Exact Sum of Prime Numbers in Matrix Form

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Abstract

This paper introduces a novel approach to represent the nth sum of prime numbers using column matrices and diagonal matrices. The proposed method provides a concise and efficient matrix form for computing and visualizing these sums, promising potential insights in number theory and matrix algebra. The innovative representation offers a new perspective to explore the properties of prime numbers in the context of matrix algebra.

1 Introduction

Prime numbers and matrices are two fundamental mathematical concepts that have profound implications and applications in various fields across the world. Prime numbers, the building blocks of arithmetic, have fascinated mathematicians for centuries due to their unique properties and importance in number theory. On the other hand, matrices, defined as arrays of numbers arranged in rows and columns, serve as powerful tools for representing and manipulating complex data and transformations in diverse disciplines.

In this context, this paper explores the significant roles that prime numbers and matrices play in different aspects of the world. It delves into the practical applications and theoretical significance of both concepts, highlighting their contributions to fields like cryptography, computer science, engineering, data science, quantum mechanics, and more. By understanding the interplay between prime numbers and matrices, we can gain valuable insights into their combined potential and impact on modern society.

Through an exploration of real-world examples and theoretical foundations, this paper aims to showcase how these two seemingly distinct mathematical concepts converge to create a profound impact on technology, science, and various practical applications. By revealing their applications and connections, this study endeavors to emphasize the importance of prime numbers and matrices in advancing knowledge and addressing complex challenges faced across different domains. Ultimately, the integration of prime numbers and matrices stands as a testament to the richness and versatility of mathematics, paving the way for further exploration and innovation in the world's ever-evolving landscape. [1][7][3]

Definition of Matrix.

A matrix is a mathematical concept used to organize and manipulate data or mathematical elements. It is represented as a rectangular array of numbers, symbols, or expressions arranged in rows and columns. The size of a matrix is denoted by "m x n," where "m" represents the number of rows, and "n" represents the number of columns.[2]

A general representation of a matrix is as follows:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Definition of Row Matrix.

A row matrix, also known as a row vector, is a specific type of matrix that contains a single row of elements. It is a one-dimensional array represented as a matrix with a single row and multiple columns. The row matrix can be denoted as.

$A = [a_1, a_2, a_3 \cdots a_n]$

A row matrix is composed of elements arranged in a single row from left to right.

Row matrices play a vital role in a diverse range of mathematical operations, including vector addition, matrix multiplication, and solving systems of linear equations. Their widespread application is evident in various fields such as linear algebra, statistics, and other branches of mathematics and science, where data requires organization and manipulation in a one-dimensional structure. The horizontal alignment of elements in a row matrix provides a practical and efficient means of representing and processing data, enabling smooth computations and analyses across a wide spectrum of applications.

Theorem

If an $n \times n$ matrix is premultiplied by an $n \times n$ elementary row matrix, the resulting $n \times n$ matrix is the one obtained by performing the corresponding elementary row-operation on A.

Proof

We will prove that forming $C = E_{ij}A$ is equivalent to interchanging rows i and j of A. By the definition of matrix multiplication, row i of C has components

$$c_{ij} = \sum_{k=1}^{n} e_{ik} a_{kp} \quad 1 \leq p \leq n$$

Among the elements $\{e_{i1}, e_{i2}, \cdots, e_{in}\}$ only $e_{ij} = 1$ is nonzero.So,

$$c_{ip} = e_{ip} a_{ip} = a_{ip} \quad 1 \leq p \leq n$$

and elements of row i are those of row j. Similarly, the elements of row j are those of row i, and the other rows are unaffected.

Definition of Column Matrix.

A column matrix, also known as a column vector, is a unique matrix type with a single column and multiple rows. It arises as a special instance when the number of columns (n) equals 1, and the number of rows (m) is greater than 1. Represented in a vertical format, a column matrix consists of a series of values or data elements stacked one on top of the other.

Column matrices play a crucial role in representing and processing sets of values or data linearly. Their vertical arrangement makes them particularly advantageous in various mathematical operations, such as vector transformations, matrix multiplications, and solving systems of linear equations. This inherent versatility and ease of manipulation lead to their extensive use in diverse fields, including linear algebra, statistics, and numerous other disciplines where organized data representation and manipulation are fundamental.

A general representation of a column matrix is as follows

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$$

In this representation, $(a_1, a_2, \ldots, a_m\ )$

column matrices consist of elements organized in a single vertical column, with each element placed one below the other from top to bottom. Similar to row matrices, they hold crucial significance in diverse mathematical operations, including vector addition, matrix multiplication, and solving systems of linear equations. Various fields, such as linear algebra, statistics, and others, extensively utilize column matrices for data organization and processing in a one-dimensional structure. The primary distinction between row matrices and column matrices lies in their orientation; row matrices are arranged horizontally, whereas column matrices adopt a vertical format. This fundamental difference in orientation renders them suitable for distinct applications and mathematical operations, streamlining data representation and manipulation with efficiency..

Definition of Diagonal Matrix.

A diagonal matrix is a special type of square matrix characterized by having zeros for all elements located outside the main diagonal. The main diagonal, running from the top-left corner to the bottom-right corner of the matrix, contains the sole non-zero elements in this specific matrix format. In general, for an $n \times n$ square matrix, a diagonal matrix is represented as:

$$\begin{bmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ 0 & 0 & d_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & d_n \end{bmatrix}$$

In this representation, $(d_1, d_2, \ldots, d_n\ )$ diagonal matrices consist of non-zero elements solely on their diagonal, while all other elements outside the main diagonal are set to zero. These matrices hold considerable importance in diverse mathematical applications, particularly in linear algebra and matrix operations. One of their key advantages is their ease of manipulation during fundamental tasks like matrix multiplication and the computation of matrix powers. Furthermore, diagonal matrices frequently arise in various scenarios, including solving systems of linear equations, determining eigenvalues and eigenvectors, and diagonalizing more complex matrices. Owing to their straightforward and structured nature, diagonal matrices significantly streamline calculations and find extensive use across numerous fields, including mathematics, engineering, and various scientific disciplines.

Matrix Multiplication.

If A is an $m \times n$ matrix and B is an $n \times p$ matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}$$

the matrix product C = AB (denoted without multiplication signs or dots) is defined to be the $m \times p$ matrix

$$\mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1p} \\ c_{21} & c_{22} & \cdots & c_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mp} \end{pmatrix}$$

such that

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj} = \sum_{k=1}^{n} a_{ik}b_{kj}$$

for i = 1, ..., m and j = 1, ..., p.

That is, the entry c_{ij} of the product is obtained by multiplying term-by-term the entries of the ith row of A and the jth column of B, and summing these n products.In other words, c_{ij} is the dot product of the ith row of A and the jth column of B.

Therefore, AB can also be written as

$$\mathbf{C} = \begin{pmatrix} a_{11}b_{11} + \cdots + a_{1n}b_{n1} & a_{11}b_{12} + \cdots + a_{1n}b_{n2} & \cdots & a_{11}b_{1p} + \cdots + a_{1n}b_{np} \\ a_{21}b_{11} + \cdots + a_{2n}b_{n1} & a_{21}b_{12} + \cdots + a_{2n}b_{n2} & \cdots & a_{21}b_{1p} + \cdots + a_{2n}b_{np} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}b_{11} + \cdots + a_{mn}b_{n1} & a_{m1}b_{12} + \cdots + a_{mn}b_{n2} & \cdots & a_{m1}b_{1p} + \cdots + a_{mn}b_{np} \end{pmatrix}$$

Thus the product AB is defined if and only if the number of columns in A equals the number of rows in B.[5][4]

In our groundbreaking new prime number theory, we introduce novel representations for the nth term of prime numbers using distinct shapes and matrices. Through inventive linear combinations, we unveil fresh perspectives, ushering in a trans formative era in number theory. Our innovative approach promises to reveal previously unseen patterns and forge new connections, pushing the boundaries of mathematical understanding. This work holds the potential to revolutionize the field and inspire further exploration in prime number research. Formula of new prime number theory we can write in the form as

$$\pi(n) = 2n + \sum_{i=1}^{n-1} g_i f(c_i) \qquad f(c_i) = (n-i) \ \ [6]$$

where

$$g_i = p_{i+1} - p_i \qquad i = 1, 2, 3, 4...(n-1)$$

now we will convert in matrix form

let

$$\mathbf{B} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{n-1} \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} n-1 & 0 & 0 & \cdots & 0 \\ 0 & n-2 & 0 & \cdots & 0 \\ 0 & 0 & n-3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & n-(n-1) \end{bmatrix}$$

now we can write formula in matrix form

$$\pi(n) = 2n + AB$$

In other form

$$\pi(n) = 2n + a_{ij} b_{i1}$$

next formula is this form

$$\pi(n) = (3n-1) + \sum_{i=2}^{n-1} g_i f(c_i) \qquad f(c_i) = (n-i) \quad i = 2, 3, 4...(n-1) \ \ [6]$$

where

$$g_i = p_{i+1} - p_i \qquad i = 2, 3, 4...(n-1)$$

let

$$\mathbf{D} = \begin{bmatrix} g_2 \\ g_3 \\ \vdots \\ g_{n-1} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} n-2 & 0 & 0 & \cdots & 0 \\ 0 & n-3 & 0 & \cdots & 0 \\ 0 & 0 & n-4 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & n-(n-1) \end{bmatrix}$$

now we can write formula in matrix form

$$\pi(n) = (3n-1) + CD$$

In other form

$$\pi(n) = (3n-1) + c_{ij}d_{i1}$$

2 Conclusion

In summary, our paper introduces a groundbreaking approach that leverages column matrices and diagonal matrices to represent the nth sum of prime numbers. This novel method offers a concise and efficient way to compute and visualize these sums, showing great potential in advancing both number theory and matrix algebra. By merging these two fields, our innovative representation unveils a fresh perspective to explore the intricacies and properties of prime numbers. We believe this work will stimulate further research and lead to exciting discoveries in the realm of mathematics and its practical applications.

References

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