The Condition for the Real Part of Dirichlet Function to be 1/2

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Abstract

Find the trigonometric function sin(n/2)π that satisfies the Dirichlet feature, and then analyze the conditions for making the real part of the L-function 1/2.

Keywords

Exceptional zero/Dirichlet feature

We know that the Dirichlet feature is a function $\chi(n)$ defined on an integer with the range of C, and now we have found a trigonometric function $\sin\frac{(n)}{2}\cdot\pi=\chi(n)$, then:

1) $\chi(n+N)=\sin\frac{(n+N)}{2}\cdot\pi$ = $\chi(n)=\sin\frac{(n)}{2}\cdot\pi$ (N is an even period constant)

2) $\chi(1)=\sin\frac{(1)}{2}\cdot\pi=1$

3) $\chi(nm)=\sin\frac{(nm)}{2}\cdot\pi$ = $\chi(n)\chi(m)=\left(\sin\frac{(n)}{2}\cdot\pi\right)\left(\sin\frac{(m)}{2}\cdot\pi\right)$ (n and m are are integers)

4) $\chi(x)=0$ $(\gcd(x,N)\neq 1)$ $\Rightarrow \chi(x)=\chi(d+e\cdot i)$

Let : $x=d+e\cdot i$ $\Rightarrow \chi(x)=\chi(d+e\cdot i)=\sin\left(\frac{d}{2}+\frac{e}{2}i\right)\cdot\pi=0$ ($x,d,e\in C$)

So[2] : $d/2=1/2+k \Rightarrow d=1+2k$ $\Rightarrow \chi(x)=\sin\left((1/2+k)+\frac{e}{2}i\right)\cdot\pi=0$ (k is an integer)

It can be seen that $\sin\frac{(n)}{2}\cdot\pi$ satisfies all four conditions, and it is a Dirichlet feature

The Dirichlet L-function is: $L(s,\chi)=\sum_{n=1}^{+\infty}\frac{\chi(n)}{n^s}$

When n is an integer ,then $\chi(n)=\sin\frac{(n)}{2}\cdot\pi=1$ or $L(s,\chi)=0$, these integer points are bothethe trivial zeros of the L-function and the Riemannian Zeta function; When $x=1+2k+e\cdot i=s/2$ and $2k=0$, then $x=1+e\cdot i=s/2$ and $L(s,\chi)=0$, these points on the complex plane are both non trivial zeros of the L-function and Riemannian Zeta function.But when $2k\neq 0$, $x=1+2k+e\cdot i$,it is not a non-trivial zero of the Riemannian Zeta function but a non-trivial zero of the L-function.

Conclusion:

when $2k=0$,then $\chi(x)=0$,the real parts of the non trivial zeros of the L-function and the Riemannian Zeta function are both all $1/2$. The Riemann hypothesis holds, but the Zero point conjecture is not completely true.

References

[1] Riemann,On the number of prime numbers less than a given quantity, Monatsberichte der Berliner Akademie, November, 1859.

[2] Xiaohui Li ,Proof that the Real Part of All Non Trivial Zeros of Riemann Zeta Functions is 1/2,2023-08-01