Any even number greater than 6 can be written as the sum of two prime numbers

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Abstract：

The so-called strong Goldbach conjecture, which means that any even number greater than 6 can be written as the sum of two prime numbers, is also known as the "strong Goldbach conjecture" or the "Goldbach conjecture about even numbers".

This paper utilizes the basic principle that the number of all odd numbers in the positive integer remains constant, and the sum of all odd numbers remains constant, and the values of odd numbers are not equal to each other. It is found that there are both odd prime numbers pr1 and pr2 in the [3, n] and (n, 2n-2) intervals, respectively. Then, the equivalent transformation is performed by using the principle that the number of all odd numbers, the sum of all odd numbers remains constant, and the values of odd numbers are not equal to each other, thereby proving that 2n=pr1+pr2.

Keywords:

Prime/Even/Odd

We know:

A prime number refers to a natural number that has no other factors than 1 and itself among natural numbers greater than 1.

Even numbers refer to integers that can be divided by 2.

Odd numbers refer to integers that cannot be divided by 2.

In the interval of $[1,2n]$, a positive integer is: $n(n > 0, n \in N^+)$

So：

The mathematical expression for even numbers is: $2n$

The mathematical expression for odd numbers is：$2n-1$

We know that in the integer interval $[3,n]$,when $n > 3$,according to the prime number theorem: $\pi(n) \sim n/\ln(n) > 2$, there must be an odd prime number in the interval $[3,n]$, we can set it as:

$$pr_1 = n - k_1 \quad (k_1 \in N^+, 0 < k_1 < n)$$

Meanwhile, according to the Bertrand Chebyshev theorem, when $n > 3$, in the interval $(n,2n-2)$, there is at least one odd prime number, we can set it as:

$$pr_2 = n + k_2 \quad (k_2 \in N^+, 0 < k_2 < n)$$

We know that in the integer interval $[1,2n]$, the number of all odd numbers and the sum of all odd numbers remain constant. Therefore, it is easy to see that the number of all odd numbers is n, the sum of all odd numbers is $s=\sum_{n=1}^{+\infty}(2n-1)$, and the set of all odd numbers can represented by S as: $S=\{1,3,5...pr_1...pr_2...(2n-1)\}$

so the sum of all odd numbers can be expanded into the following expression:

$$s=\underbrace{1+3+5+...pr_1+...+pr_2+...+(2n-1)}_{n} \tag{1}$$

We can find that any odd number in set S can be represented by a universal expression, which is:

$$(2x_i-1)\ \left(1\le x\le n, x\in N^+\right)$$

So the expansion of the sum of all odd numbers $s=\sum_{n=1}^{+\infty}(2n-1)$ can be expressed as follows:

$$s=\underbrace{(2x_1-1)+(2x_2-1)+(2x_3-1)+...+(2x_i-1)+...+(2x_n-1)}_{n} \tag{2}$$

Since both pr_1 and pr_2 are odd prime numbers, then: $pr_2-pr_1=k_1+k_2$, then the value of (k_1+k_2) must be even, meaning that k_1 and k_2 are both odd or even, then the value of (k_2-k_1) must also be positive even or negative even or zero, and $|k_2-k_1|<n$.

Now let me analyze the value of k_2-k_1:

We have concluded that the value of k_2-k_1 must be positive even or negative even or zero. Therefore, an odd or prime number can be found in the set S, with a value of $[(2x_i-1)+(k_2-k_1)]$ and $[(2x_i-1)+(k_2-k_1)]\in S$. Therefore, the expansion of the sum of all odd numbers $s=\sum_{n=1}^{+\infty}(2n-1)$ can be equivalently transformed into:

$$s_1=\underbrace{(2x_1-1)+(2x_2-1)+(2x_3-1)+...+[(2x_i-1)+(k_2-k_1)]+...+(2x_n-1)}_{n} \tag{3}$$

It is not difficult to see from equation (3) above that if $k_1 \neq k_2$, then $s_1 \neq s$, which is obviously contradictory, we can immediately conclude that:

$$k_2 - k_1 = 0$$

Now let's set the integer $k = k_1 = k_2$, so the prime numbers pr_1 and pr_2 can be represented as:

$$pr_1 = n - k$$

$$pr_2 = n + k$$

From the above, we can conclude that:

Any even number $2n(n > 3, n \in N^+)$ can be represented by the sum of prime numbers pr1 and pr2, namely:

$$2n = pr_1 + pr_2 = (n-k) + (n+k)$$

Conclusion: Any even number greater than 6 can be written as the sum of two prime numbers, that is, the strong Goldbach conjecture holds!

References

[1] Pan Chengdong, Pan Chengbiao. Elementary Proof of the Prime Number Theorem. Shanghai: Shanghai Science and Technology Press, February 1988, 1st edition: 1-8

[2] Riemann,On the number of prime numbers less than a given quantity, Monatsberichte der Berliner Akademie, November, 1859.

[3] Xiaohui Li ,Proof of the Goldbach's Conjecture.http://vixra.org/abs/2307.0158，2023-07-29

Comment:

Some one may find that:

Similarly， an odd or prime number can be found in the set S, with a value of $[(2x_i - 1) + (k_2 + k_1)]$ and $[(2x_i - 1) + (k_2 + k_1)] \in S$

According to the method of this paper,so the value of $k_2 + k_1$ must be zero, that is： $k_2 + k_1 = 0$,

then: $k_2 = k_1 = 0$, or $k_1 = -k_2$

Obviously, in the $[3, n]$ and $(n, 2n-2)$ intervals, when k_1 and k_2 are both zero or $k_1 = -k_2$, it is not possible to find that both pr_1 and pr_2 are prime numbers.