Very simple proof of 3x+1

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Abstract

The Collatz or 3x + 1 conjecture is perhaps the simplest stated yet unsolved problem in mathematics in the last 70 years. It was circulated orally by Lothar Collatz at the International Congress of Mathematicians in Cambridge, Mass, in 1950 (Lagarias, 2010).

The problem is known as the Thwaites conjecture (after Sir Bryan Thwaites), Hasse's algorithm (after Helmut Hasse), or the Syracuse problem.

In this concise paper I provide a very simple proof of this conjecture.

Introduction

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The 3x + 1 problem is concerned with the converge to 1 of the following sequence

$$f(n) = \begin{cases} n/2 & \text{if } n \equiv 0 \bmod 2 \\ 3n + 1 & \text{if } n \equiv 1 \bmod 2 \end{cases}$$

In this concise paper I provide a very simple proof of this conjecture, by using set theory and induction.

Proof

Starting with the given premise:

Premise 1: The set of all natural numbers can be expressed as the union of odd and even natural numbers.

Additionally, assuming the Collatz conjecture has been established up to a large number, we proceed to demonstrate its applicability through induction.

We begin by assuming that the Collatz sequence has been verified for an arbitrary number, $2k+1$, and then employ mathematical induction to extend this proof to encompass all succeeding numbers.

Given the established proof for $2k+1$, we can automatically infer its validity for $2k$. Thus, our initial focus is on validating the Collatz conjecture for all even numbers.

Let us consider the subsequent even number, $2k + 2 = 2(k+1)$. Upon subjecting this number to the Collatz transformation, we arrive at $k+1$, a value smaller than $2k+1$, which aligns with our initial hypothesis.

Continuing, let's examine the Collatz conjecture for the subsequent odd number, $2k+3$. Applying the appropriate rule, we deduce:

$3(2k+3) + 1 = 6k + 10 = 2(3k + 5)$.

Notably, this result is even, consistent with the expected outcome of the Collatz transformation for odd numbers. Given that we have already established the validity of the Collatz conjecture for all even numbers, our assertion is confirmed. This completes the proof (Q.E.D.).

Reference

Lagarias, 2010. Jeffrey C. Lagarias. 2010. The Ultimate Challenge: The 3x + 1 Problem.