The Fine Structure Constant: revisited

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Abstract

A comparison between hydrodynamics (NSE) and gauge theory vector potential gauge flow (e.g. Hopf solution of NSE), yields alpha as the Reynolds number: eddies as Feynman loops etc. It explains the QED grading by alpha and lifetimes of particles (graded by powers of alpha) as a dissipation process.

The theory of alpha can be formulated via the Schrodinger operator spectrum for Hydrogen atom and Boltzmann partition function, when related to Hopf fibration (Kepler problem on S3, magnetic topological monopole in the gauge theory formulation as an exact solution of NSE) for one loop (electronic orbital).

The computation of the fine structure constant uses finite symmetry groups corroborated with H. Jehle's loopforms model of electron (Hopf fibration with connection and vector potential flow).

The article brings together research material towards achieving such a goal. A program emerges: Physics Laws as Period Laws, and alpha an element of Pi-groups of periods.

1 Introduction

Understanding the fine structure constant $\alpha \approx 1/137$ is a test of really understanding the basic structure of matter and interactions.

There is enough material by now to do so: interpret, model and compute α.

1.1 Interpretation of Alpha

α is in fact the *Quantum Reynolds Number* of EM (flow of the vector potential), appearing in Bohr's model of the Hydrogen atom, even before the corrections needed for "fine structure" by Sommerfeld, that led to its name. It determines the change of dynamics from linear (tree-level Feynman Diagrams in QED) to turbulent (formation of loops, as analogue of eddy currents).

Continuing the mechanical-EM analogy, this is the ratio between inertial forces (electric field, divergent and capable of work) and "viscous forces" (magnetic field, responsible of curving the trajectories):

$$v_{Bohr} = \alpha \cdot c = \frac{e}{h/e}.$$

The Dimensional Analysis and Buchkingham's Pi Theorem will provide the general, abstract (formal), framework.

1.2 Quantum Amplitudes as Periods

Feynman scattering amplitudes are periods, as well as corresponding to Dessins d'Enfant, which in turn determine modes of vibration (compare with cymatics: 2D/3D and Schrodinger orbitals as "drum modes") which are Johnson solids (generalizations of Platonic and Archimedian). These can be represented as Belyi maps, as a model of how baryons "vibrate" in excited states [10] etc.

1.3 It is The Strength of Electric vs. Magnetic Forces

Yet α is much more primary: when an EM "loop" is formed, the distribution of energy in the partition function changes, with the ratio between Electric (open lines of force, divergent transformations) and Magnetic (closed lines of force, rotation transformations) α.

Recall also that *quark fields* are of EM type ($SU(2)$-gauge theory; 3 quarks form an RGB-Cartan frame).

1.4 The Mathematical Period

In another stage of this research, the author proposed the idea that α is a period, representable as a finite series graded by π [5, 22] (and in another, that it may be related to the prime zeta value at 2 [20]). Finding the reference [1], and within it [2], as a "new clue", gave a feeling of confirmation.

Indeed [12] gives:

$$1/\alpha = 137.035999084 \pm 0.000000021 \quad rel.\ uncertenty = 1.510^{-10},$$

while the "pastime-calculation" of alpha [2] selected by [1], the "period candidate" for us, gave:

$$P = \pi(4\pi^2 + \pi + frm[o]--) \approx 137.036303776, \quad P - \alpha^{-1} = 0.00030469187, \tag{1}$$

with a relative approximation error:

$$(P - \alpha^{-1})/\alpha^{-1} = 0.00000222344^{1}.$$

The running constant issue is a different, complex issue, and here we rely on Quantum Hall Effect more precise measurements (Low Energy Physics[4]), with a clearer understanding of the theory, then the HEP.

In the concluding section the plan to continue its "reverse engeneering", in terms of *periods* [19] is presented and a relation with elliptic curves is anticipated.

2 Further developments

The plan to further analyze alpha in a larger framework, is sketched below.

Recall that the previous research indicates that alpha results from finite symmetries of the Hopf fibration structure *alone* [8, 21] (E vs. M). Even the *positronium*, as an unstable exotic atom, (alternatively, an electron-positron created loop / "eddy current" in QED), exhibits the same ratio between energies (orbitals vs. "internal mass" due to interaction), as the Hydrogen atom [13], determined by alpha.

We hope that putting this material together will be helpful in the study of α.

In what follows, the theory of the Hydrogen atom is revisited, with a preliminary analysis of the solution, involving α.

3 The Hydrogen Atom

The new key approach to compute the *fine structure constant* uses the Hydrogen atom solution of Schrodinger's equation, which yields the spectrum of the Hamiltonian:

$$E_n = -\alpha \cdot \frac{1}{N(n)} \cdot E_0, \quad N(n) = \sum_{l,m} 1 = 2n^2, \quad E_0 = m_e c^2, \tag{2}$$

where n is the principal quantum number corresponding to one of the the quantum phase resonant frequancy $Z/n \to U(1)$ (finite subgroup in 2D) and the constant Boltzmann / quantum partition function for the corresponding spherically symmetric (degenerate) states (spherical harmonics: 3D finite geometries / rep of $SU(2)$):

$$N(n) = \sum_{0 \le l < n} (2l + 1), \quad \leftrightarrow \quad l \in Z/n, \quad -l \le m \le l.$$

An important point is, that, although this is a first approximation: Kepler's problem for the center of mass and equivalent mass, no spin or magnetic momentum contributions etc., it nevertheless "reveals"

[1] Relative precission: two in a million!

what the fine structure constant is, in a geometric and quantum framework (gauge theory hidden in Schrodinger's eq.), as a comparison with the original Sommerfeld definition for alpha demonstrates.

To motivate the reader, we include a few comments and "clues" early on in the introduction, to be substantiated later on.

3.1 The Separation of Schrodinger Equations and of the solution

Note the $SO(3)$ isotropy assumption on the central potential of Coulomb type, which rules out the effects of the quark structure of the proton.

As a bound state without external interactions, SE separates "time" from "space" (non-relativistic regime), and then the "radial" vs. angular factors (polar decomposition of conformal group structure).

3.1.1 Relativistic vs. Non-relativistic

This also explains why the non-relativistic model is "good enough" to exhibit the role of alpha: one loop (toroidal structure due to the Hopf fibration), yet laminar flow (no additional "eddy currents": magnetic vortices).

A bound system, like the Hydrogen atom, can be modeled as non-relativistic, when the binding energy (here EM: 13.6 eV) is small vs. "internal energy", i.e. mass of electron $511000 eV$.

3.1.2 The Hierarchy of Structure

The Hydrogen atom is an irreducible system (if you want it to function in EM regime; like an electronic oscillator, with RLC and transistors), with a hierarchy of structure. The 2nd level of structure, i.e. the proton with quarks and their fields is approximated via electric charge and rest mass, which dominates, hence a non-relativistic model is quite adequate[2].

We should expect a laminar flow in the NSE / Reynolds constant viewpoint: "inertial forces", here due to EM binding, dominating, when compared to "viscous forces", due to quark fields interaction corrections. Yet the EM flow is closed, as an orbital, hence a "topological eddy current" is formed. This is a "preliminary" justification for Reynold's number "borderline" case, here the fine structure constant:

$$E_{n=0} = \alpha E_0.$$

See also [5] for additional insight into the role of alpha as a ratio of impedances: $\alpha = Z_{free}/Z_{bound}$.

3.2 Electron orbital and Proton Quark Field

From above we see that we cannot understand α only from this first level of structure of the Hydrogen atom: we have to understand *the baryon field of the proton, which generates the electron orbital as in a condenser between two plates*[3].

This is where the Hopf bundle and finite Platonic groups enter the picture, following H. Jehle's intuition regarding the electron's toroidal model.

3.2.1 EM-Mechanic analogy revisited

As it is well known, the meaning of alpha as a ratio of electric charge (source of electric, work capable, force field) and (quantized) magnetic charge (fluxon h/e, as a unit of "curvature" / monodromy of the EM connection):

$$\frac{e}{h/e} = \alpha c, \quad \textit{Bohr model} : \alpha = v/c,$$

also corresponds to a linear momenta ratio (m_e would cancel in the above form from the right: ratio of mechanic momentum (inertial / external configuration space picture) to Compton momentum $m_e c$ (internal, gauge theory model).

[2] See the more complicated relativistic Sommerfeld model, for comparison.
[3] Too tempting to think of Casimir effect here ...

4 Analysis of the Formulas

We call α^2 the *the Bohr structure constant*, since it is a ratio (invariant) of the electron-proton structure of the fields (orbital vs. quark structure; associated to the masses of electron and proton, as other unexplained parameters):

$$E_E/E_M = \alpha^2 N(n)$$

. We will explore a few directions towards modeling ...

Here $N(n) = 1/2n^2$ is a factor disclosing the number of (degenerate) states for geometries with frequancy" n, of the electronic cloud". The $l \in Z/n$ denotes the possible (finite) orientations relative to a given direction of the angular momentum; or possibly a relative quantum phase of spinning vs. whirling in a toroidal model ofo the orbital (winding number for Hopf fibration; Hopf index).

For each l the $2l+1$ projections of the magnetic vector (spin? $S = eB$) reveals also the finite Klein geometry involved (of Platonic / Archimedian type).

Hence α is a separate issue" (electron vs. proton structure) from the finite geometry interpretation, associated with the numeric integral factor $N(n)$ (possible finite geometries).

4.1 The "Fine structure" Constant

The usual meaning of alpha is related to the fine structure, removing the degeneracy (quark field / magnetic and spin related) [13], §5.2, p.165.

There, $\alpha^2/N(n)$ factor is involved, in a correction of the *relative energy levels*, regarding *transitions*:

$$E_{photon}/E_0 = \alpha^2(\frac{1}{N(n_f)} - \frac{1}{N(n_i)^2}),$$

i.e. a redistribution of the same energy (conservation) among a different number of *equiprobable states* (regular solid" / configuration space symmetries).

Or rather think a geometry of the orbital with possible orientations in a magnetic field, when studied ...

4.2 Fine structure and Quality Factor

Note that the fine structure energy levels:

$$\Delta E_{fs}/m_0c^2 = (\alpha^2/2n^2)^2 \cdot M(j), \quad N(j) = \frac{2n}{j+1/2} - \frac{3}{2}.$$

This is a different issue, looking like a quality factor (timelife related: $\tau = 1/Q$):

$$"Q" = \frac{\Delta E_{fs}}{E_n} = \alpha^2 \cdot M(n,j).$$

It looks like a *resonant state*, with a certain *lifetime*, like for elementary particles called *resonances* (3D-cymatics of of baryon nodes and meson channels[4]), but it is a perturbation due to a *refinement* of the geometry involved (Hopf fibration vs. split extension $S^1 \times S^2$? TB addressed later on).

Note that in this ratio E_0 (rest energy as a comparison) and the number of principal level" n cancel.

4.3 What is M(n,j)?

The multiplicity factor in the fine structure of energy is:

$$M(j) = \frac{4n-3}{2j+1}, \quad j = l+s, \quad s = \pm 1/2.$$

Here $j = l + s$ is total angular momentum of the electron (orbital plus spin).

$2j+1 = dim(V_j)$, the corresponding irreducible rep of $SU(2)$ within the tensor product for V_l and V_s.

$d = 2(2n+1) = 4n+2$ is the number of directions for n, as if it is an angular momentum times the two spin orientations (as directions); but why we have $4n-3 = d-5$ ... not sure why it is there ...

The spin of the proton is a total spin of three quarks ...

[4] In a previous article, a unification between leptons and mesons was proposed: lepto-meson channels [11].

4.4 The perturbation series

Tentatively, the structure of the energy of the (n, l, m, s) state is:

$$E = E_0 + E_0\alpha^2 c_1 + E_0\alpha^4 c_2 ...$$

where c_i are the corresponding integral factors ("combinatorial" / dimensions of reps and eigenspaces), and a constant term E_0 was added ("rest energy"?).

5 Multi-facets of Alpha

5.1 A bit of history

The famous constant approximately 1/137 (at low energies)occurs in several quite different contexts, from high energy physics, atomic physics or low temperature physics, e.g. quantum Hall effect.

Since it involves both quantum constants and emergent space-time constants (speed of light), it has several interpretations, some involving a mechanic analogy, others pure EM.

Previous attempts of the author to understand it [3], led to a reasonable bulk" of material, which in principle should be enough to establish the missing link with the core theory underlying its origin: finite groups of symmetry and the Hopf bundle, the structure underlying gauge theory of EM and quarks fields of a baryon.

5.2 Alpha for a bound state

The original *fine structure constant* introduced by A. Sommerfeld in refining Bohr's model of the hydrogen atom, defined $\alpha = v/c$, based on the mechanical analogy used at that date: speed of electron relative the speed of light. The accepted standard form" of the fine structure constant, in MKS units [6], is:

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c.}$$

The Bohr model of the atom, improved by Sommerfeld, uses space-time variables, which are not observable. One may switch to a pure EM formulation, using the general momemntum $P = mv + eA$ (later on).

5.2.1 ... and QED

There are various other places where it occurs, with slight variations regarding what that relevant constant is, e.g. in QED:

$$g_{QED} = \sqrt{4\pi\alpha} \approx 0.303.$$

Here g_{QED} is a coupling constant grading the EM processes by the number of loops; hence it is measure of degrees of freedom introduced by a loop vs. a free propagator" (tree level).

5.3 Electric/Magnetic strength ratio

From quantum Hall effect physics, alpha is essentially $e/(h/e)$, the ratio between electric charge and quantized magnetic "charge" (source of "vorticity"/monodromy: fluxon).

In this area of study elements of Quantum Circuit are used, and the corresponding variables: L, C, ω etc. Hall conductance or von Klitzing constant are tightly related with α.

So far the "best" interpretation of alpha seams to be its interpretation as a proportionality between electric and magnetic impedance of the vacuum (in "free space") and in bound states [5, 6]. This interpretation is in nned of a Math-Physics model, which was initiated by H. Jehle, and can be completed by considering the finite structures of the Hopf bundle $U(1) \to SU(2) \to S^2$, as explained by the present author.

5.4 Analogy with: Pi

The "real" numbers corresponding to real quantities and experimental measurements are the so called *periods*: integrals of rational functions over algebraic domains.

The main reason is that measurements reflect resonant states of systems consisting in discrete structures ("world is quantum").

A prototypical example of a "fundamental constant" in mathematics is π, or arguibly 2π, the ratio between the bound symmetric geometry, the circle and its linear scale, its radius.

It seams that all other periods can be represented as a series graded by π. A similar behaviour is exhibited by α; not only it is a grading in QED corresponding to number of loops, but also in EPP [9]: the lifetimes of elementary particles obey a grading in powers of alpha.

This suggests that the finite Platonic geometries underlying these states (TOI subgroups of $SU(2)$) are directly correlated to alpha; hence the reverse, alpha should be computable from such geometries.

5.5 Jehle's work and Hopf bundle

This was crudely attempted by H. Jehle (see the dodecahedron use to estimate the number of loopform fibrations $N \approx 207$ [8]). A rigorous method is to quantize the Hopf fibration: Platonic groups of the base S^2 combined with "face" frequencies, which correspond to quantum phase $exp(i\omega t)$, the subgroups Z/n of S^1.

5.6 3rd Quantization

We aim to show that alpha expresses the ratio of number of basic states in a finite symmetry groups approach to the SM.

The use of finite groups of symmetry led to progress in the SM since the 1990s, allowing to explain the three generations of fermions, what quark flavors are, allowing to compute the CKM and PMNS matrices and Weinberg angle [7].

5.6.1 Everything is (must be) Quantized

The fact that finite groups control de SM is a natural conclusion from a final "leap" in the 100 years of "quantization": reality is quantum, locally finite. This requires not only the "quantization of the qubit space $S^3 \cong SU(2)$, but also of its symmetries: Mobius transformations $SL_2(Z) \to SL_2(C)$. This final step is also suggested by the connection with modular curves approach to modeling weak decays [10].

5.6.2 Quantization of Angular Momentum

As a direct consequence, in a "particle" interaction there may be only finitely many relative directions to consider: angular momentum results as quantized.

In this direction it is worth analyzing the Platonic Bell inequalities [24], and apply to a Platonic model of baryons (quark model with generations corresponding to the Platonic groups of symmetry: TOI), conform work by F. Potter and the author.

This avenue of developing the Standard Model also suggests a relation between spin and flavor (wave functions) for baryons. It is also consistent with the H. Jehle toroidal model of fermions and quarks (baryons), when developed based on the Hopf fibration.

5.6.3 What Geometries are involved?

The Johnson polyhedra model the "vibrations" (excited states) of a baryon. When considering a meson interaction, e.g. as a nuclear bond between two nucleons, possibly higher genus Riemann surfaces are involved. The finite aspect would correspond to Platonic tessellations and Belyi maps and morphisms.

6 Conclusions and Further Developments

The fine structure constant is in fact the EM-Reynolds Number, and a ratio between Electric and Magnetic "strength" (repartition of energy in the partition function, between divergent / work states / potential and rotation / curvature "energy" - mass).

The Hydrogen atom model exhibits alpha as suggested above.

6.1 A Systematic Search for The Period

A period formula for α seems to have been found.

Further study will address an interpretation of α in terms of Hopf fibration and de Rham periods, and its relation to dimension analysis.

Further developments will address the following:

1) Dimensional analysis and Backingham Pi Theorem [25];

2) Geometric - Period analysis of the above formula, or perhaps better, of the "Coulomb/QED form" (the RHS as a quadratic form):

$$4 \times \frac{1}{4\pi\alpha} \quad \leftrightarrow \quad 4\pi^2 + \pi + 1 = N(\pi),\ N(z) = 4z^2 + z + 1^5.$$

3) Comparison with "loopforms" of H.Jehle, theory which in fact is a *Loop Quantum EM* (not Gravity). An relation between α and EC is expected;

4) A comparison with the *finite modes* due to Platonic (3D) and Hyper-Platonic (4D) symmetries and Klein Geometries. Note that $S^1 \to SU(2) \to S^2$ is a homogeneous space (Klein Geometry / Cartan), $SU(2) \cong S^3$ and the Hydrogen atom has $SO(4)$ extended symmetries (that of S^3).

6.2 ... and Elliptic Curves

We will include a few ideas and comments, to be developed later on.

6.3 Electric vs. Magnetic Energy (Mass)

From Equation 2 §3, the total "electric energy" (due to assuming the Coulomb law field at this stage), over the magnetic energy, which presumably sums up the energy of all internal states, is (discarding the negative sign):

$$\frac{\sum E - n}{E_0} = \alpha \frac{1}{2}\zeta(2), \quad \zeta(2) = \pi^2/6.$$

6.4 Elliptic Periods

The toroidal model of the EM field of "elementary particles" is established by now (HJ, Hopf fibration, QC and many more sources). Tentatively we rethink the quadratic factor in the Equation 1 as follows:

$$4\pi^2 + \pi + 1 = (2\pi)^2 + \frac{1}{2}(2\pi) + 1 = (2\pi)^2(1 - \frac{\omega_1}{\pi})(1 - \frac{\omega_2}{\pi}),$$

with the "EC periods" ω_i solution of:

$$z^2 - \frac{1}{2} + 1 = 0, \quad z_i = 1/4(1 \pm i\sqrt{15}).$$

[5] Or $N(z) = 4\pi^2 + \pi z + z^2, N(1)$ etc.

6.5 Mass and The Monster Group

Obviously there are are better "guesses" to relate to the periods of meaningfull EC, and relate to F. Potter's observation that the fermion masses (e, muon, tau) seam related to the j-invariant, as Klein defined to understand the quintic (icosahedron symmetry as the 3rd generation symmetry group) [10].

Indeed, the *Legendre relation* between the periods of an EC can be written as (see Wiki [26] for further details on quasi-periods etc.):

$$det(\omega\ \eta) = 2\pi i, \quad \omega = (\omega_1, \omega_2), \eta = (\eta_1, \eta_2),$$

providing support for the above tentative of reinterpreting the quadratic form.

In other words, α could be an algebraic number "extension" of a period, or maybe π^3/α ... [6]

6.6 Physics Laws as Period Laws

More importantly, a growing amount of evidence (Feynman amplitudes as periods, corresponding to dessins d'enphant, which correspond to Belyi maps as a model for baryon modes and flavor geometries etc.) suggests that Physics Laws as Period Laws, in the sense of de Rham period matrix. This was suggested as early as 1977 by Post [14], based on work done by Kiehn (see ref. cit.).

6.7 Physics as Mathematics

The general idea is that Buckingham's Pi Theorem leads much deeper:Natural units are related to the pi-group of dimensional constants which *are periods*, and Physics can be rewritten *not in geometric units* by forcing $h = 1, c = 1$ etc. but rather in *cohomological natural units* (de Rham periods) [17].

The role of dimensional constants are reminiscent of projective representations, and of quantum periods (Berry phase, wilson loops, central extensions etc.).

Then Stasheff's *Cohomological Physics* [31], would become the "King" joining Gauss's Queen of Mathematics: Number Theory ...

The circle would close, and "Number (does) Rule the Universe" and God only had to "give us" the prime numbers ... The author still believes that α is somehow related to the prime zeta function [20], and since the Riemann zeros are in fact, experimentally, dual to the prime numbers [28], they should play a role in the explanation of *alpha*, in regards to the structure of $E_0 = m_e c^2$, related to the spectrum of the baryon (counting Hopf fibration finite gauge fields, or so [23])[7].

Whether we compute alpha sooner rather then later, the program is to rewrite the foundations of Quantum Physics as Period Laws, as for instance the "split" of the quantum of action in "electric" and "magnetic" coupling constants (see Post and Kiehn for the spin 3-form):

$$h = e \cdot g, \quad \frac{v_B}{c} = \alpha = \frac{e}{g}.$$

6.8 Chern-Simons Theory and Langlands Program

The spin 3-forom of Kiehn, used by Post, is related to the CS-Theory [27] (to be investigated), perhaps as:

$$\oint A \wedge F, \quad F = dA + A \wedge A^{8}$$

This leads to even much deeper questions, related to Montonen-Olive duality, S-duality etc. and Langlands program [30, 29], justified by the above "Program" in Phsyics ... But keeping the "bridge" open between meaningfully, pictorial Physics and Mathematical summits (beyond the clouds / veil for most of us [15]) is important.

[6] A pastime puzzle for the Free Mathematician - paraphrasing [2] ...

[7] Incidentally, that might have to do with rethinking real numbers as sequences of modular Mobius-Lorentz group $SL_2(Z)$ [16] ...

[8] Not clear why 2/3 ... for now.

A Kepler Problem and Alpha

The Bohr model of the hydrogen atom is the Kepler problem for a central conservative force (pointwise singularity with a harmonic potential, source of an "electric" force (capable of work) with the Sommerfeld quantization condition $\oint_C pdq = n\hbar$, i.e. the symplectic potential is quantized (monodromy $2\pi\hbar$ - analog to a Cauchy residue for dz/z).

A.1 Coulomb Law

The Coulomb law is uniquely determined by the condition that the force is (instantaneous) and central (pointwise sourse) conservative (no "leaks" in other forms, dimensions etc.). If the two charges orbit one around the other:

$$F = e^2/r^2,$$

in appropriate units (charge and distance), determining the unit of force.

The $1/r^2$ inverse square law is mandated by the fundamental solution of the Poisson equation $\Delta u = 0$, in vacuum (although the "singularity $r = 0$ has a mass" in this classic setup, hence the "vacuum" is everywhere else).

e^2 assumes the two particles have the same charge magnitude, but of opposite signs, while assuming the law of signs[9] e.g. the *positronium problem* [13], e^+ and e^-.

A.2 Bohr or Sommerfeld quantization

Now assuming the quantization condition we are in the position of deriving Bohr energy levels:

$$E_n/E_0 = \alpha \frac{1}{2n^2}, \quad E_0 = mc^2.$$

Another way to think about this is:

$$E_{Electric} = \alpha E_{magnetic}, \qquad E_{electric} = 2n^2 \cdot E_n,\ E_{magnetic} = m_0 c^2,$$

where $E_e lectric$ is the total energy of all $2n^2$ states in the partition function, each of energy E_0, with equal probability (depending on l angular momentum and magnetic projection: [13], Ch. 5).

We intuitively know that "mass" is of magnetic origin, needing the full description of the 3-quarks with fields of EM type and gluons, the analog of photon, mediating the EM-type of interactions.

But note that in the positronium example, only the electron's mass enters the computations, hence, α is characteristic of EM only, the ratio between the *electric energy/mode* (open / divergent lines) and *magnetic energy / mode* (closed loops)[10].

Hence the above relation confirms that alpha is in fact a ratio of the number of modes, radial (2D-Z/n) vs. angular.

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