cubic ellipsoid nucleus - part 1: the model and its mass formula

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Abstract

This paper examines the hypothesis that the structure of the nucleus determines that of the atom and its properties and attempts to construct a geometric model of the nucleus that contributes to this hypothesis.
The model proposed here suggests that the structure of the nucleus is, in general, an ellipsoid with the nucleons connected by cubic bonds and the nucleus shells correlate with those of the atom.
In accordance with the model, a simplified theoretical mass formula was created to compare it with the experimental data; the test included 82 stable nuclei from Ar_{18}^{40} to Pb_{82}^{210}.
The mass formula depends on two terms:

- E_b: the total binding energy between the nucleons in the nucleus.
- E_c: the total electric energy of the nucleus.

and has two parameters:

- d_0: the minimum distance between two neighboring nucleons in the cubic structure of the nucleus.
- e_b: the binding energy between these neighboring nucleons.

The results for the calculation parameters were:

- d_0 = 1.62±0.03 fm
- e_b= 5.72±0.03 Mev

The results for the relative errors of the mass formula calculation were:

relative error	maximum	average	standard dev.
	1.9%	0.6%	0.5%

If we consider the nucleons for simplicity as rigid bodies, then we get in addition a rough estimation for d_0 through the radii of the proton and neutron:

- $r_n \approx 0.80$ fm, $r_p \approx 0.84$ fm, $d_0 \approx (r_n + r_p)$ =1.64 fm
- *relative deviation of* d_0: $\left|\frac{d_0-(r_n+r_p)}{(r_n+r_p)}\right| \approx 1.3\%$

These results for the mass calculation and d_0 strengthen the model assumptions.

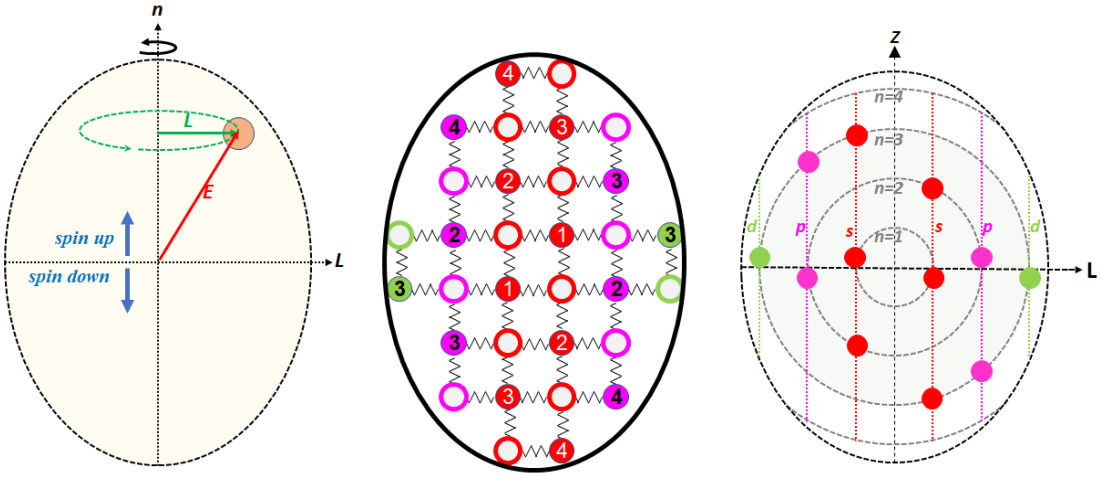
Content

The model at a glance

According to the model these are the shape and properties of the nucleus:

- the nucleus has an ellipsoid shape.
- the nucleon bonds build a cubic system.
- protons are connected to neutrons (**p**-**n**).
- neutrons are connected mainly to protons.
- the protons are populated and organized in shells in the nucleus in analogy to those of the electrons in the atom.
- the energy layers (principal quantum number **n**) grow with the distance from the origin.
- the perpendicular distance from the **z**-axis in the **x**-**y**-plane reflects the angular momentum (**L**, sub-orbitals).
- the upper half of the ellipsoid is referred to as spin-up and the lower part as spin-down.
- the nucleus possibly rotates around its **z**-axis.

The following drawings describe the idea via cross sections in the **x**-**z**-plane of the nucleus.



1: a nucleon in the nucleus *2: the bonds between the nucleons* *3: the energy levels of the nucleus*

1. a nucleon (**circle**) is observed inside the ellipsoid (dashed line) that encloses the nucleons and schematically defines the nucleus surface:
   - the distance from the origin represents its energy **E**.
   - the distance from the **z**-axis depicts it angular momentum **L**.
   - the nucleons in the upper half have spin-up, and in the lower one spin-down.
2. the bonds between the nucleons are shown for visibility as springs.
   - **protons**: full circles of the **s**, **p** and **d** sub-orbitals. **neutrons**: hollow circles.
3. the circles of equal energy states **n** in the ellipsoid.
   - the lines mark the development of the **s**, **p** and **d** sub-orbitals along the **z**-axis.
   - the **s** line crosses all **n** circles from 1 to 4 (**s1** to **s4**).
   - the **p** line begins by **n**=2 and reaches till **n**=4 (**p2** to **p4**).
   - the **d** line begins by **n**=3 and reaches the ellipsoid border, before it reaches the **n**=4 circle, and therefore there are no **d4** states at this stage (only **d3**).

Introduction

The nucleus and the atom are governed by different forces, have a size difference of about 5 orders of magnitude and according to current physics the order of the nucleus in shells is different than that of the atom [10].
The hypothesis, that this research investigates, is that the structure of the nucleus determines the one of the atom; therefore an attempt was made to find a geometric model that could describe this and, at the same time, meet the requirements and constraints of the current theories of nuclear and atomic physics to justify this new perspective without contradictions.
The starting point was that the hypothesis holds and so, in the opposite direction, it is possible to learn and deduce from the atom about the structure of the nucleus.
Once such a model was obtained it was tested and compared with experimental data.
The methods used in this work to analyze the nucleus are essentially those of classical physics.

Requirements

The nucleus shape

- The structure of the nucleus shall "make sense" physically.
- The nuclear density (meaning the distance between two neighboring nucleons) is assumed to be (at least nearly) constant and the structure of the nuclear bonds is homogeneous and periodic.
- A proton is connected only to neutrons (**p-n** bond) because we assume that the **p-p** bond has a too strong electric repulsion; otherwise we could expect to observe a stable He_2^2 atom for instance.
- A neutron is preferably connected with protons (**p-n** bond) because it is assumed that the proton stabilizes the neutron and that the **n-n** bond alone (with no protons involved) is unstable; otherwise we could expect to observe a stable **n-n** nucleus.

Reflection of the atom properties
If the nucleus influences the atom, then it should reflect the atomic structure:

- the atomic energy levels or shells.
- the orbitals and sub-orbitals and their population sequence.
- the correct number of neutrons for each isotope.
- the total nuclear spin.
- Pauli's exclusion principle.
- Hund's rules of electronic states population may apply similarly to protons.

Comparison with experimental data

- A theoretical mass formula suitable for the model shall be constructed.

Results

The model

We get the following model, which is developed and explained in detail below:

- The structure of the nucleus:
  - the nucleus is in general an ellipsoid.
  - it is composed of nucleons connected in a cubic system.
  - a proton is connected to neutrons.
  - a neutron is preferably connected to protons.
  - the excess neutrons, beyond the number equal to that of the protons, are in the envelope of the ellipsoid.
- Properties:
  - the energy levels grow with their distance from the origin. *
  - the perpendicular distance from the **z**-axis (i.e. in the **x**-**y**-plane) depicts the angular momentum (and so the sub-orbitals). *
  - the upper side of the ellipsoid is arbitrarily defined as spin-up and the lower part as spin-down. *
  - the model assumes that the nucleus possibly rotates around its main axis (the **z**-axis).*
- The model tries to deliver the following:
  - a justification of the electron shells, the energy levels, the orbitals and sub-orbitals and an explanation of the structure of the atom and the periodic table.
  - the right number of protons and neutrons in the nucleus and the correct nuclear spin.
  - in addition:
    - It doesn't contradict Pauli's exclusion principle.
    - like in the atomic physics the population sequence of the protons is possibly according to Hund's rules in the range where the electronic states follow the L-S coupling.*
- Examining the model:
  - the ellipsoid shape makes sense physically.
  - a theoretical mass formula was created and delivered good results:
    - nuclei mass with an average relative error <1%.
    - combined radii of proton and neutron with a relative error <2%.

* Topics that are not essential to the first study and do not contradict the model, but help in its development and construction. They shall be developed in following studies in order to expand and establish the model.

The Mass formula

The mass formula was developed to match the model and test its feasibility:

$$m_{calc_x} = Z_x \cdot m_p + N_x \cdot m_n - \frac{(E_{b_x} - E_{c_x})}{c^2}$$

- m_{calc_x}: the calculated mass of the nucleus x.
- Z_x: the atomic number of the nucleus x (number of protons).
- m_p: the mass of the proton.
- N_x: the number of neutrons in the nucleus x (number of nucleons A_x minus Z_x).
- m_n: the mass of the neutron.
- E_{b_x}: the total energy of the nucleon bonds in the nucleus x.
- E_{c_x}: the total electric energy (between all protons) in the nucleus x.
- c: the speed of light.

The binding energy of the nucleus is:

$$E_{b_x} = e_b \cdot n_{b_x}$$

- e_b: the energy of a single nucleon-nucleon bond in the nucleus (assuming they are equal for all bonds in all nuclei).
- n_{b_x}: the number of nucleon-nucleon bonds in the nucleus x.

The electric energy of the nucleus is:

$$E_{c_x} = \frac{e^2}{4\pi\epsilon_0}\,\frac{1}{d_0}\left\{\frac{1}{2}\sum_i^{Z_x}\,\sum_{j\neq i}^{Z_x}\frac{1}{d_{i,j}}\right\} = \frac{e^2}{4\pi\epsilon_0}\,\frac{1}{d_0}\,e_{c_x} \quad \text{where} \quad e_{c_x} \coloneqq \frac{1}{2}\sum_i^{Z_x}\,\sum_{j\neq i}^{Z_x}\frac{1}{d_{i,j}}$$

- d_0: the minimum distance between two neighboring nucleons in femtometer (assuming all nuclei have the same cubic structure and distance between their nucleons).
- $d_{i,j}$: the unitless distance between the protons of the indices *i* and *j* measured in multiples of d_0: $d_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$
- e_{c_x}: the unitless total electric energy of the nucleus (sum of the reciprocal distances).

The absolute relative error of the calculation for the nucleus x is:

$$rel\_err_x = \left| \frac{m_{calc_x} - m_{meas_x}}{Z_x \cdot m_p + N_x \cdot m_n - m_{meas_x}} \right| = \left| \frac{m_{calc_x} - m_{meas_x}}{mass\_defect_x} \right|$$

- m_{meas_x}: the measured mass of the nucleus x.
- rel_err_x is represented here in percentage.
- $mass\_defect_x$: $Z_x \cdot m_p + N_x \cdot m_n - m_{meas_x}$ is the mass defect of the nucleus x.

The mass formula, in this preliminary simplified form, depends thus only on the two variables:

- e_b: the energy of a single nucleon-nucleon bond.
- d_0: the minimum distance between two neighboring nucleons.

The implementation requires two preliminary calculation steps for all nuclei:

- Drawing the nucleus and counting the number of nucleon-nucleon bonds n_{b_x}.
- Calculating the relative total energy of the nucleus e_{c_x} (sum of reciprocal distances).

Results of the mass formula calculations

This section discusses the relative error of the mass formula calculation depending on the binding energy, e_b, and the distance between two neighboring nucleons, d_0, for 120 nuclei of common isotopes of elements from Li_3 to Pu_{94} (for several elements more than one isotope was taken). experimental data from [1].
The lighter nuclei till approximately Ar_{18} have larger relative errors than those of larger nuclei and are therefore shown in a different table.
The results of the mass formula calculation for 82 stable nuclei from Ar_{18}^{40} to Pb_{82}^{210}:

maximum	average	st. dev.	≤ 2% *	≤ 1%	≤ 0.5%
1.9%	0.6%	0.5%	100%	81%	58%

* the amount of nuclei with a relative error smaller than or equal to 2%.

- e_b = 5.72±0.03 Mev
- d_0 = 1.62±0.03 fm

these values are within a reasonable range [5].

If we consider the nucleons for simplicity as rigid bodies, then we get a rough estimation for d_0 through the radii of the proton and neutron: r_n [3] *(Neutron radius)*, r_p [4] *(Proton radius)*: $d_0 \approx (r_n + r_p)$.
Setting these values we get a result within a reasonable range:

- $r_n \approx 0.80$ fm, $r_p \approx 0.84$ fm, $d_0 \approx (r_n + r_p)$ =1.64 fm
- *relative deviation for* d_0: $\left|\frac{d_0 - (r_n + r_p)}{(r_n + r_p)}\right| = |\frac{1.62 - 1.64}{1.64}| \approx 1.3\%$

This estimation could strengthen the hypothesis of the model.

Discussion of the results and conclusion

The theory of the cubic ellipsoid nucleus offers a different perspective that apparently doesn't contradict current physics, but could expand its understanding and open new research direction, not only in nuclear physics, but also in atomic and other fields.

The model aims to achieve the following:

- a tangible geometric shape for the nucleus and the connection between the structure of the nucleus and the atom.
- a reflection of the structure of the periodic table in terms of the shells, the number protons and neutrons for each isotope, and qualitatively for the energy levels and orbitals; it was built that way from the beginning, but here it was shown to be possible.
- a theoretical mass formula, that relates directly to the theory. [11]
- the mass formula calculates the nuclear masses in good agreement with the experimental data.
- the distance d_0 between two neighboring nucleons agrees with the sum of the neutron and proton radii; this strengthens the model assumption and the concept of the mass formula.
- the chemical properties of an atom are independent of its isotopes; we therefore assume that its protons have the same spatial structure for all of its isotopes; this justifies the model assumption, that the excess neutrons are located in the envelope.
- the model delivers the correct total nuclear spin.

Remarks

- at this stage the "exact" organization of the core of the nucleus (its inner part, pairs of protons and neutrons) is known only for nuclei with complete sub-orbital, but we believe to have a good estimation also for the other nuclei.
- Light nuclei (below Argon and especially below Nitrogen) have a larger deviation from the mass formula calculation; the reason is assumed to be their structure that is not perfectly cubic or their density, meaning the distance between neighboring nucleons, that is slightly larger than their value by "well ordered" nuclei. Further research shall consider this.

part 1 - Sources and references

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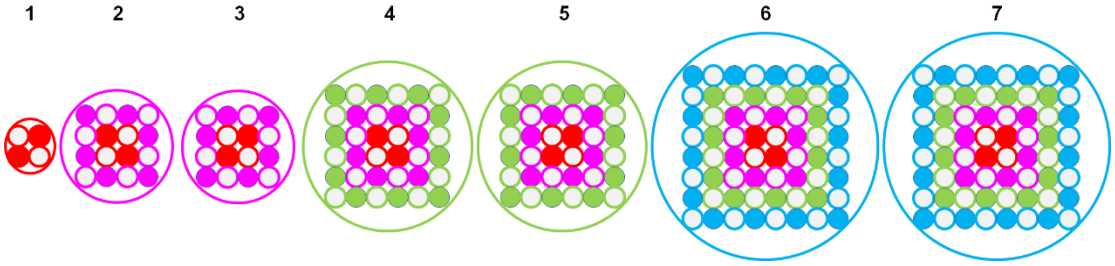
2. *Charge Radius: International Atomic Energy Agency (IAEA)*
3. *Neutron radius: Povh, B.; Rith, K.; Scholz, C.; Zetsche, F. (2002). Particles and Nuclei: An Introduction to the Physical Concepts. Berlin: Springer-Verlag. p. 73*
4. *Proton radius: Yong-Hui Lin, Hans-Werner Hammer and Ulf-G. Meißner: New insights into the nucleon's electromagnetic structure; Physical Review Letters, https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.128.052002*
5. *Reid, R. V. (1968). "Local phenomenological nucleon–nucleon potentials". Annals of Physics. **50** (3)*
6. *P. Roy Chowdhury; C. Samanta; D. N. Basu (January 26, 2006). "α decay half-lives of new superheavy elements". Physical Review C. **73** (1): 014612*
7. *C. Samanta; P. Roy Chowdhury; D. N. Basu (April 6, 2007). "Predictions of alpha decay half lives of heavy and superheavy elements". Nuclear Physics A. **789** (1–4): 142–154*
8. *G. Royer; K. Zbiri; C. Bonilla (2004). "Entrance channels and alpha decay half-lives of the heaviest elements". Nuclear Physics A. **730** (3–4): 355–376*
9. *Hermann Haken, Hans Christoph Wolf: Atom- und Quantenphysik – Einführung in die experimentellen und theoretischen Grundlagen. 5. Auflage. Springer, Berlin 1993, S. 329*
10. *LibreTexts physics: Nuclear Shell Model*
11. *LibreTexts physics: Binding energy and Semi-empirical mass formula*

Appendix

The development of the model

We describe in the following a visual way for the model development in order to make the idea easier to grasp.
The number of protons for closed layers of the periodic table is: **2**, **8**, **8**, **18**, **18**, **32**, **32**.
We assume that the protons and neutrons are ordered in pairs, so the number of nucleons is: **4**, **16**, **16**, **36**, **36**, **64**, **64**, or as square powers: $\mathbf{2^2}$, $\mathbf{4^2}$, $\mathbf{4^2}$, $\mathbf{6^2}$, $\mathbf{6^2}$, $\mathbf{8^2}$, $\mathbf{8^2}$.
We get square layers in a cubic structure placed on top of each other to form a square pyramid shape as shown in the following figure:



***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**.*
***neutrons**: hollow circles with colors according to their orbital.*

or as a table (**p**-protons, **n**-neurons, **A**=**p**+**n**: nucleons):

layer	p	n	A=p+n	p total	A total	S	P	D	F
1	2	2	4	2	4	2			
2	8	8	16	2+8=10	16+4=20	2	6		
3	8	8	16	10+8=18	20+16=36	2	6		
4	18	18	36	18+18=36	36+35=72	2	6	10	
5	18	18	36	36+18=54	72+36=108	2	6	10	
6	32	32	64	54+32=86	108+64=172	2	6	10	14
7	32	32	64	86+32=118	172+64=236	2	6	10	14

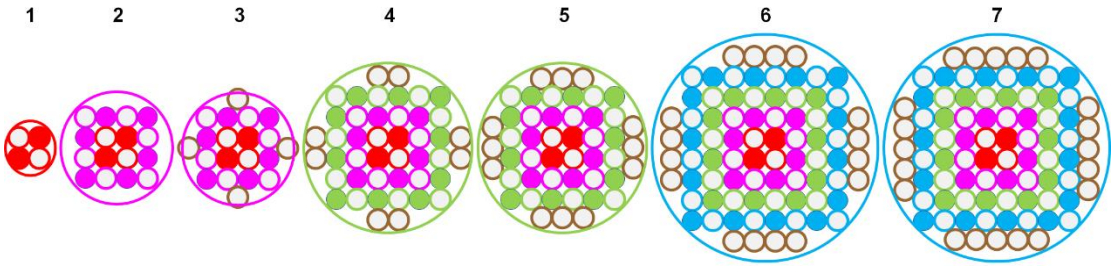
The interim result received for the noble gases is:
He_2^4, Ne_{10}^{20}, Ar_{18}^{36}, Kr_{36}^{72}, Xe_{54}^{108}, Rn_{86}^{172}, Og_{118}^{236}.

- The number of nucleons **A** (atomic mass) is correct for Helium and Neon.
- Argon has a stable isotope with 36 nucleons, but its most common isotope consists of 40 nucleons. [1]
- For nuclei larger than Argon the number of nucleons is larger than what we found.

Next assumption is that as the nucleus grows, excess neutrons are required to stabilize it. The model shall thus be expanded to explain the number of excess neutrons in each layer. This begins after Neon, i.e. from the third layer of the periodic table.

Completing the nucleus layers

In order to stabilize the nucleus, excess neutrons are added from the third layer.
We demand that each layer has more neutron than its predecessor under the assumption that this increases the nucleus stability.
We receive the following pyramid shape:



***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**.*
***neutrons**: hollow circles with colors according to their orbital.*
***excess neutrons**: beyond the number of the protons.*

The number of excess neutrons for each layer is **0**, **0**, **4**, **8**, **12**, **16**, **20** respectively, meaning a total of **4**, **16**, **20**, **44**, **48**, **80**, **84** nucleons at each layer.

layer	p	n=p+add.	A=p+n	p tot.	A total	S	P	D	F
1	2	2	2+2=4	2	4	2			
2	8	8+**0**=8	8+8=16	10	4+16=20	2	6		
3	8	8+**4**=12	8+12=20	18	20+20=40	2	6		
4	18	18+**8**=26	18+26=44	36	40+44=84	2	6	10	
5	18	18+**12**=34	18+34=48	54	84+48=132	2	6	10	
6	32	32+**16**=48	32+48=80	86	132+80=212	2	6	10	14
7	32	32+**20**=52	32+52=84	118	212+84=296	2	6	10	14

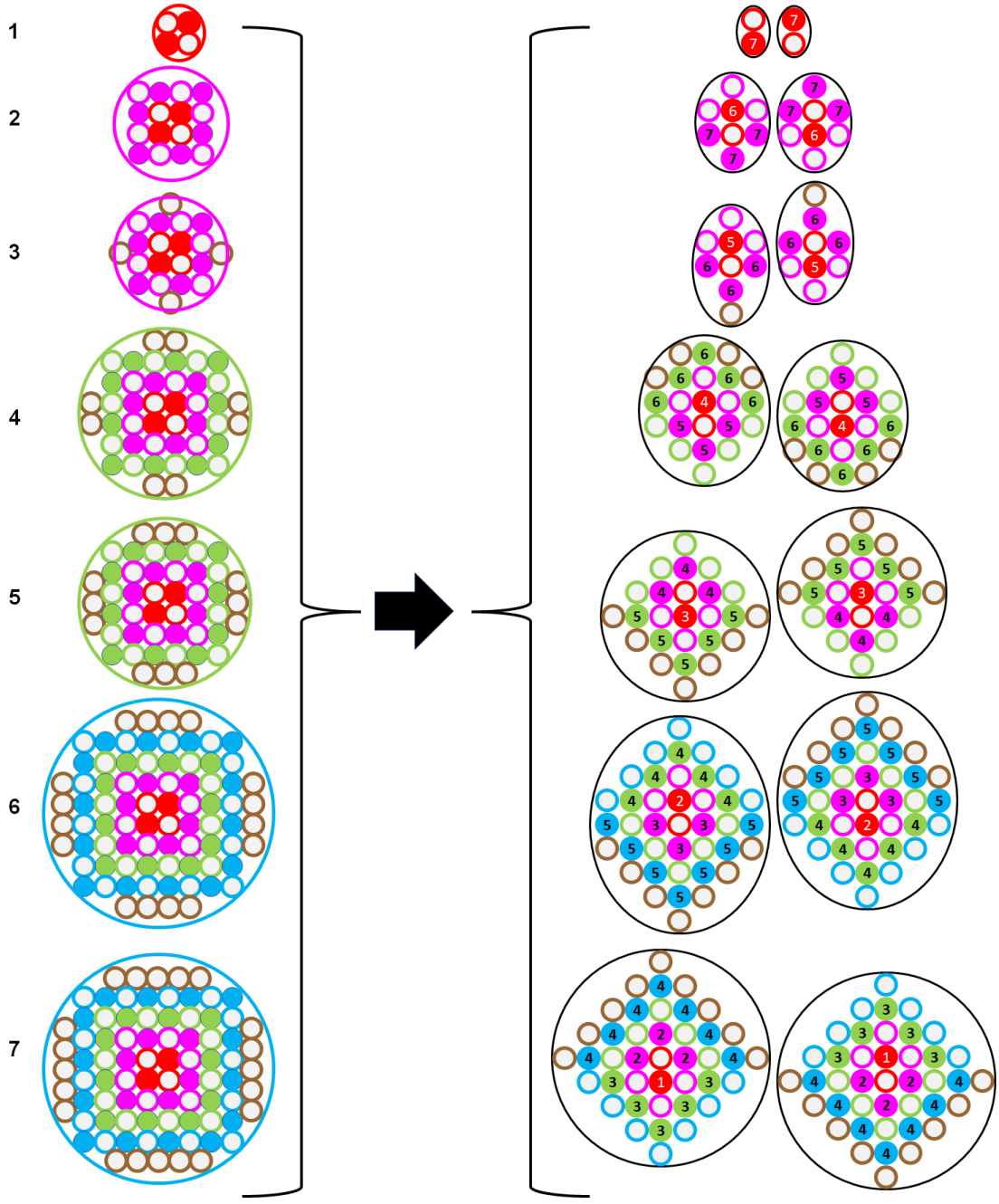
The results received for the noble gases are:

He_2^4, Ne_{10}^{20}, Ar_{18}^{40}, Kr_{36}^{84}, Xe_{54}^{132}, Rn_{86}^{212}, Og_{118}^{296}.

for the stable nuclei up to Xenon these results are correct. Radon and Oganesson are radioactive and are large. For Radon Rn_{86}^{212} is an isotope with a half-life in the range of minutes [1]. About Oganesson there is not much data, but Og_{118}^{296} is expected to be one of its isotopes [6, 7, 8].

Splitting the layers to form an ellipsoid

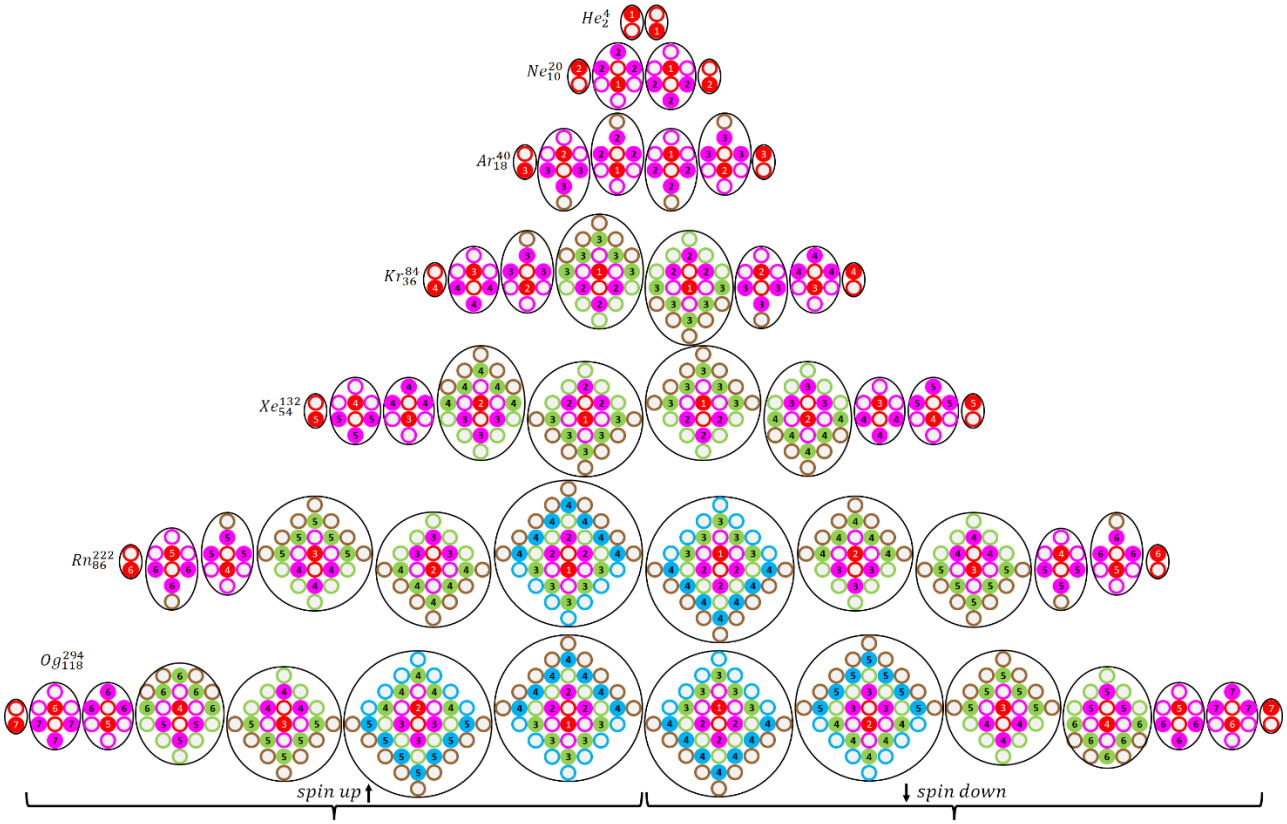
The pyramid form is not symmetrical with respect to the nucleus center and does not fit the energy levels nor the liquid drop model; therefore, we split each layer in two, one at the positive side of the **z**-axis and the other at its negative side.
This is the shape of the nucleus, that the model suggests.



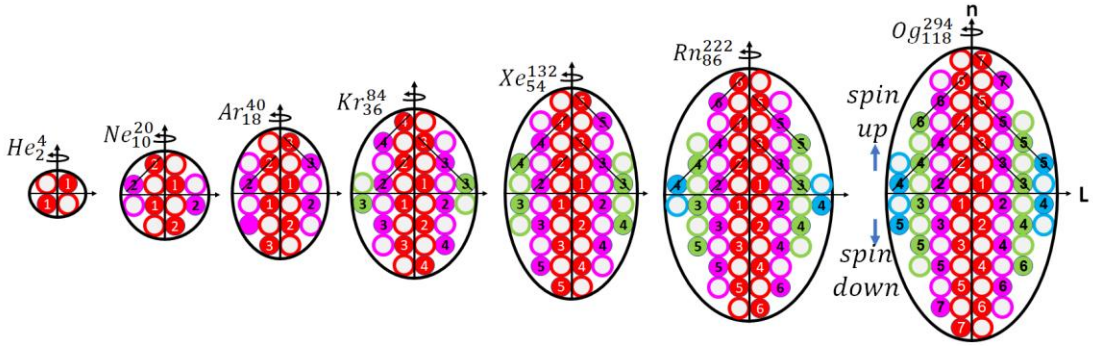
***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**. **Numbers**: energy levels.*
***neutrons**: hollow circles with colors according to their orbital.*
***excess neutrons**, beyond the number equal to the protons (unpaired neutrons).*

The nuclei of the noble gases

As a result of the model structure we've reached in the last section, we can draw the nuclei of the noble gases:



***Nucleus layers**: cross sections in the **x-y** planes along the **z** axis.*
***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**. **Numbers**: energy levels.*
***neutrons**: hollow circles with colors according to their orbital.*
***excess neutrons**: beyond a numb equal to the protons.*

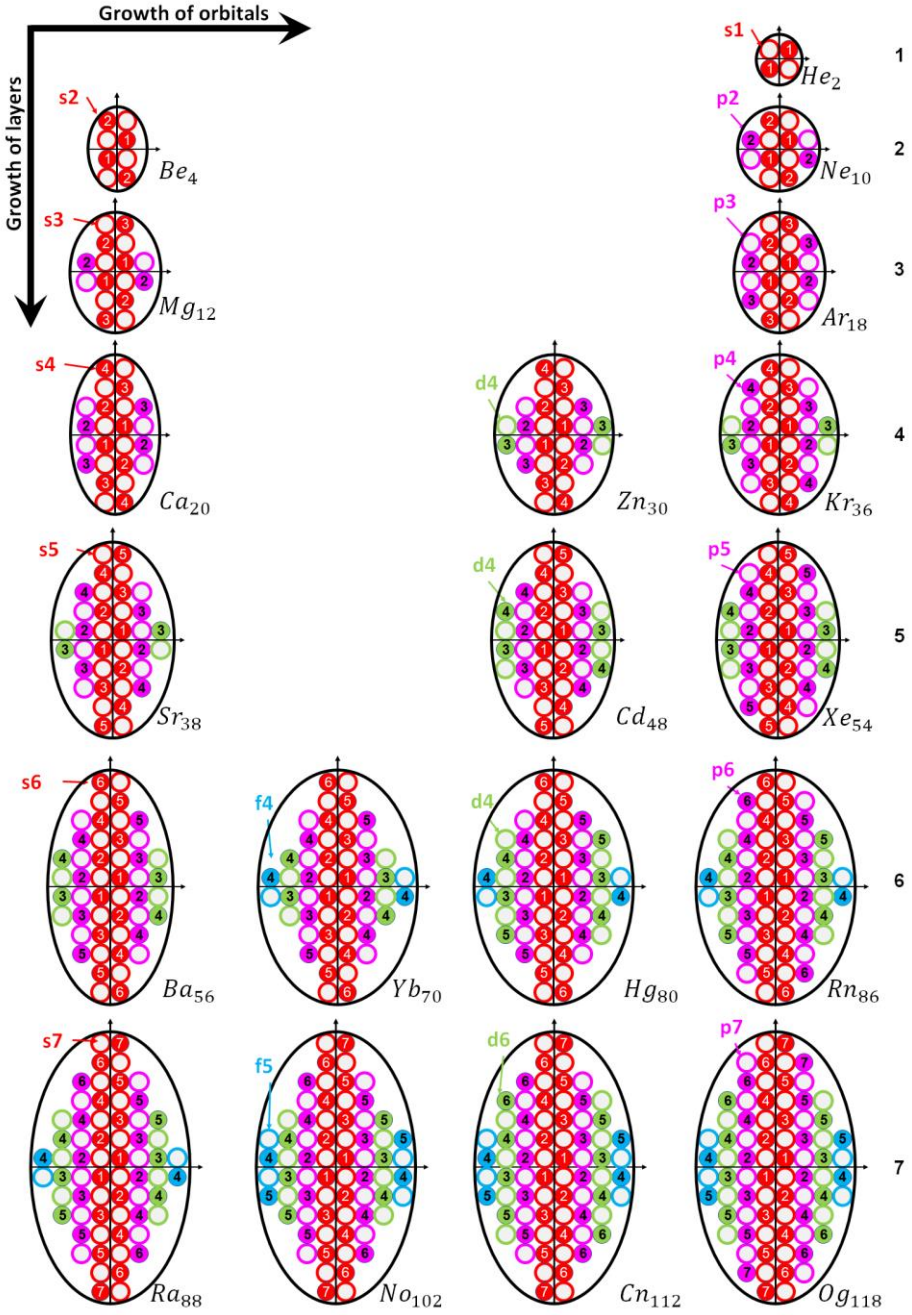


***The layers and energy levels of the nuclei:** a cross section in the **x-z** plane.*
***excess neutrons**, beyond a number equal to the protons, are not shown.*

The ellipsoids of the full sub-orbitals of the periodic table

In order to better understand the model, the ellipsoids of the full sub-orbitals are shown and ordered as they appear in the periodic table. The orbitals grow from left to right and the layers top to bottom; each colored arrow refers to the orbital that was filled last.

Cross sections in the x-z plane of the ellipsoids of the full sub-orbitals



***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**. **Numbers**: energy levels.*
***neutrons**: hollow circles with colors according to their orbital.*
***excess neutrons**, beyond a number equal to the protons, are not shown.*

The implementation of the mass formula calculation

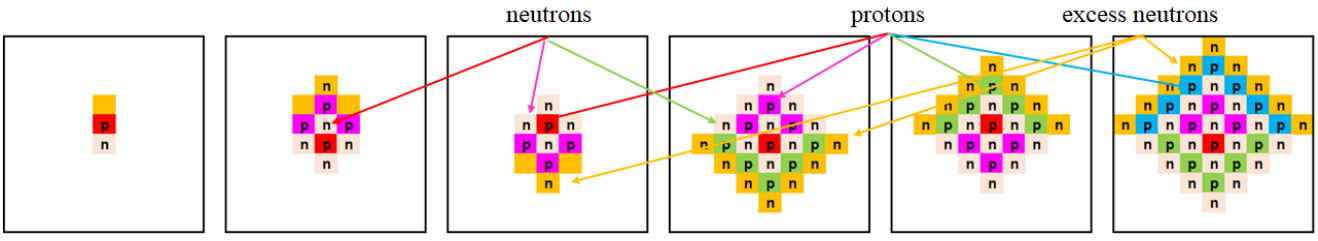
This section explains how the number of nucleon-nucleon bonds n_{b_x} and the relative total energy of the nucleus e_{c_x} were calculated.

Drawing the nucleus and counting the number of nucleon bonds n_{b_x}

The counting of the number of nucleon-nucleon bonds n_{b_x} of each nucleus was implemented by drawing all the nuclei in Excel sheets that automatically run this counting.
The sequence of the process was the following:

- First the Oganesson nucleus Og_{118}^{294} is built, because this is the nucleus that closes the noble gases, meaning it has closed orbital and as such is more probable to draw it correctly according to the model.
- The nuclei with closed sub orbitals are derived from Oganesson:
  - Og_{118}^{294} is copied and the number of protons and neutrons is adjusted to create the next full sub-orbital nucleus below it, which is Copernicium Cn_{112}^{282}.
  - In a similar manner Cn_{112}^{282} is copied and Nobelium No_{102}^{256} is drawn.
  - This process continues to form all the nuclei with closed sub orbitals (S, P, D, F).
- Beginning with every nucleus that closes a sub-orbital, the nuclei below it are built in a similar process to the above till the nucleus above the next closed sub-orbital is reached:
  - Oganesson is copied to the next below it, which is Tennessine.
  - Then Oganesson is copied to build Livermorium and so on till Nihonium is reached.
  - Then a similar process is done by using the Copernicium Cn_{112}^{282} nucleus for all nuclei below it and above Nobelium No_{102}^{256}.
  - The process continues till every nucleus with a closed sub-orbital creates all the nuclei in its sub-orbital.

During this process the structure of the nuclei was analyzed and studied and ideas were developed on how to assess what configuration is more probable.
Next drawing shows the Excel sheet of Radon Ra_{86}^{222} (only the left half, spin-up, is shown due to its large size).



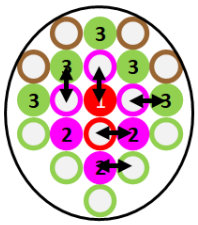
Legend: *protons (**p**) of the orbitals **S**, **P**, **D**, **F**. neutrons (**n**).*

Calculating the relative total energy of the nucleus e_{c_x}

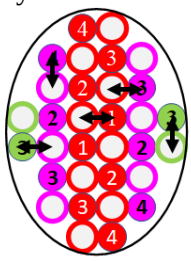
We first build an Excel sheet that calculates the sum of all proton-proton relative energies for Oganesson (the reciprocal distances):

$$e_{c_x} = \frac{1}{2}\sum_i^{Z_x} \sum_{j\neq i}^{Z_x} \frac{1}{d_{i,j}} \quad \text{with} \quad \frac{1}{d_{i,j}} = \frac{1}{\sqrt{(x_j-x_i)^2+(y_j-y_i)^2+(z_j-z_i)^2}}$$

then in a somewhat similar manner to the process of creating the Excel sheets that count n_{b_x}, as described in the section above, the closed sub-orbitals are created and each of them creates the nuclei in its sub-orbital below it.
Following drawings show the minimum distance between two neighboring nucleons d_0 through cross sections of the nucleus (marked by arrows in several positions):

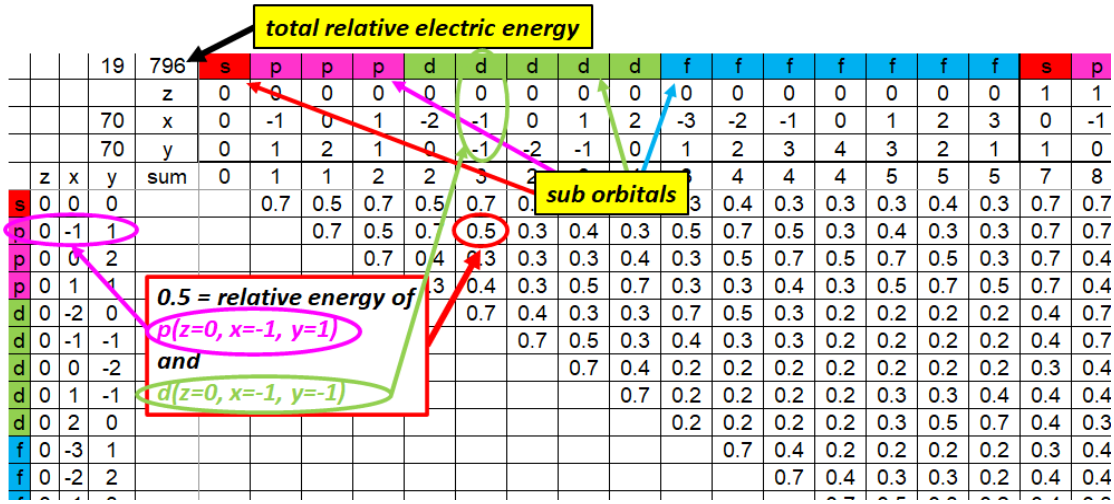


$\boldsymbol{d_0}$ *in the x-y plane*



$\boldsymbol{d_0}$ *in the x-z plane*

The Excel sheet of Yb_{70} explains the calculation of the relative electric energy e_{c_x} (the reciprocal distances):



			19	796	s	p	p	p	d	d	d	d	d	f	f	f	f	f	f	f	s	p
				z	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
			70	x	0	-1	0	1	-2	-1	0	1	2	-3	-2	-1	0	1	2	3	0	-1
			70	y	0	1	2	1	0	-1	-2	-1	0	1	2	3	4	3	2	1	1	0
	z	x	y	sum	0	1	1	2	2	3	2				4	4	4	5	5	5	7	8
s	0	0	0			0.7	0.5	0.7	0.5	0.7	0.			3	0.4	0.3	0.3	0.3	0.4	0.3	0.7	0.7
p	0	-1	1				0.7	0.5	0.7	0.5	0.3	0.4	0.3	0.5	0.7	0.5	0.3	0.4	0.3	0.3	0.7	0.7
p	0	0	2					0.7	0.4	0.3	0.3	0.3	0.4	0.3	0.5	0.7	0.5	0.7	0.5	0.3	0.7	0.4
p	0	1	1						0.3	0.4	0.3	0.5	0.7	0.3	0.3	0.4	0.3	0.5	0.7	0.5	0.7	0.4
d	0	-2	0							0.7	0.4	0.3	0.3	0.7	0.5	0.3	0.2	0.2	0.2	0.2	0.4	0.7
d	0	-1	-1								0.7	0.5	0.3	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.4	0.7
d	0	0	-2									0.7	0.4	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.4
d	0	1	-1										0.7	0.2	0.2	0.2	0.2	0.3	0.3	0.4	0.4	0.4
d	0	2	0											0.2	0.2	0.2	0.2	0.3	0.5	0.7	0.4	0.3
f	0	-3	1												0.7	0.4	0.2	0.2	0.2	0.2	0.3	0.4
f	0	-2	2													0.7	0.4	0.3	0.3	0.2	0.4	0.4

The coordinates of each proton appear once above and once from left; for each pair of (different) protons the relative electric energy (their reciprocal distance) is calculated at the point of their intersection.

Executing the mass formula calculation; analyzing and learning the model

The exact structure of the nucleus is crucial for the study of the model and for the correct execution of the mass formula. This structure determines:

- n_{b_x}: the number of bonds in the nucleus.
- e_x: the total electric energy of the nucleus.

The electric energy, e_x, is less sensible towards small changes of the nucleus structure, but for n_{b_x} small variations result in a large impact (of one percent or more) on the relative error of the mass formula calculation, so while running the tests the "correct" value of n_{b_x} for each nucleus was sought. "Correct" means logical in the physical sense and compared with other nuclei, so this isn't something that can be proved yet by this paper.
The process of filling up the nucleus with nucleon runs as follows:

- The following nuclei are selected to begin with:
    - nuclei of filled-up sub-orbitals (S, P, D, F) because their real shape is assumed to be more probable to be created correctly:
      Be_4, Ne_{10}, Mg_{12}, Ar_{18}, Ca_{20}, Zn_{30}, Kr_{36}, Sr_{38}, Cd_{48}, Xe_{54}, Ba_{56}, Yb_{70}, Hg_{80}, Rn_{86}, Ra_{88} (Hydrogen and Helium are not discussed in this paper due to their small number of bonds, that seem to deviate from the cubic structure; a separate future research shall deal with them).
- The nuclei are filled up with:
    - the protons: there is only one possible configuration.
    - the paired neutrons: also here there is only one possible configuration.
    - the excess neutrons are set in a way that reaches maximum symmetry and compared between various nuclei to assess the probable configuration.
- The mass formula is calculated and the best parameters are found.
- The nuclei that show larger deviation of their relative errors are checked again and their structure is being studied and possibly changed via comparison with other nuclei.
- At a second step all other nuclei are being created and the process repeats itself.

After creating all nuclei, trials are run in iterations with the construction of the nuclei.
This process is delicate, because we shall avoid the tendency to adapt the structure so that better results are achieved.
The results are not necessarily accurate as we cannot be sure of the correct structure of the nuclei, but a rough estimation seems possible.

Mass formula calculation: data

This section shows the data according to which the mass of the nuclei was calculated via the mass formula.
The parameters with which the mass formula are shown here are those of the best results achieved while running the calculation:

- d_0 = 1.62 fm
- e_b = 5.72 Mev

The legend of the table below is given in the order of the columns from left to right:

- *nuc*: the nucleus (name)
- Z_x: atomic number of the nucleus of the element x. the number of protons.
- A_x: mass number of the nucleus of the element x. The number of nucleons.
- $N_x := A_x - Z_x$: the number of neutrons of the nucleus of the element x.
- n_k: the number of nucleon-nucleon bonds in the nucleus as it was calculated by the relevant Excel file. *
- e_c: total relative electric energy of the nucleus as it was calculated by the relevant Excel file. *
- *meas. [amu]*: measured mass of the nucleus in [amu].
- *Base:* $Z_x \cdot m_p + N_x \cdot m_n$ *[amu]*: base mass in [amu]:

  number of protons · proton mass + number of neutrons · neutron mass

- *calc. m [amu]*: calculated mass according to the mass formula in [amu]
  $m_{calc_x} = Z_x \cdot m_p + N_x \cdot m_n - \frac{(E_{b_x} - E_{c_x})}{c^2}$.
- Δ_1*: [amu] calc. - meas.*: calculated mass - measured mass in [amu]
- Δ_2*: [amu] base - meas.*: base mass - measured mass in [amu]
- *rel. err.* Δ_1*:* Δ_2: the relative error in percent:
  $rel.err. = \frac{calculated\ mass - measured\ mass}{base\ mass - measured\ mass}$

* see section: The mass formula

nuc.	Z_x	A_x	n_k	e_c	meas. [amu]	calc. m [amu]	rel. error Δ_1:Δ_2	base $Z_x*m_p+N_x*m_n$ [amu]	Δ_1: calc - meas.	Δ_2: base - meas.
H	1	2	1	0.0	2.014	2.010	233.7%	2.016	-0.004	0.002
H	1	3	2	0.0	3.016	3.012	43.5%	3.025	-0.004	0.009
He	2	3	2	0.7	3.016	3.012	61.4%	3.023	-0.004	0.007
He	2	4	4	0.7	4.003	4.008	18.4%	4.032	0.005	0.029
Li	3	6	6	1.9	6.015	6.013	7.1%	6.048	-0.002	0.033
Li	3	7	7	1.9	7.016	7.015	1.6%	7.056	-0.001	0.040
Be	4	9	11	3.4	9.012	9.008	6.7%	9.072	-0.004	0.060
B	5	10	12	5.7	10.013	10.011	2.2%	10.080	-0.001	0.067
B	5	11	14	5.7	11.009	11.008	1.8%	11.088	-0.001	0.079
C	6	12	16	9.0	12.000	12.006	6.3%	12.096	0.006	0.096
C	6	13	18	9.0	13.003	13.002	1.0%	13.104	-0.001	0.101
N	7	14	20	12.7	14.003	14.001	2.0%	14.112	-0.002	0.109
N	7	15	21	12.7	15.000	15.003	2.8%	15.120	0.003	0.120
O	8	16	24	16.7	15.995	15.996	0.9%	16.128	0.001	0.133
O	8	17	25	16.7	16.999	16.999	0.3%	17.136	0.000	0.137
O	8	18	26	16.7	17.999	18.001	1.4%	18.145	0.002	0.146
F	9	19	29	20.8	18.998	18.994	2.9%	19.152	-0.004	0.154
Ne	10	20	32	26.1	19.992	19.988	2.7%	20.159	-0.005	0.167
Ne	10	21	33	26.1	20.994	20.990	2.0%	21.168	-0.003	0.174
Ne	10	22	34	26.1	21.991	21.993	0.8%	22.177	0.002	0.185
Na	11	23	36	30.0	22.990	22.992	0.9%	23.184	0.002	0.194
Mg	12	24	38	34.1	23.985	23.991	2.7%	24.191	0.005	0.206
Mg	12	25	40	34.1	24.986	24.987	0.5%	25.200	0.001	0.214
Mg	12	26	42	34.1	25.983	25.983	0.3%	26.209	0.001	0.226
Al	13	27	45	39.3	26.982	26.977	1.9%	27.216	-0.004	0.234
Si	14	28	46	44.8	27.977	27.984	2.7%	28.223	0.007	0.246
Si	14	29	48	44.8	28.976	28.980	1.3%	29.232	0.003	0.255
Si	14	30	50	44.8	29.974	29.976	1.0%	30.241	0.003	0.267
P	15	31	52	53.0	30.974	30.979	1.9%	31.248	0.005	0.274
S	16	32	56	59.7	31.972	31.968	1.4%	32.255	-0.004	0.283
Cl	17	35	63	66.4	34.969	34.956	4.0%	35.280	-0.013	0.311
Ar	18	36	66	74.4	35.968	35.953	4.7%	36.287	-0.015	0.319
Ar	18	40	70	74.4	39.962	39.963	0.1%	40.322	0.000	0.359
K	19	39	70	79.9	38.964	38.958	1.6%	39.312	-0.006	0.348
Ca	20	40	72	85.6	39.963	39.958	1.2%	40.319	-0.004	0.356
Ca	20	42	74	85.6	41.959	41.963	1.3%	42.336	0.005	0.378
Ca	20	44	78	85.6	43.955	43.956	0.2%	44.353	0.001	0.398
Sc	21	45	81	92.8	44.956	44.952	1.0%	45.361	-0.004	0.405
Ti	22	46	84	103.7	45.953	45.951	0.3%	46.368	-0.001	0.415
Ti	22	48	88	103.7	47.948	47.944	0.9%	48.385	-0.004	0.437
Ti	22	50	90	103.7	49.945	49.949	1.0%	50.403	0.004	0.458
V	23	51	93	112.4	50.944	50.946	0.5%	51.410	0.002	0.466
Cr	24	52	96	123.1	51.941	51.945	1.0%	52.417	0.005	0.477
Cr	24	54	100	123.1	53.939	53.938	0.2%	54.435	-0.001	0.496
Mn	25	55	103	132.2	54.938	54.936	0.5%	55.442	-0.002	0.504
Fe	26	54	102	141.5	53.940	53.941	0.2%	54.432	0.001	0.492
Fe	26	56	106	141.5	55.935	55.933	0.3%	56.449	-0.002	0.514

nuc.	Z_x	A_x	n_k	e_c	meas. [amu]	calc. m [amu]	rel. error Δ_1:Δ_2	base $Z_x*m_p+N_x*m_n$ [amu]	Δ_1: calc - meas.	Δ_2: base - meas.
Co	27	59	112	153.3	58.933	58.932	0.1%	59.474	-0.001	0.541
Ni	28	58	112	164.5	57.935	57.933	0.4%	58.464	-0.002	0.528
Ni	28	60	114	164.5	59.931	59.938	1.3%	60.481	0.007	0.550
Ni	28	62	118	164.5	61.928	61.931	0.4%	62.498	0.002	0.570
Cu	29	63	120	174.6	62.930	62.935	1.0%	63.506	0.006	0.576
Zn	30	64	122	185.1	63.929	63.941	1.9%	64.513	0.011	0.584
Zn	30	66	126	185.1	65.926	65.933	1.2%	66.530	0.007	0.604
Zn	30	68	130	185.1	67.925	67.926	0.2%	68.548	0.001	0.623
Ga	31	69	132	195.0	68.926	68.931	0.8%	69.555	0.005	0.629
Ge	32	74	142	205.1	73.921	73.921	0.1%	74.597	0.000	0.676
As	33	75	144	218.5	74.922	74.928	1.0%	75.604	0.007	0.682
Se	34	80	154	230.0	79.917	79.920	0.5%	80.646	0.003	0.729
Br	35	79	154	241.7	78.918	78.921	0.4%	79.636	0.003	0.718
Br	35	81	158	241.7	80.916	80.914	0.3%	81.653	-0.002	0.737
Kr	36	82	160	253.5	81.913	81.920	0.9%	82.661	0.007	0.747
Kr	36	84	164	253.5	83.911	83.913	0.2%	84.678	0.001	0.766
Kr	36	86	166	253.5	85.911	85.918	0.9%	86.695	0.007	0.785
Rb	37	85	167	264.1	84.912	84.912	0.0%	85.685	0.000	0.773
Sr	38	84	164	271.1	83.913	83.927	1.8%	84.675	0.014	0.762
Sr	38	86	168	271.1	85.909	85.920	1.3%	86.692	0.010	0.783
Sr	38	88	172	271.1	87.906	87.912	0.9%	88.710	0.007	0.804
Y	39	89	176	283.2	88.906	88.907	0.1%	89.717	0.001	0.811
Zr	40	90	180	295.5	89.905	89.901	0.4%	90.724	-0.004	0.820
Nb	41	93	186	313.4	92.906	92.906	0.0%	93.749	0.000	0.843
Mo	42	98	196	329.6	97.905	97.902	0.4%	98.791	-0.003	0.885
Tc	43	98	197	343.5	97.907	97.908	0.1%	98.789	0.000	0.882
Ru	44	102	204	357.5	101.904	101.911	0.8%	102.823	0.007	0.918
Rh	45	103	208	373.9	102.906	102.910	0.5%	103.830	0.004	0.925
Pd	46	106	214	384.8	105.903	105.908	0.5%	106.855	0.004	0.951
Ag	47	107	218	405.6	106.905	106.910	0.6%	107.862	0.005	0.957
Cd	48	112	228	421.0	111.903	111.906	0.3%	112.904	0.003	1.001
In	49	115	234	433.9	114.904	114.906	0.2%	115.928	0.002	1.025
Sn	50	120	244	447.1	119.902	119.899	0.3%	120.970	-0.003	1.068
Sb	51	121	247	463.9	120.904	120.904	0.0%	121.978	0.000	1.074
Te	52	126	256	478.4	125.903	125.904	0.1%	127.020	0.001	1.116
I	53	127	260	493.4	126.904	126.901	0.3%	128.027	-0.003	1.122
Xe	54	132	268	508.0	131.904	131.908	0.3%	133.069	0.004	1.165
Cs	55	133	270	519.5	132.905	132.914	0.7%	134.076	0.009	1.171
Ba	56	138	278	531.0	137.905	137.918	1.0%	139.118	0.013	1.213
La	57	139	280	545.4	138.906	138.927	1.7%	140.125	0.020	1.219
Ce	58	140	284	566.6	139.905	139.930	2.0%	141.133	0.024	1.227
Pr	59	141	288	582.7	140.908	140.928	1.6%	142.140	0.020	1.232
Nd	60	144	298	603.2	143.910	143.910	0.0%	145.164	0.000	1.254
Pm	61	145	298	621.6	144.913	144.935	1.8%	146.172	0.023	1.259
Sm	62	152	310	636.1	151.920	151.935	1.1%	153.231	0.015	1.311
Eu	63	153	316	659.5	152.921	152.927	0.5%	154.238	0.006	1.317
Gd	64	158	328	683.3	157.924	157.918	0.4%	159.280	-0.006	1.356

nuc.	Z_x	A_x	n_k	e_c	meas. [amu]	calc. m [amu]	rel. error Δ_1:Δ_2	base $Z_x*m_p+N_x*m_n$ [amu]	Δ_1: calc - meas.	Δ_2: base - meas.
Tb	65	159	330	700.8	158.925	158.930	0.4%	160.287	0.005	1.362
Dy	66	164	340	718.4	163.929	163.928	0.1%	165.329	-0.002	1.400
Ho	67	163	340	735.8	162.929	162.934	0.4%	164.319	0.005	1.391
Er	68	166	348	756.6	165.930	165.929	0.1%	167.344	-0.001	1.414
Tm	69	169	354	777.9	168.934	168.937	0.2%	170.369	0.003	1.434
Yb	70	172	362	796.0	171.936	171.930	0.4%	173.393	-0.006	1.457
Lu	71	175	368	815.1	174.941	174.936	0.3%	176.418	-0.005	1.477
Hf	72	178	374	834.0	177.944	177.942	0.1%	179.442	-0.002	1.499
Ta	73	181	383	859.3	180.948	180.936	0.8%	182.467	-0.012	1.519
W	74	184	390	882.6	183.951	183.939	0.7%	185.492	-0.012	1.541
Re	75	185	393	903.1	184.953	184.948	0.3%	186.499	-0.005	1.546
Os	76	192	404	923.6	191.961	191.959	0.1%	193.558	-0.002	1.597
Ir	77	193	410	947.2	192.963	192.952	0.7%	194.565	-0.011	1.602
Pt	78	194	412	971.0	193.963	193.970	0.4%	195.573	0.007	1.610
Au	79	197	420	992.4	196.967	196.966	0.1%	198.597	-0.001	1.631
Hg	80	198	426	1,014.0	197.967	197.957	0.6%	199.605	-0.010	1.638
Hg	80	200	430	1,014.0	199.968	199.949	1.1%	201.622	-0.019	1.654
Hg	80	202	432	1,014.0	201.971	201.955	1.0%	203.639	-0.016	1.669
Tl	81	205	436	1,032.2	204.974	204.972	0.1%	206.664	-0.002	1.689
Pb	82	208	442	1,050.5	207.977	207.977	0.0%	209.688	0.001	1.712
Bi	83	208	444	1,072.4	207.980	207.984	0.3%	209.687	0.005	1.707
Po	84	208	448	1,092.0	207.981	207.977	0.2%	209.686	-0.004	1.704
At	85	209	451	1,111.1	208.986	208.984	0.1%	210.693	-0.002	1.707
Rn	86	222	470	1,132.0	222.018	221.999	1.1%	223.804	-0.019	1.787
Fr	87	223	471	1,146.8	223.020	223.014	0.3%	224.811	-0.006	1.792
Ra	88	226	476	1,162.0	226.025	226.022	0.2%	227.836	-0.003	1.811
Ac	89	227	481	1,183.3	227.028	227.019	0.5%	228.843	-0.008	1.816
Th	90	232	488	1,204.8	232.038	232.039	0.0%	233.885	0.001	1.847
Pa	91	231	493	1,235.2	231.036	231.027	0.5%	232.875	-0.009	1.839
U	92	238	502	1,263.5	238.051	238.058	0.4%	239.935	0.007	1.884
Np	93	237	503	1,289.6	237.048	237.067	1.0%	238.924	0.019	1.876
Pu	94	244	520	1,309.9	244.064	244.041	1.2%	245.984	-0.023	1.920