A Note on the Laws of Newtonian Mechanics.

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Abstract.

Here attention is drawn to the assumptions behind some very elementary results of Newtonian mechanics. It is noted that incorrect conclusions may be drawn if these are not remembered when applying or criticising these laws.

Introduction.

In recent times, questions have arisen concerning the validity of at least some of Newton's celebrated laws of motion. This has been generated by increased interest in the idea of inertial propulsion. It is the intention here to examine once again some of Newton's most basic equations of motion and try to understand if the criticism is justified but, if not, why not?

Some comments relating to Newton's laws of motion.

In the normal approach to considering motion in a straight line, it is customary these days to start by supposing the acceleration, a, constant and to write this in the form

$$\frac{d^2s}{dt^2} = a$$

with t representing time as usual.

This equation is then integrated to give the familiar result:

$$v = u + at$$

where v and u represent the final and initial speeds respectively.

However, one could easily consider the slightly different situation where v is the speed achieved from an initial speed u under the action of an *average* acceleration, a, in time t. In this slightly different situation, the average acceleration would be given by the difference between the final and initial speeds divided by the time taken; that is

$$\text{average acceleration} = (v - u)/t$$

or, purely in symbols,

$$a = (v - u)/t$$

where, in this case, a represents an average acceleration.

Hence, formally the same equation applies to both the case of a constant acceleration and that of an average acceleration. This is a seemingly almost trivial point but it does indicate that care should be taken always when applying well known formulae. It is always worth checking in detail the assumptions made before effectively relying totally on a purely mathematical result.

Again, it is worth emphasising that while mathematics retains a real beauty and fascination by itself, when it is used to help solve a problem in physics, it must always be the physics that takes the lead; in such cases, the mathematics is purely and simply a tool to be used and any results obtained must be scrutinised with real care to see if they reflect a true reflection of the physical situation being investigated.

Incidentally, the above would mean that, in the latter case, Newton's law relating force and acceleration, $F = ma$, would be replaced by one linking the *average* force with the *average* acceleration

Other seemingly basic results which need to be considered in the present context are the so-called conservation laws. It is interesting to note that, in the classic text on mechanics by Synge & Griffith [1], only conservation of energy is listed in the index. This is, of course, a law at the heart of much in physics and is extended in, for example, thermodynamics to include the concept of heat but can be extended to include other forms of energy too. However, the so-called conservation laws of linear momentum and angular momentum are not in the list. Why is this so? The answer is, of course, that these 'laws' are not general; conservation of linear momentum in a given direction holds only if there is no resultant component of force acting in that direction and conservation of angular momentum only holds if the moment of the force acting has a zero component in the given fixed direction. Hence, care must the taken when invoking these two 'laws' and care must also be exercised if someone is tempted to criticise Newtonian mechanics because they claim to have found violations of these 'laws'.

A further point is worth making and that refers to the so-called Third Law; that is, the popular statement that 'action and reaction are equal and opposite'. A little thought shows that this seemingly simple statement must be treated with great care. Obviously if two bodies are pushing together and neither is moving, it would seem reasonable to say that the forces of each on the other are equal but what does the law really mean when one body pushing on another causes the second to move. In this latter case, the reaction must include the resulting movement and so, what at first sight might be thought a law of statics takes on a totally different meaning. Once again very careful thought must precede any conclusions drawn about the validity, or otherwise, of this law of Newton.

Conclusions.

These are a few seemingly trivial points to raise but serve to illustrate extremely well how important it is to check on the initial assumptions made in the construction of any theoretical model. So often unnecessary mistakes occur because this simple check has not been made. In the particular instance discussed here, attention is drawn to these basic points simply to attempt to put some ongoing arguments into perspective. Finally, it might be reiterated that, when dealing with problems in physics, it is the physics which must take the lead, always remembering that the mathematics is, in these situations, merely an extremely useful tool but one to be utilised with care.

Reference.

1. Synge, J. L. & B. A Griffith; 1959, *Principles of Mechanics*. McGraw-Hill, New York.