_{i\neq j}^{Z}\frac{(x_i - x_j)}{d_{i,j}{}^3}$

where:

- d_0: the minimum distance between two neighboring nucleons in femtometer (assuming all nuclei have the same cubic structure and distance between their nucleons).
- $d_{i,j}$: the unitless distance between the protons of the indices *i* and *j* measured in multiples of d_0: $d_{i,j} = \sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2 + \left(z_j - z_i\right)^2}$
- Z: atomic number (number of protons).

Similarly we get the relative forces in the $\boldsymbol{y}$ and $\boldsymbol{z}$ direction: f_{j_y} and f_{j_z}.

The total relative force on the proton $\boldsymbol{j}$ in the envelope is: $f_j = \sqrt{f_{x,j}{}^2 + f_{y,j}{}^2 + f_{z,j}{}^2}$ (absolute value).

The range of excess neutrons

In order to calculate the number of excess neutrons in the envelope we define:

- Z_{env} : the number of neutrons in the envelope (the excess neutrons).
- $n_{min} = \sum_{i}^{Z_{env}}\begin{cases}1: & f_{max} \le f_i \\ 0 & \end{cases}$: minimum number of excess neutrons.
- $n_{opt} = \sum_{i}^{Z_{env}}\begin{cases}1: & f_{opt} \le f_i \\ 0 & \end{cases}$: intermediate or optimum number of excess neutrons.
- $n_{max} = \sum_{i}^{Z_{env}}\begin{cases}1: & f_{min} \le f_i \\ 0 & \end{cases}$: minimum number of excess neutrons.

The relative forces $f_{min}, f_{opt}, f_{max}$ are found by trial, while learning the model. We note to not confuse: n_{min} is calculated via f_{max} whereas n_{max} via f_{min}.
We roughly estimate these limits here with:

- $f_{min} = 0.295$
- $f_{opt} = 0.310$
- $f_{max} = 0.325$

This is only a support tool in the development of the population rules. It helps us by the estimation of the upper and lower limits of excess neutrons for stable nuclei, but it is not exact and in order to determine the precise locations additional rules shall be considered.

The population rules of excess neutrons

The rules of how to fill up the excess neutrons are determined via iterations of a semi empirical process that compares the results for the various nuclei with the experimental data.

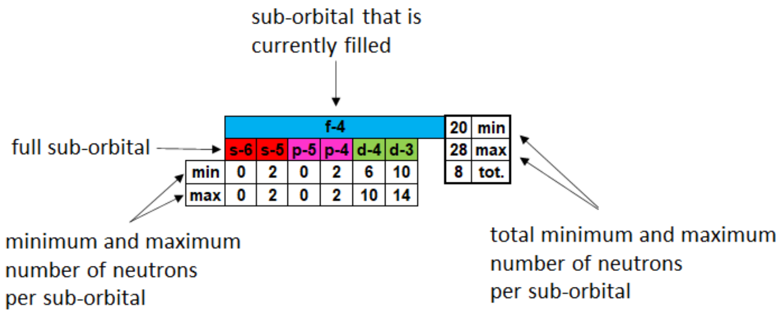
- The first step was to calculate the mass formula and get the values for its parameters:
  - d_0: the minimum distance between two neighboring nucleons in the cubic structure of the nucleus.
  - e_b: the binding energy between two neighboring nucleons.
- The next step was to complete the drawings for all stable nuclei:
  - the above data was used in cases where more than one position for the excess neutron is possible.
  - the total nucleus spin was considered.
  - a comparison between the various nuclei was made to determine which positions are more likely to be the correct ones.
- The results of this process were then compared also with the positions of surface protons on which larger electric forces act.

The next section presents the results and delivers the guidelines for the population process.

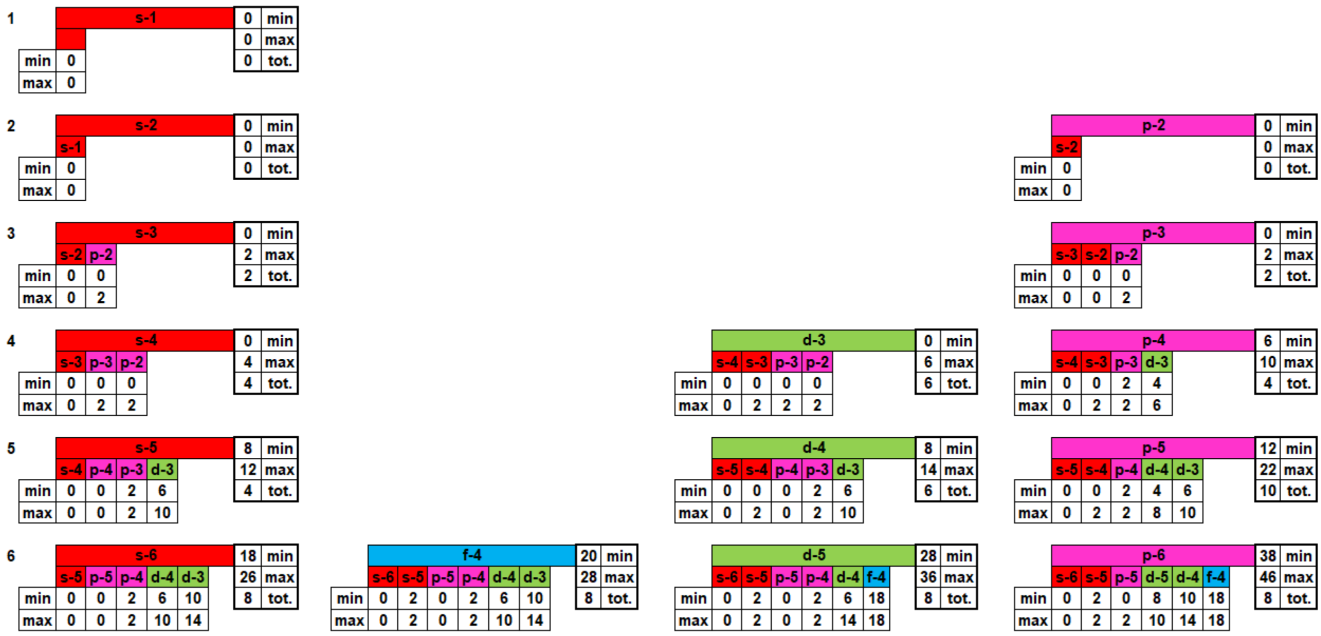
Remark: according to the model all excess neutrons are located in the envelope, but not all neutrons that are located in the envelope are excess neutrons. [5]

The population of sub-orbitals of stable nuclei

The following drawings shows the population process of the excess neutrons along the sub-orbitals of the periodic table.
The outer most sub-orbital of the layer that is currently in filling process is the upper one in the drawing and is marked with its name and color (in the example below: row/layer: 6, sub-orbital: **f-4**). Below it the population of the outermost sub-orbitals of every layer of the ellipsoid are shown with their minimum and maximum number of excess neutrons.
On its right the total minimum and maximum number of excess neutrons is shown.



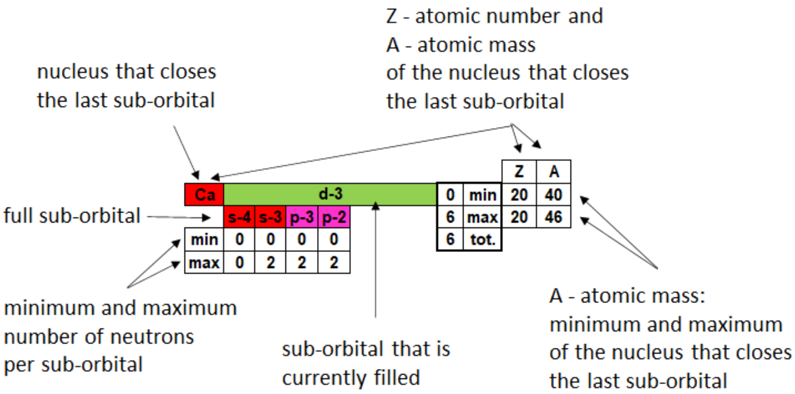
Explanation on how to read the table below.



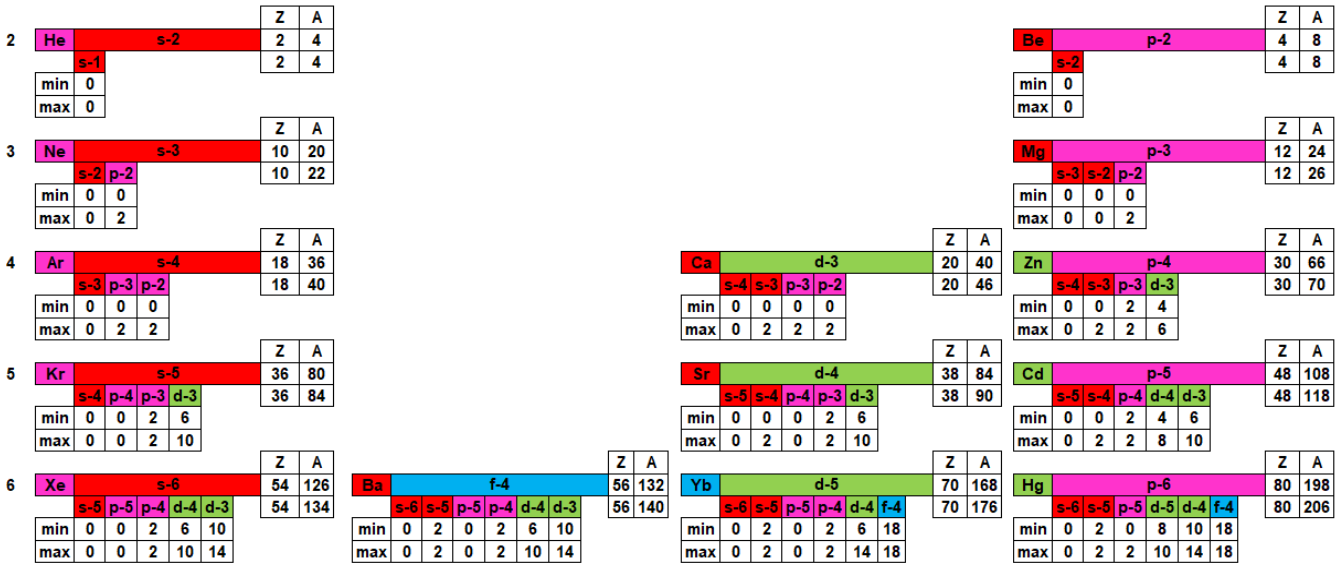
The number of excess neutrons per sub-orbital along the process of filling in the elements of the periodic table.
s *sub-orbital,* ***p*** *sub-orbital,* ***d*** *sub-orbital,* ***f*** *sub-orbital.*

The sub-orbitals of stable isotopes

The following drawings are similar to those of the last section, only the nucleus of the element that closed the last sequence (the sub-orbital of the last layer that was filled) is shown on the left and on the right the minimum and maximum values of its atomic mass A are calculated.



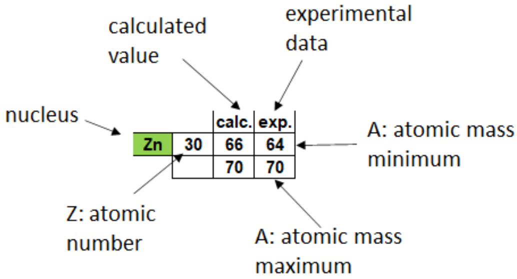
Explanation on how to read the table below.



The number of excess neutrons per sub-orbital along the process of filling in the elements of the periodic table with the nuclei of the elements that closed the last sequence (the sub-orbital of the last layer that was filled).
s *sub-orbital,* ***p*** *sub-orbital,* ***d*** *sub-orbital,* ***f*** *sub-orbital.*

The number of stable isotopes for nuclei with full sub-orbitals

The range of stable isotopes with closed sub-orbital was calculated in the last section.
Here it is shown in comparison with the experimental data.
The result verifies the population process and strengthen the model.



Explanation on how to read the table below.

			calc.	exp.
2	Be	4	8	9
			8	9
	Ne	10	20	20
			22	22
3	Mg	12	24	24
			26	26
	Ar	18	36	36
			40	40
4	Ca	20	40	40
			46	46
	Zn	30	66	64
			70	70
	Kr	36	78	80
			86	86
5	Sr	38	84	86
			90	88
	Cd	48	108	106
			118	114
	Xe	54	126	124
			134	134
6	Ba	56	132	130
			140	138
	Yb	70	168	168
			176	176
	Hg	80	198	196
			206	204

A comparison between the calculated and experimental data for the upper and lower limits of stable isotopes of the nuclei of the elements with closed sub-orbitals.
s *sub-orbital,* ***p*** *sub-orbital,* ***d*** *sub-orbital,* ***f*** *sub-orbital.*

Discussion of the results and conclusion

Analyzing the stable nuclei according to the cubic ellipsoid model, leads to the creation of a set of rules for the population of the excess neutrons (the neutrons beyond the number equal to that of the protons) that are located in the envelope of the nucleus.
The rules were gained by an iterative process.
We can see the pattern of the rules through the tables that describe it.
The calculations of the electrical forces on the surface protons help us to justify the rules, but this is still not a general description of the process and its justification.
To summon up, we describe in this paper the rules and explain them, but we don't deliver a clear proof for why this is done exactly so.
The main assumptions are that it has to do with:

- lower limit for excess neutrons: the stability of protons on the envelope of the nucleus that requires excess neutrons (proton emission [4]).
- upper limit for excess neutrons: the stability of excess neutrons on the envelope requires protons, and beyond a certain number of neutrons there are not enough protons anymore to stabilize them (neutron emission [3]).

These mechanisms shall be further developed in other research.

Sources and references

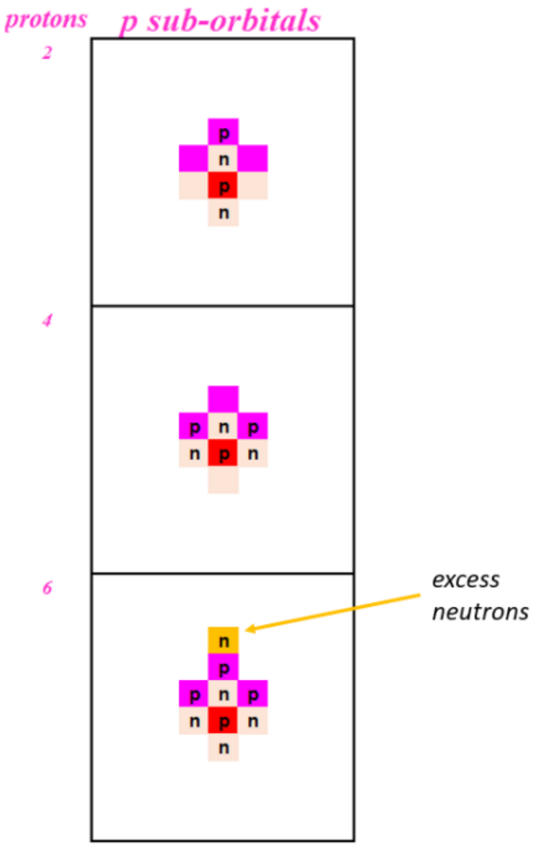
1. *Tables of Nuclear Data: Japan Atomic Energy Agency (JAEA)*
2. *Valley of stability - (LibreTexts)*
3. *Neutron emission - (Wikipedia)*
4. *Proton emission - (Wikipedia)*
5. *a cubic ellipsoid geometric model of the atomic nucleus and its mass formula - Ronen Yavor - (viXra)*

Appendix

The order of filling the sub-orbitals

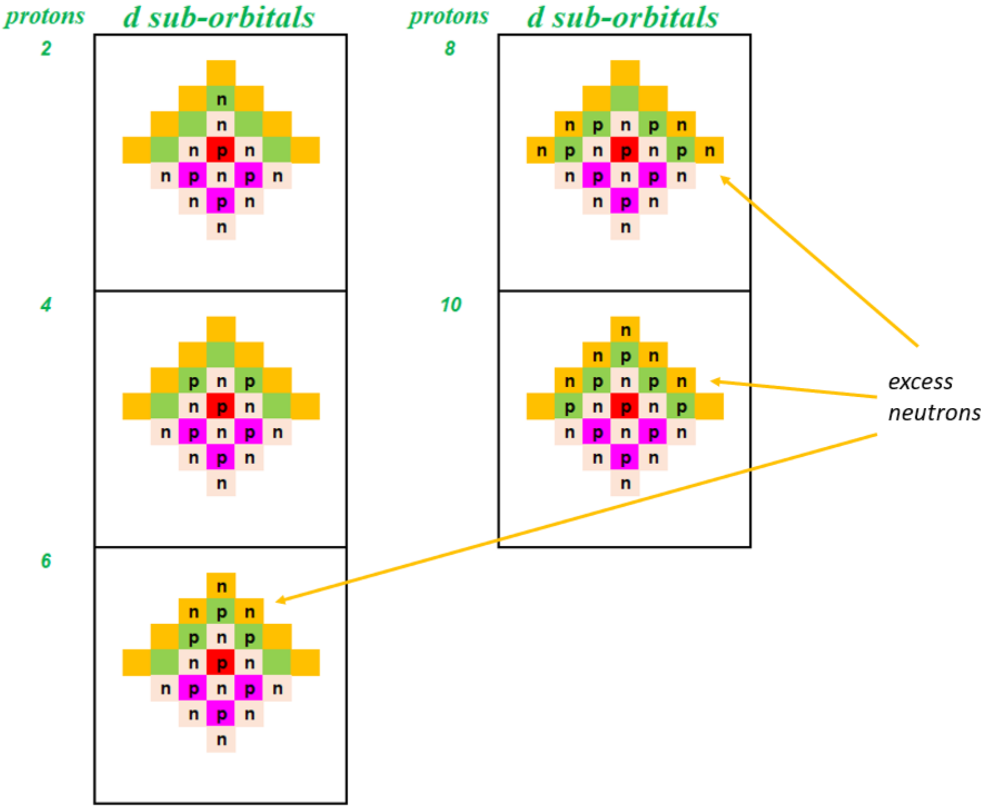
We want to learn the sequence by which each sub-orbital is filled.
We observe only nuclei with even number of protons, because the symmetry of the nucleus makes the study more visible and easier to understand.
The rules are not strict but give a general direction. Further study shall make it more precise.

The p sub-orbitals



***Legend**: **p**: proton, **n**: neutron*
***s** sub-orbital, **p** sub-orbital, **d** sub-orbital, **f** sub-orbital*
excess neutrons

The d sub-orbitals

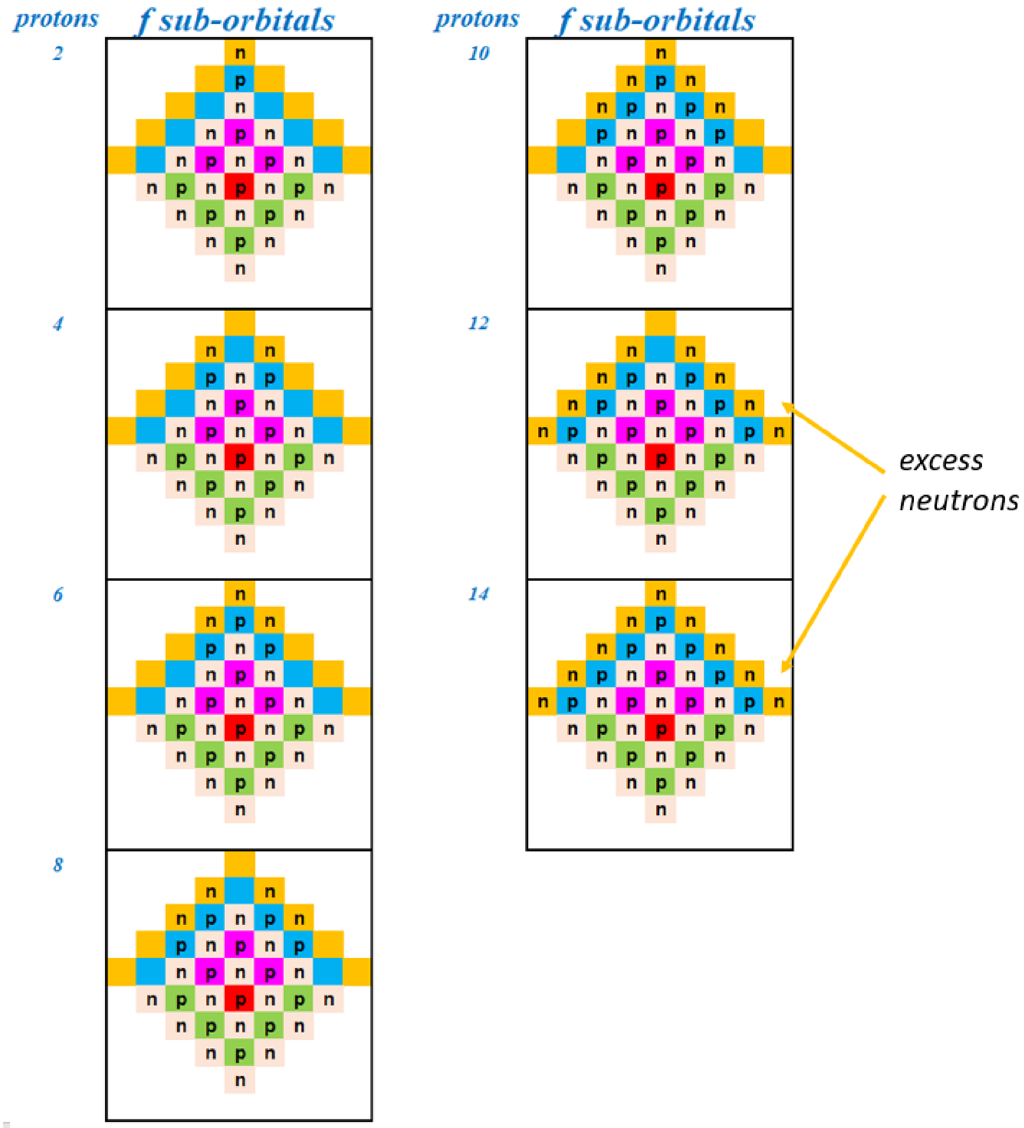


Legend: ***p***: *proton*, ***n***: *neutron*

s *sub-orbital*, ***p*** *sub-orbital*, ***d*** *sub-orbital*, ***f*** *sub-orbital*

excess neutrons

The f sub-orbitals



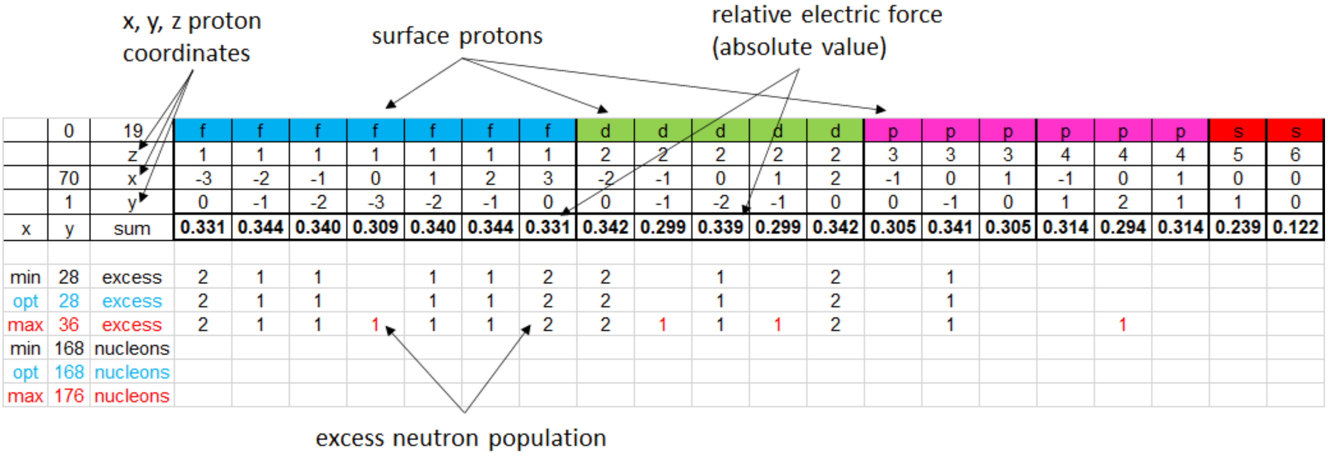
Legend: ***p***: *proton*, ***n***: *neutron*

s *sub-orbital*, ***p*** *sub-orbital*, ***d*** *sub-orbital*, ***f*** *sub-orbital*

excess neutrons

The range of stability for the stable nuclei with closed sub-orbitals

The following drawings show the expected range of excess neutron for stable nuclei with closed sub-orbitals according to the model.



	0	19	f	f	f	f	f	f	f	d	d	d	d	d	p	p	p	p	p	p	s	s
		z	1	1	1	1	1	1	1	2	2	2	2	2	3	3	3	4	4	4	5	6
	70	x	-3	-2	-1	0	1	2	3	-2	-1	0	1	2	-1	0	1	-1	0	1	0	0
	1	y	0	-1	-2	-3	-2	-1	0	0	-1	-2	-1	0	0	-1	0	1	2	1	1	0
x	y	sum	**0.331**	**0.344**	**0.340**	**0.309**	**0.340**	**0.344**	**0.331**	**0.342**	**0.299**	**0.339**	**0.299**	**0.342**	**0.305**	**0.341**	**0.305**	**0.314**	**0.294**	**0.314**	**0.239**	**0.122**
min	28	excess	2	1	1		1	1	2	2		1		2		1						
opt	28	excess	2	1	1		1	1	2	2		1		2		1						
max	36	excess	2	1	1	1	1	1	2	2	1	1	1	2		1			1			
min	168	nucleons																				
opt	168	nucleons																				
max	176	nucleons																				

population according to the forces on the surface protons.

General guidelines for the population of excess neutrons

- the **s** sub-orbital is usually not populated at all or with two excess neutrons at most; usually if **s** is populated, it is only when there is another **s** sub-orbital above it.
- the **p** sub-orbital is populated with two excess neutrons at most; usually it is populated when there is another **p** sub-orbital above it.
- the **d** sub-orbital has usually between 4 to 8 excess neutrons, if there is no **d** sub-orbital above it; if there is a **d** sub-orbital above it, it is populated with 6 to 14 excess neutrons.
- the **f** sub-orbital is usually fully populated with excess neutrons (18 neutrons).

Remarks

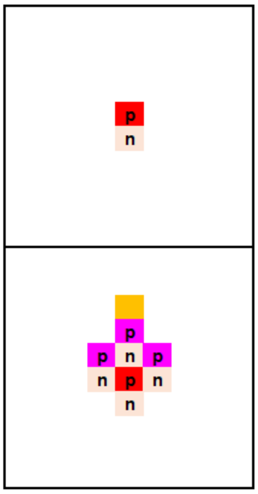
- the calculation while using the electric forces supports us by assessing the range of excess neutrons, but doesn't show where to locate them.
- we show only the upper half of the nucleus, so for each neutron here there are actually two in the nucleus.
- every layer has a number of nucleons that is equal to or smaller than the number of nucleons of the layer below it.

Neon Ne_{10} – surface protons and excess neutrons

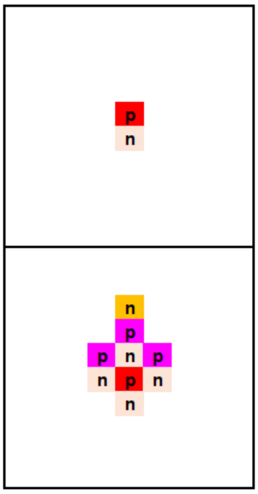
	0	3	p	p	p	s	s
		z	1	1	1	-1	2
	10	x	-1	0	1	0	0
	1	y	0	-1	0	1	0
x	y	sum	**0.293**	**0.278**	**0.293**	**0.278**	**0.278**
min	0	excess					
opt	0	excess					
max	2	excess		1			
min	20	nucleons					
opt	20	nucleons					
max	22	nucleons					

Neon Ne_{10} – stable nuclei

Ne-10-20



Ne-10-22



Legend: ***p***: *proton*, ***n***: *neutron*

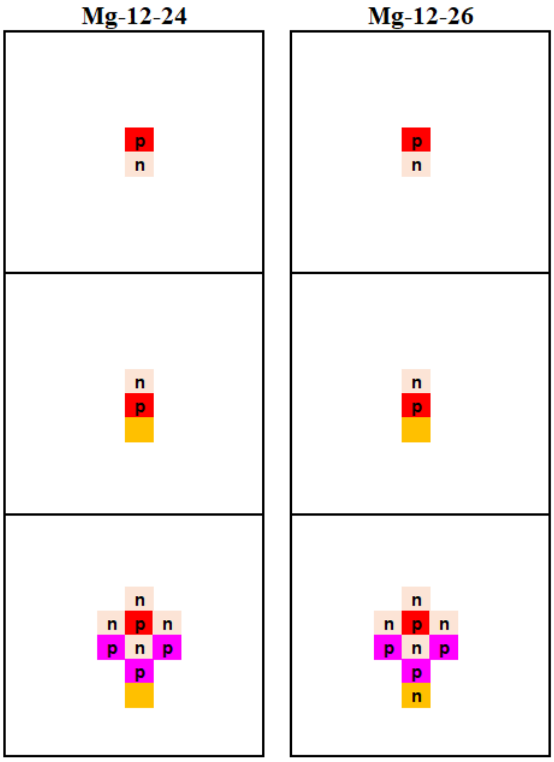
s *sub-orbital*, ***p*** *sub-orbital*, ***d*** *sub-orbital*, ***f*** *sub-orbital*

excess neutrons

Magnesium Mg_{12} – surface protons and excess neutrons

	0	3	p	p	p	s	s
		z	1	1	1	2	3
	12	x	-1	0	1	0	0
	1	y	0	-1	0	0	1
x	y	sum	**0.295**	**0.281**	**0.295**	**0.227**	**0.118**
min	0	excess					
opt	0	excess					
max	2	excess		1			
min	24	nucleons					
opt	24	nucleons					
max	26	nucleons					

Magnesium Mg_{12} – stable nuclei



Legend: ***p***: *proton*, ***n***: *neutron*

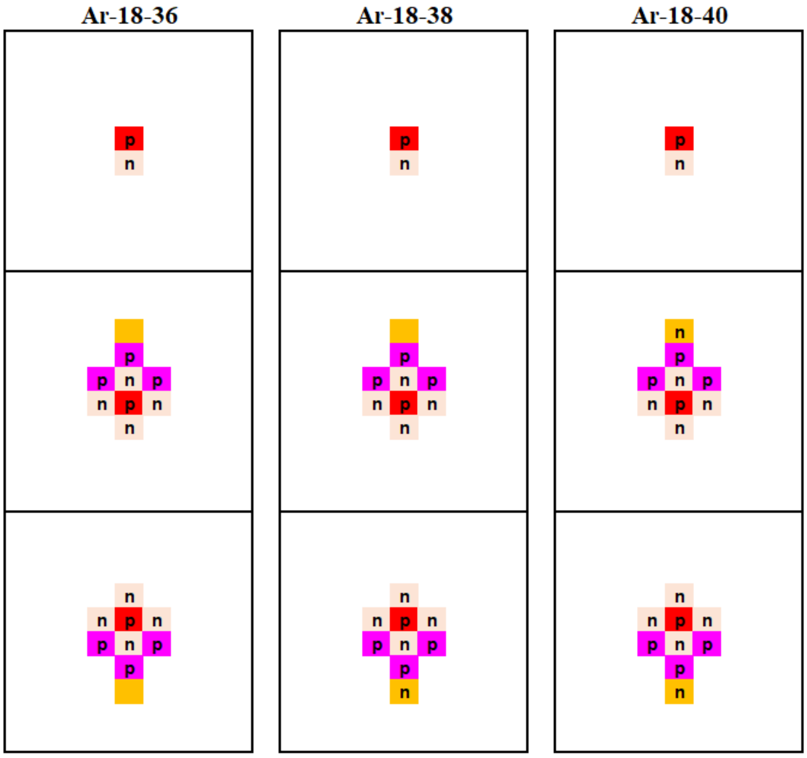
s *sub-orbital*, ***p*** *sub-orbital*, ***d*** *sub-orbital*, ***f*** *sub-orbital*

excess neutrons

Argon Ar_{18} – surface protons and excess neutrons

	0	5	p	p	p	p	p	p	s
		z	1	1	1	2	2	2	3
	18	x	-1	0	1	-1	0	1	0
	1	y	0	-1	0	1	2	1	1
x	y	sum	**0.322**	**0.297**	**0.322**	**0.305**	**0.287**	**0.305**	**0.284**
min	0	excess							
opt	2	excess		1					
max	4	excess		1			1		
min	36	nucleons							
opt	38	nucleons							
max	40	nucleons							

Argon Ar_{18} – stable nuclei



***Legend**: **p**: proton, **n**: neutron*

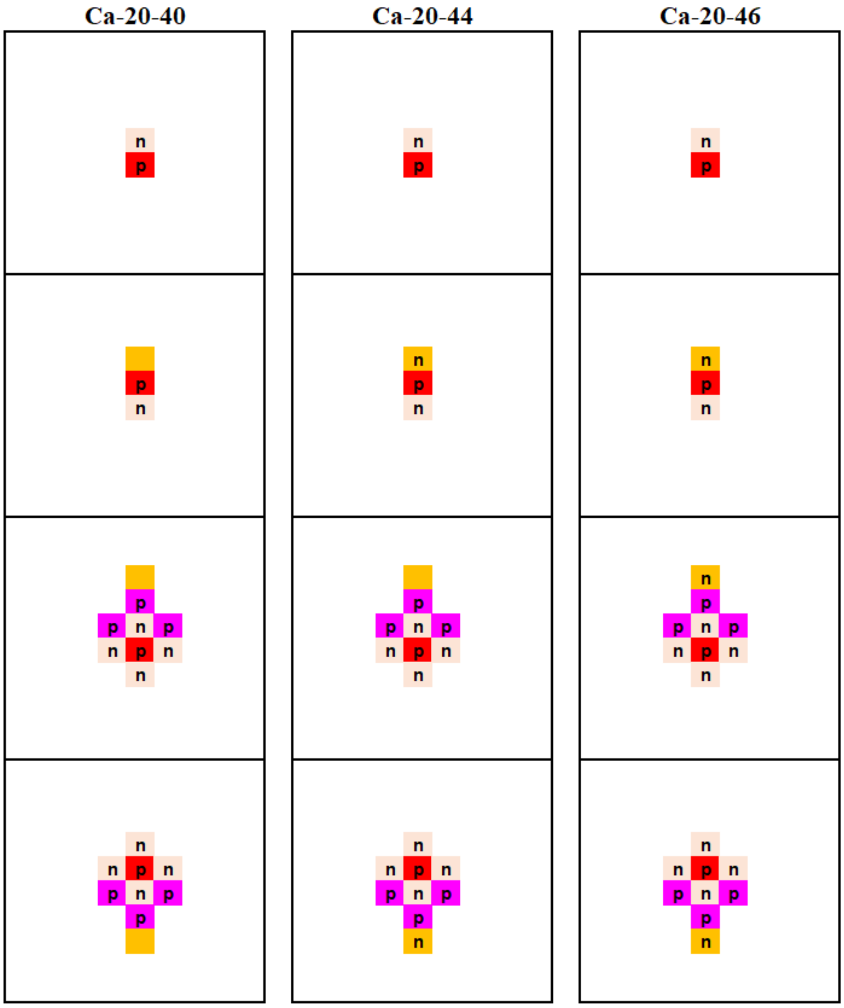
s sub-orbital, p sub-orbital, d sub-orbital, f sub-orbital

excess neutrons

Calcium Ca_{20} – surface protons and excess neutrons

	0	5	p	p	p	p	p	p	s	s
		z	1	1	1	2	2	2	3	4
	20	x	-1	0	1	-1	0	1	0	0
	1	y	0	-1	0	1	2	1	1	0
x	y	sum	**0.323**	**0.298**	**0.323**	**0.307**	**0.288**	**0.307**	**0.233**	**0.120**
min	0	excess								
opt	2	excess		1						
max	4	excess		1			1			
min	40	nucleons								
opt	42	nucleons								
max	44	nucleons								

Calcium Ca_{20} – stable nuclei



Legend: ***p***: *proton,* ***n****: neutron*

s *sub-orbital,* ***p*** *sub-orbital,* ***d*** *sub-orbital,* ***f*** *sub-orbital*

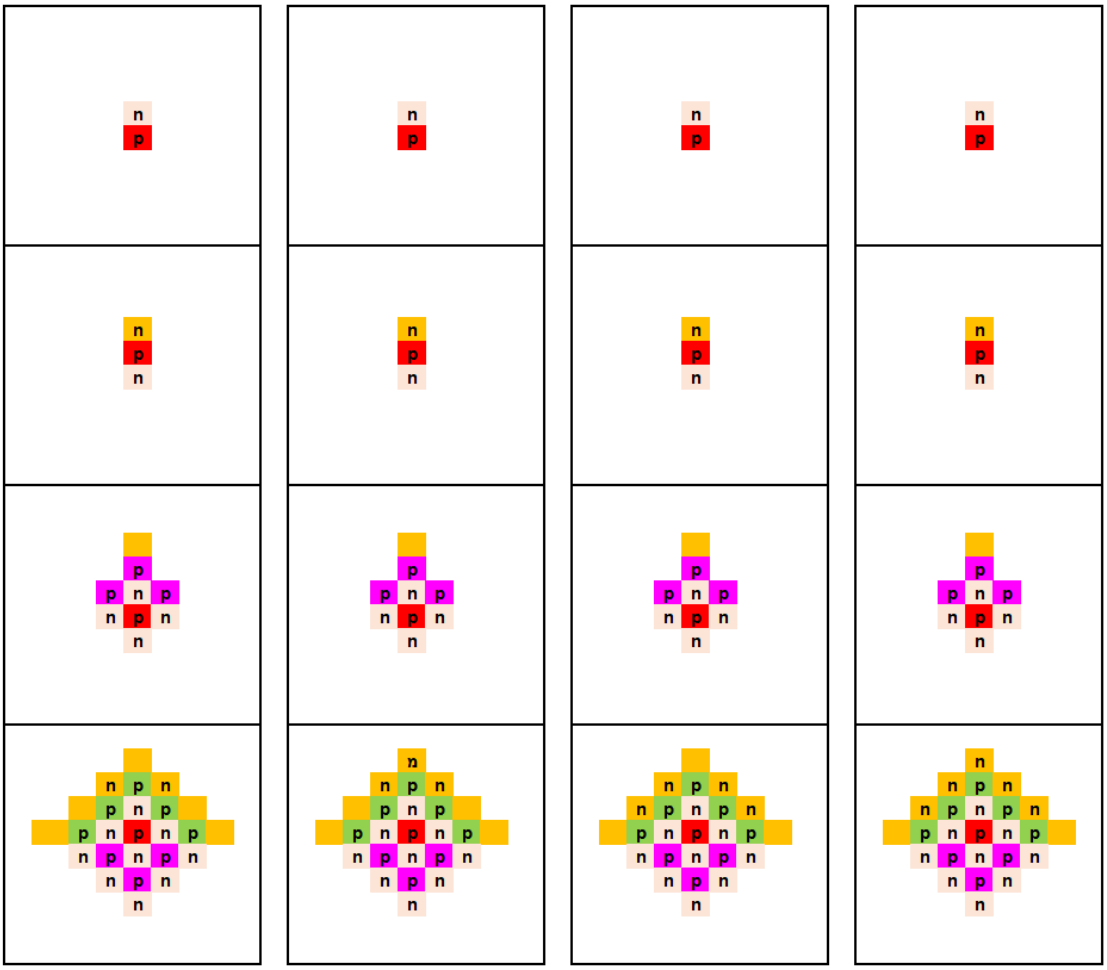
excess neutrons

Zink Zn_{30} – surface protons and excess neutrons

	0	8	d	d	d	d	d	p	p	p	s	s
		z	1	1	1	1	1	2	2	2	3	4
	30	x	-2	-1	0	1	2	-1	0	1	0	0
	1	y	1	2	3	2	1	1	2	1	1	0
x	y	sum	**0.322**	**0.327**	**0.302**	**0.327**	**0.322**	**0.325**	**0.324**	**0.325**	**0.248**	**0.123**
min	4	excess		1		1						
opt	10	excess	1	1		1	1		1			
max	12	excess	1	1	1	1	1		1			
min	64	nucleons										
opt	70	nucleons										
max	72	nucleons										

Zink Zn_{30} – stable nuclei

Zn-30-66	Zn-30-68	Zn-30-70	Zn-30-72



Legend: ***p***: *proton*, ***n***: *neutron*

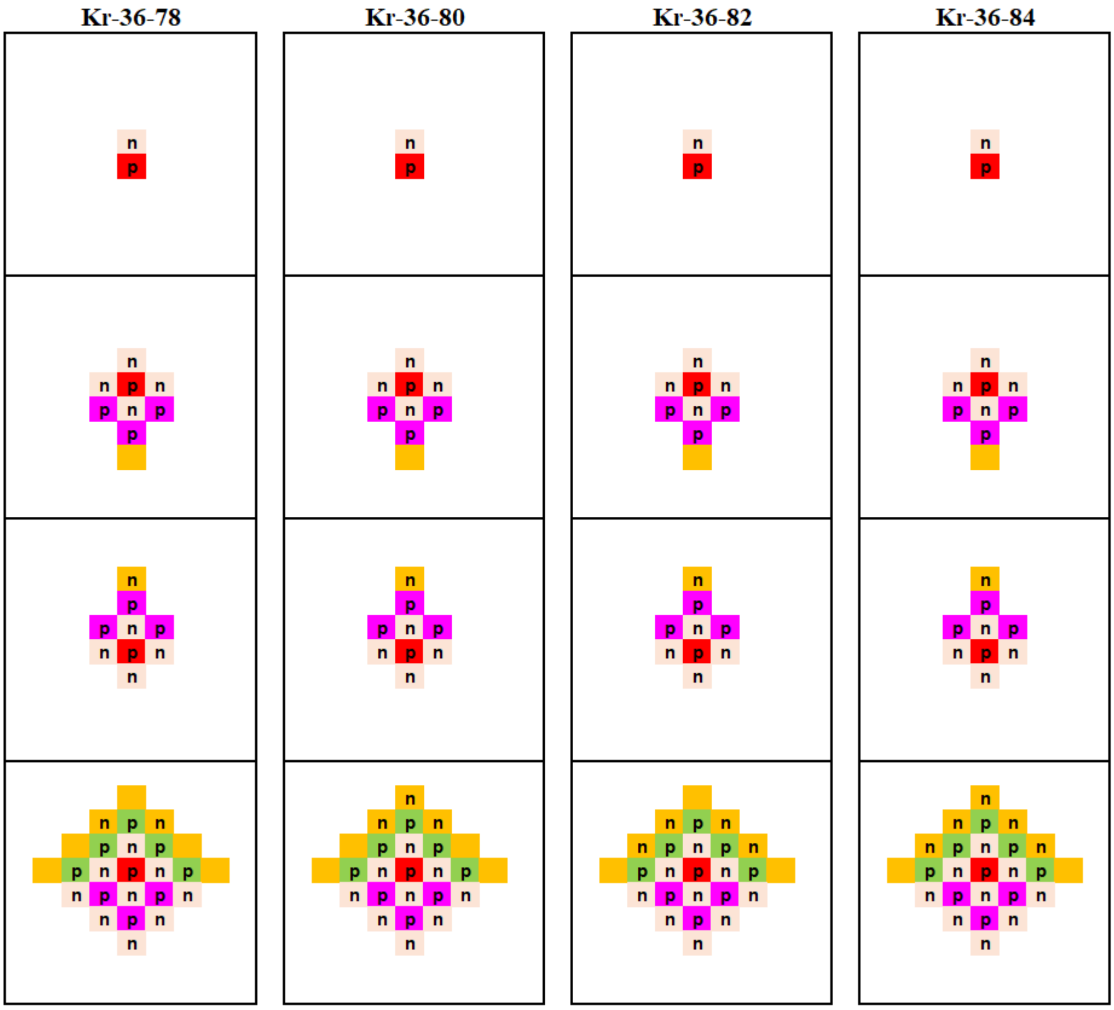
s *sub-orbital*, ***p*** *sub-orbital*, ***d*** *sub-orbital*, ***f*** *sub-orbital*

excess neutrons

Krypton Kr_{36} – surface protons and excess neutrons

	0	10	d	d	d	d	d	p	p	p	p	p	p	s	s
		z	1	1	1	1	1	2	2	2	3	3	3	-3	4
	36	x	-2	-1	0	1	2	-1	0	1	-1	0	1	0	0
	1	y	1	2	3	2	1	1	2	1	0	-1	0	1	0
x	y	sum	**0.327**	**0.331**	**0.305**	**0.331**	**0.327**	**0.298**	**0.329**	**0.298**	**0.311**	**0.291**	**0.311**	**0.289**	**0.289**
min	10	excess	1	1		1	1		1						
opt	10	excess	1	1		1	1		1						
max	14	excess	1	1	1	1	1		1			1			
min	82	nucleons													
opt	82	nucleons													
max	86	nucleons													

Krypton Kr_{36} – stable nuclei



Legend: ***p***: *proton*, ***n***: *neutron*

s *sub-orbital*, ***p*** *sub-orbital*, ***d*** *sub-orbital*, ***f*** *sub-orbital*

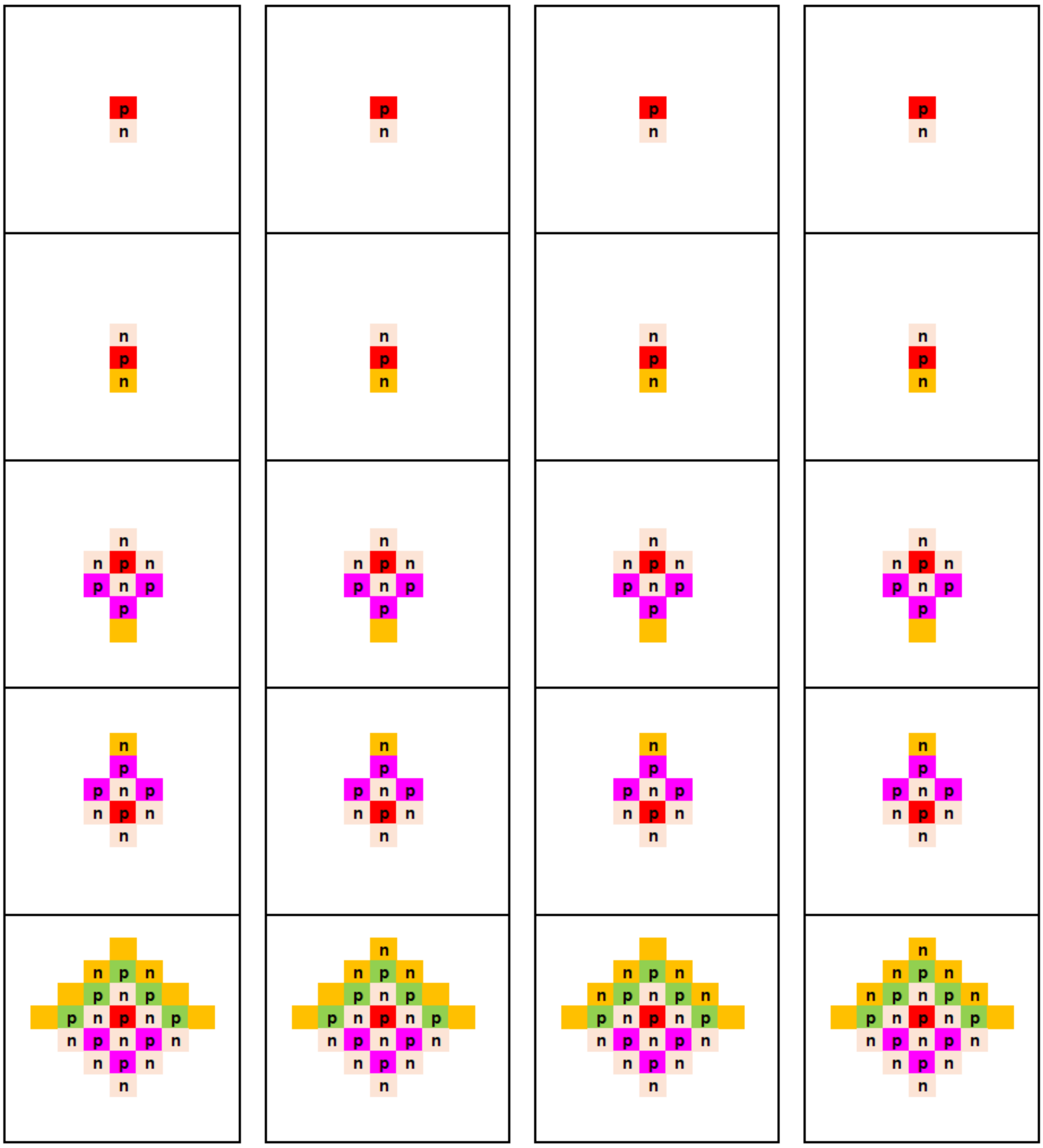
excess neutrons

Strontium Sr_{38} – surface protons and excess neutrons

	0	10	d	d	d	d	d	p	p	p	p	p	p	s	s
		z	1	1	1	1	1	2	2	2	3	3	3	4	5
	38	x	-2	-1	0	1	2	-1	0	1	-1	0	1	0	0
	1	y	1	2	3	2	1	1	2	1	0	-1	0	0	1
x	y	sum	**0.327**	**0.331**	**0.305**	**0.331**	**0.327**	**0.298**	**0.329**	**0.298**	**0.312**	**0.292**	**0.312**	**0.237**	**0.121**
min	10	excess	1	1		1	1		1						
opt	10	excess	1	1		1	1		1						
max	14	excess	1	1	1	1	1		1			1			
min	86	nucleons													
opt	86	nucleons													
max	90	nucleons													

Strontium Sr_{38} – stable nuclei

Sr-38-84 | **Sr-38-86** | **Sr-38-88** | **Sr-38-90**



Legend: ***p***: *proton*, ***n***: *neutron*

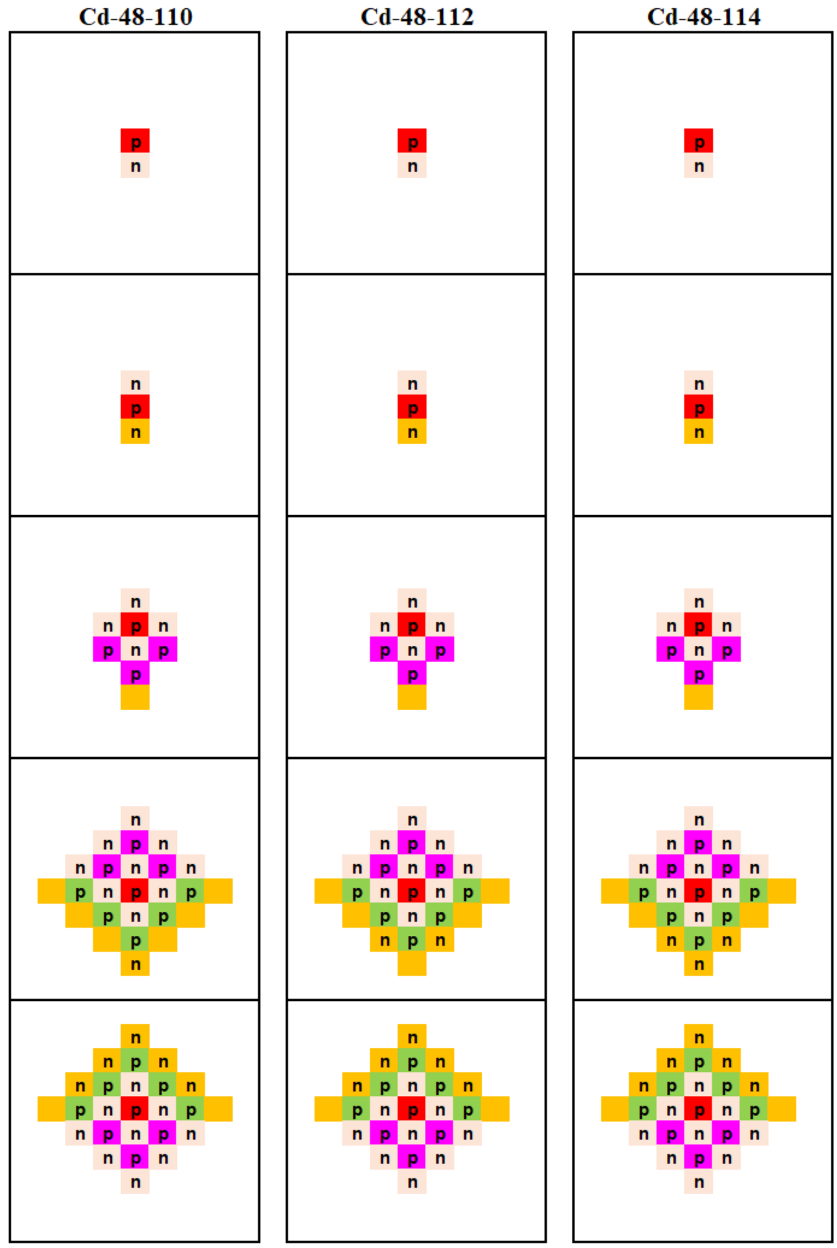
s *sub-orbital*, ***p*** *sub-orbital*, ***d*** *sub-orbital*, ***f*** *sub-orbital*

excess neutrons

Cadmium Cd_{48} – surface protons and excess neutrons

	0	13	d	d	d	d	d	d	d	d	d	d	p	p	p	s	s
		z	1	1	1	1	1	2	2	2	2	2	3	3	3	4	5
	48	x	-2	-1	0	1	2	-2	-1	0	1	2	-1	0	1	0	0
	1	y	1	2	3	2	1	0	-1	-2	-1	0	0	-1	0	0	1
		x-y-z	**0.348**	**0.339**	**0.314**	**0.339**	**0.348**	**0.334**	**0.333**	**0.312**	**0.333**	**0.334**	**0.330**	**0.330**	**0.330**	**0.250**	**0.124**
min	18	excess	1	1		1	1	1	1		1	1		1			
opt	22	excess	1	1	1	1	1	1	1	1	1	1		1			
max	22	excess	1	1	1	1	1	1	1	1	1	1		1			
min	114	nucleons															
opt	118	nucleons															
max	118	nucleons															

Cadmium Cd_{48} – stable nuclei



Legend: ***p***: *proton*, ***n***: *neutron*

s *sub-orbital*, ***p*** *sub-orbital*, ***d*** *sub-orbital*, ***f*** *sub-orbital*

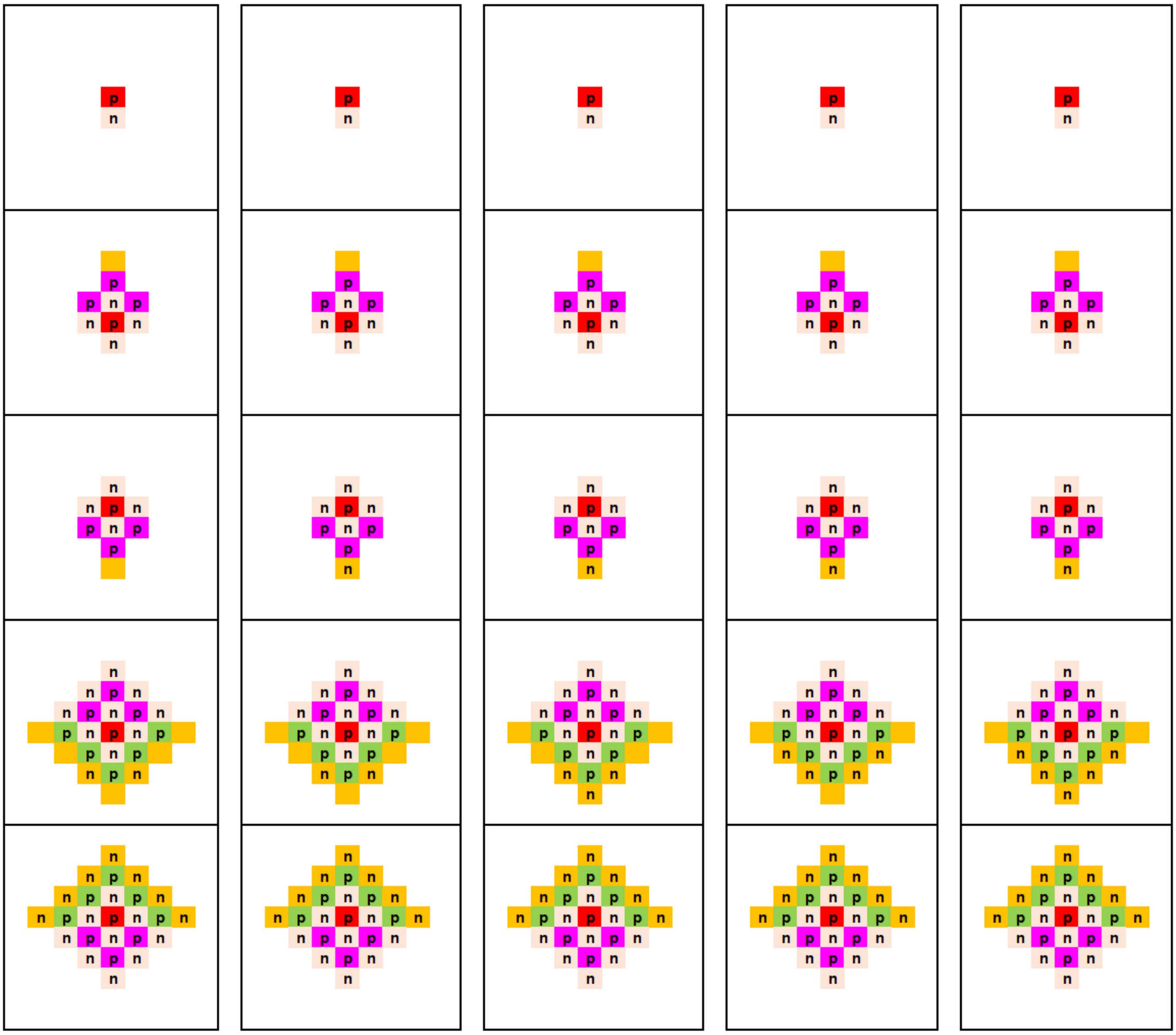
excess neutrons

Xenon Xe_{54} – surface protons and excess neutrons

	0	15	d	d	d	d	d	d	d	d	d	d	p	p	p	p	p	p	s
		z	1	1	1	1	1	2	2	2	2	2	3	3	3	4	4	4	5
	54	x	-2	-1	0	1	2	-2	-1	0	1	2	-1	0	1	-1	0	1	0
	1	y	1	2	3	2	1	0	-1	-2	-1	0	0	-1	0	1	2	1	1
x	y	sum	**0.349**	**0.340**	**0.315**	**0.340**	**0.349**	**0.337**	**0.336**	**0.313**	**0.336**	**0.337**	**0.301**	**0.334**	**0.301**	**0.312**	**0.291**	**0.312**	**0.290**
min	16	excess	1	1		1	1	1	1		1	1							
opt	22	excess	1	1	1	1	1	1	1	1	1	1		1					
max	24	excess	1	1	1	1	1	1	1	1	1	1		1			1		
min	124	nucleons																	
opt	130	nucleons																	
max	132	nucleons																	

Xenon Xe_{54} – stable nuclei

Xe-54-126 **Xe-54-128** **Xe-54-130** **Xe-54-132** **Xe-54-134**



Legend: ***p***: *proton*, ***n***: *neutron*

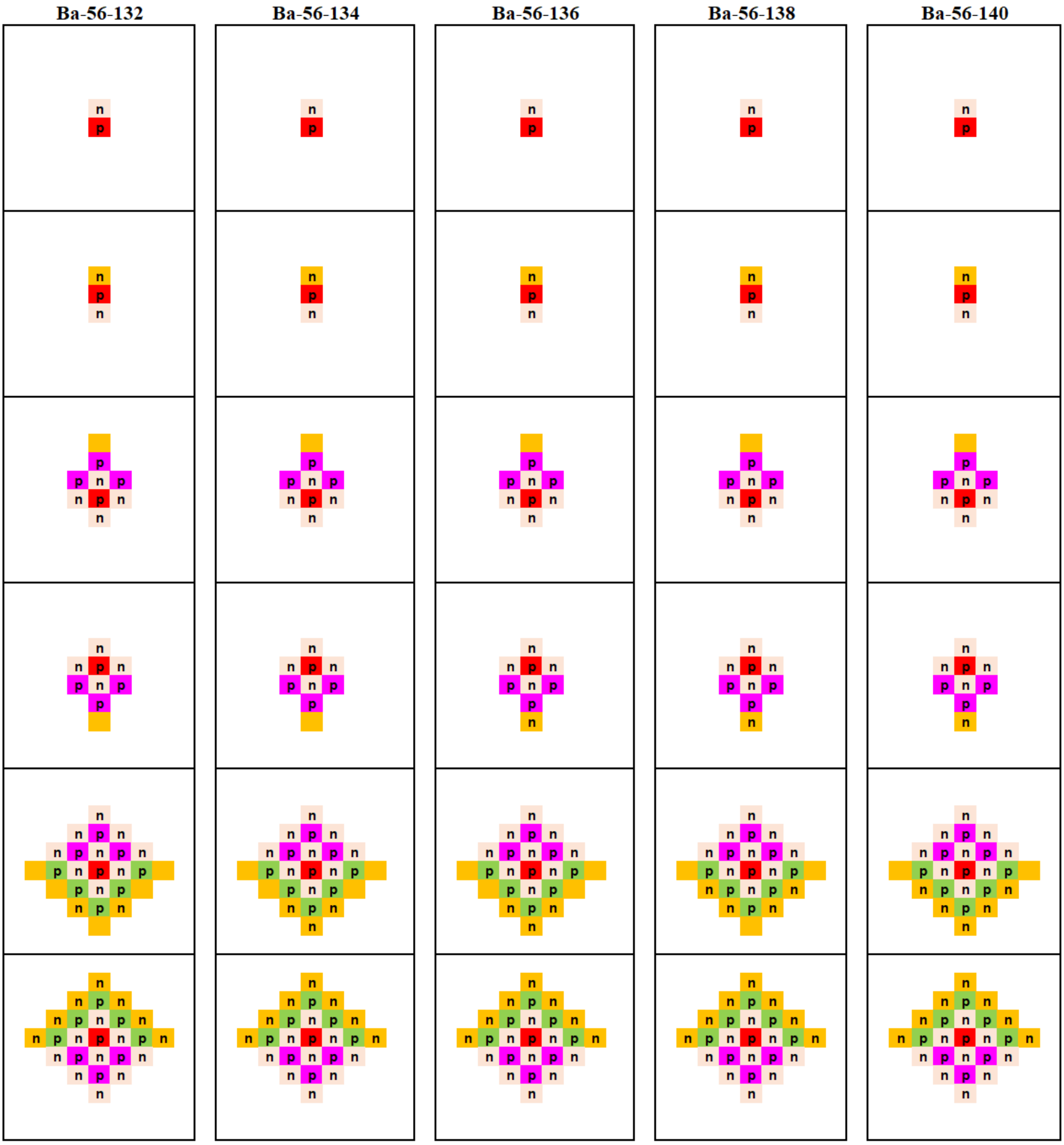
s *sub-orbital*, ***p*** *sub-orbital*, ***d*** *sub-orbital*, ***f*** *sub-orbital*

excess neutrons

Barium Ba_{56} – surface protons and excess neutrons

	0	15	d	d	d	d	d	d	d	d	d	d	p	p	p	p	p	p	s	s
		z	1	1	1	1	1	2	2	2	2	2	3	3	3	4	4	4	5	6
	56	x	-2	-1	0	1	2	-2	-1	0	1	2	-1	0	1	-1	0	1	0	0
	1	y	1	2	3	2	1	0	-1	-2	-1	0	0	-1	0	1	2	1	1	0
x	y	sum	**0.350**	**0.340**	**0.315**	**0.340**	**0.350**	**0.337**	**0.336**	**0.313**	**0.336**	**0.337**	**0.301**	**0.334**	**0.301**	**0.313**	**0.293**	**0.313**	**0.238**	**0.122**
min	22	excess	2	1		1	2	1	1		1	1		1						
opt	26	excess	2	1	1	1	2	1	1	1	1	1		1						
max	28	excess	2	1	1	1	2	1	1	1	1	1		1			1			
min	134	nucleons																		
opt	138	nucleons																		
max	140	nucleons																		

Barium Ba_{56} – stable nuclei



***Legend**: **p**: proton, **n**: neutron*

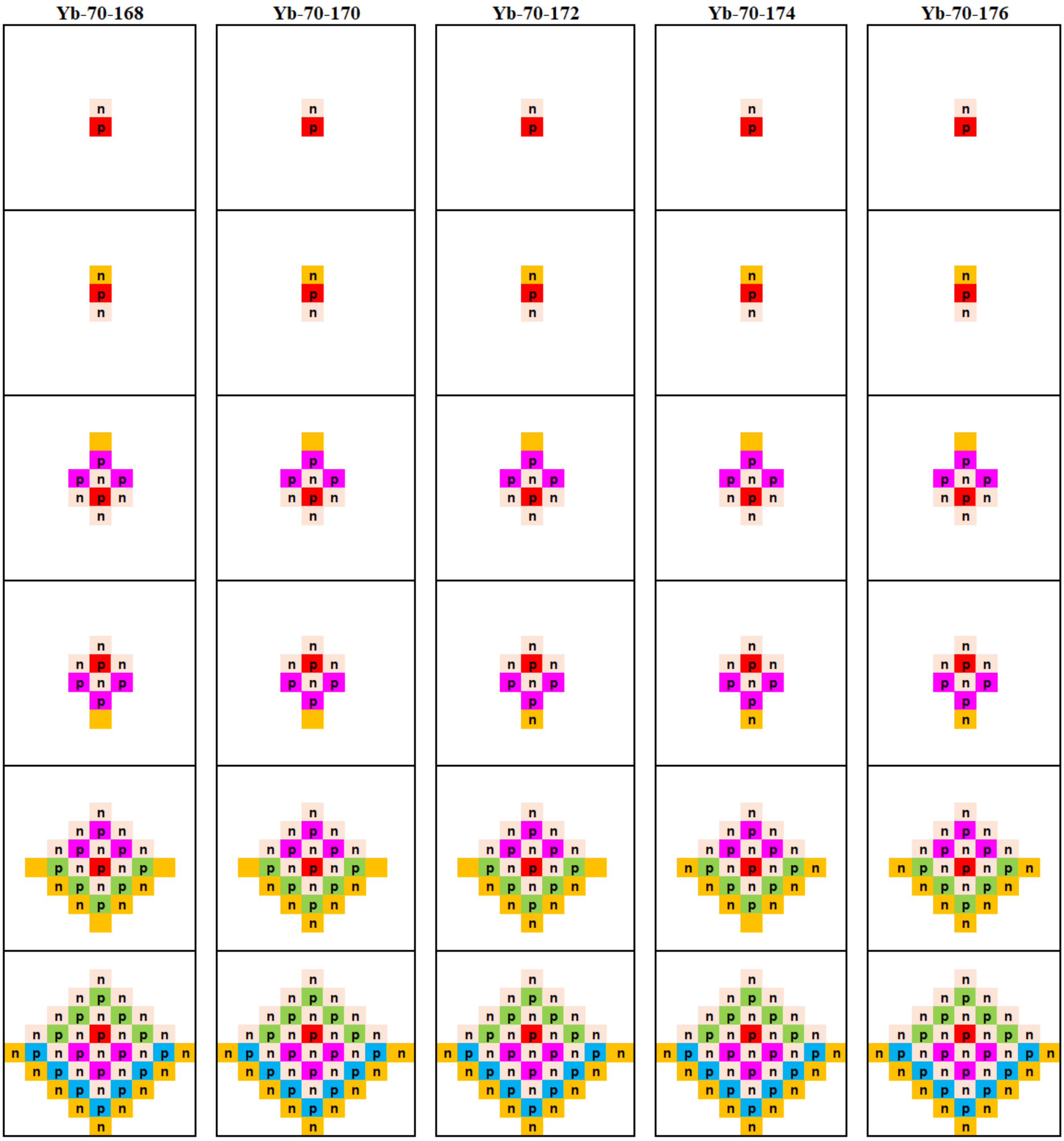
***s** sub-orbital, **p** sub-orbital, **d** sub-orbital, **f** sub-orbital*

excess neutrons

Ytterbium Yb_{70} – surface protons and excess neutrons

	0	19	f	f	f	f	f	f	f	d	d	d	d	d	p	p	p	p	p	p	s	s
		z	1	1	1	1	1	1	1	2	2	2	2	2	3	3	3	4	4	4	5	6
	70	x	-3	-2	-1	0	1	2	3	-2	-1	0	1	2	-1	0	1	-1	0	1	0	0
	1	y	0	-1	-2	-3	-2	-1	0	0	-1	-2	-1	0	0	-1	0	1	2	1	1	0
x	y	sum	**0.331**	**0.344**	**0.340**	**0.309**	**0.340**	**0.344**	**0.331**	**0.342**	**0.299**	**0.339**	**0.299**	**0.342**	**0.305**	**0.341**	**0.305**	**0.314**	**0.294**	**0.314**	**0.239**	**0.122**
min	28	excess	2	1	1		1	1	2	2		1		2		1						
opt	28	excess	2	1	1		1	1	2	2		1		2		1						
max	36	excess	2	1	1	1	1	1	2	2	1	1	1	2		1			1			
min	168	nucleons																				
opt	168	nucleons																				
max	176	nucleons																				

Ytterbium Yb_{70} – stable nuclei



Legend: ***p***: *proton*, ***n***: *neutron*

s *sub-orbital*, ***p*** *sub-orbital*, ***d*** *sub-orbital*, ***f*** *sub-orbital*

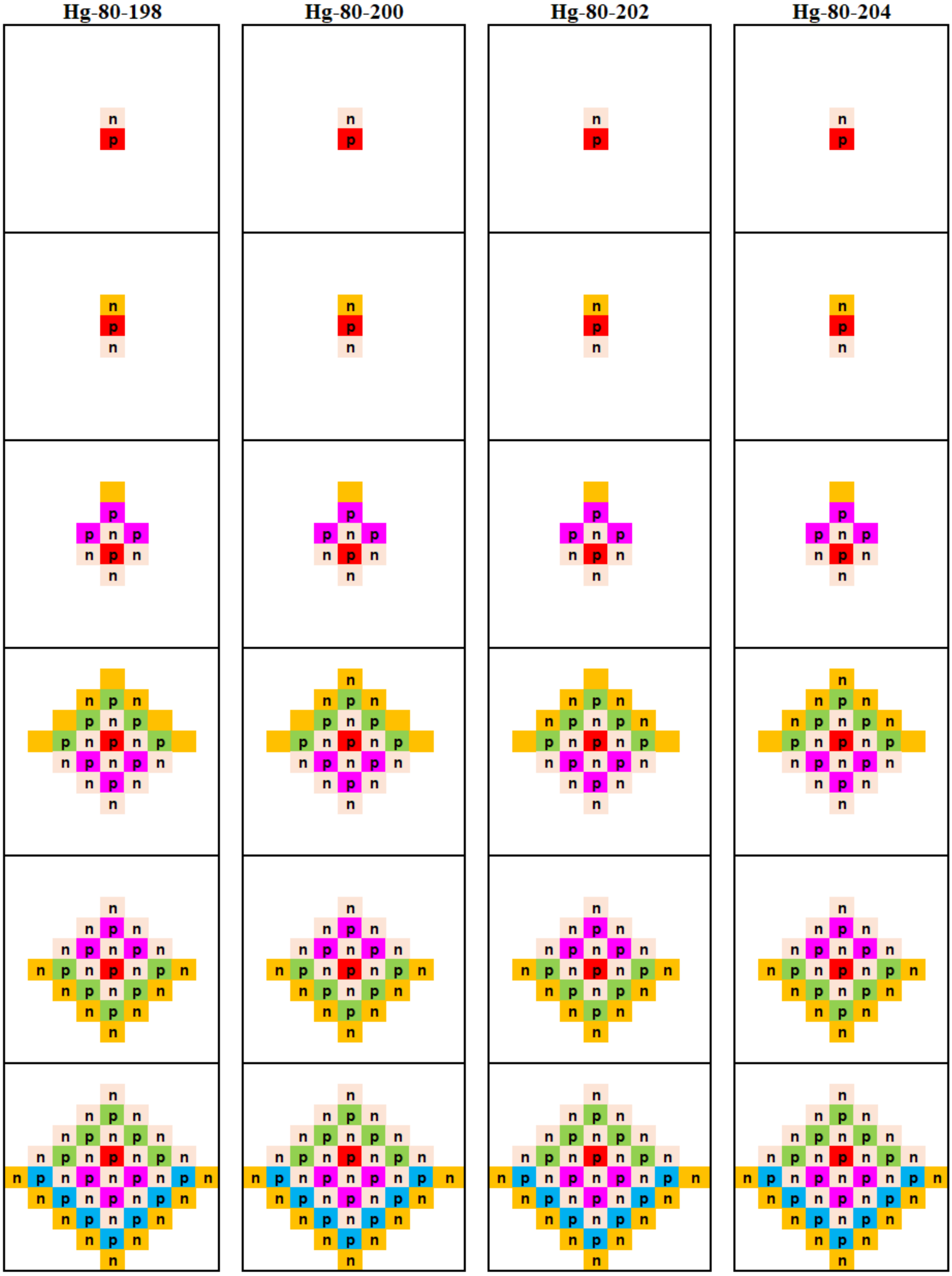
excess neutrons

Mercury Hg_{80} – surface protons and excess neutrons

	0	21	f	f	f	f	f	f	f	d	d	d	d	d	d	d	d	d	d	p	p	p	s	s
		z	1	1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	5	6
	80	x	-3	-2	-1	0	1	2	3	-2	-1	0	1	2	-2	-1	0	1	2	-1	0	1	0	0
	1	y	0	-1	-2	-3	-2	-1	0	0	-1	-2	-1	0	1	2	3	2	1	1	2	1	1	0
x	y	sum	**0.335**	**0.348**	**0.342**	**0.311**	**0.342**	**0.348**	**0.335**	**0.316**	**0.302**	**0.342**	**0.302**	**0.316**	**0.338**	**0.335**	**0.314**	**0.335**	**0.338**	**0.333**	**0.333**	**0.333**	**0.252**	**0.125**
min	32	excess	2	1	1		1	1	2			1			2	1		1	2		1			
opt	40	excess	2	1	1	1	1	1	2	1		1		1	2	1	1	1	2		1			
max	44	excess	2	1	1	1	1	1	2	1	1	1	1	1	2	1	1	1	2		1			
min	192	nucleons																						
opt	200	nucleons																						
max	204	nucleons																						

Mercury Hg_{80} – stable nuclei

Hg-80-198 | **Hg-80-200** | **Hg-80-202** | **Hg-80-204**



Legend: ***p***: *proton*, ***n***: *neutron*

s *sub-orbital*, ***p*** *sub-orbital*, ***d*** *sub-orbital*, ***f*** *sub-orbital*

excess neutrons