# A Conjecture On $\sigma(n)$ Function

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**Abstract**

We know many Arithmetical Functions [1] like $\phi(n), \sigma(n), \tau(n)$ etc. In this paper we will discuss about $\sigma(n)$ and will see a phenomenal observation.And later we will claim this observation as a conjecture.

**Keywords:** $\sigma(n)$ , Conjecture, Goldbach's Conjecture, Arithmetical Functions

**2000 Mathematics Subject Classification:** 11Axx, 11A25, 11A67

## 1 Introduction

Before entering the topic we first discuss about $\sigma(n)$ function. In Number Theory $\sigma(n)$ is defined by sum of all positive divisors of n *i.e*;

$$\sigma(n) = \sum_{d|n} d$$

For prime $p$ we have only two divisors 1 and $p$ , So $\sigma(p) = p + 1$ and $\sigma(1) = 1$

For Example: $\sigma(6) = \sum_{d|6} d = 1 + 2 + 3 + 6 = 12$

## 2 Observations

Let us look at some Examples:

$4 = \sigma(1) + \sigma(2)$

$5 = \sigma(1) + \sigma(3)$

$6 = \sigma(2) + \sigma(2)$

$7 = \sigma(1) + \sigma(5) = \sigma(2) + \sigma(3)$

$8 = \sigma(1) + \sigma(4) = \sigma(3) + \sigma(3)$

$9 = \sigma(1) + \sigma(7) = \sigma(2) + \sigma(5)$

$10 = \sigma(2) + \sigma(4) = \sigma(3) + \sigma(5)$

$11 = \sigma(2) + \sigma(7) = \sigma(3) + \sigma(4)$

$12 = \sigma(3) + \sigma(7) = \sigma(5) + \sigma(5)$

$13 = \sigma(1) + \sigma(6) = \sigma(4) + \sigma(7) = \sigma(5) + \sigma(4)$

$14 = \sigma(1) + \sigma(9) = \sigma(5) + \sigma(7) = \sigma(4) + \sigma(4)$

$15 = \sigma(1) + \sigma(13) = \sigma(2) + \sigma(6) = \sigma(4) + \sigma(7)$

$16 = \sigma(1) + \sigma(8) = \sigma(2) + \sigma(9) = \sigma(3) + \sigma(6)$

$17 = \sigma(2) + \sigma(13) = \sigma(3) + \sigma(9)$

$18 = \sigma(2) + \sigma(8) = \sigma(3) + \sigma(13) = \sigma(5) + \sigma(6)$

$19 = \sigma(1) + \sigma(10) = \sigma(3) + \sigma(8) = \sigma(5) + \sigma(9)$

$20 = \sigma(5) + \sigma(13) = \sigma(4) + \sigma(9) = \sigma(7) + \sigma(6)$

$21 = \sigma(1) + \sigma(19) = \sigma(2) + \sigma(10) = \sigma(5) + \sigma(8)$

$22 = \sigma(3) + \sigma(10) = \sigma(4) + \sigma(8) = \sigma(7) + \sigma(13)$

$23 = \sigma(2) + \sigma(19) = \sigma(7) + \sigma(8)$

$24 = \sigma(3) + \sigma(19) = \sigma(5) + \sigma(10) = \sigma(6) + \sigma(6)$

$25 = \sigma(1) + \sigma(23) = \sigma(4) + \sigma(10) = \sigma(6) + \sigma(9)$

$$26 = \sigma(7) + \sigma(10) = \sigma(9) + \sigma(9) = \sigma(11) + \sigma(13)$$

$$27 = \sigma(2) + \sigma(15) = \sigma(4) + \sigma(19) = \sigma(6) + \sigma(8)$$

$$28 = \sigma(3) + \sigma(15) = \sigma(7) + \sigma(19) = \sigma(8) + \sigma(9)$$

$$29 = \sigma(1) + \sigma(12) = \sigma(8) + \sigma(13)$$

$$30 = \sigma(5) + \sigma(15) = \sigma(6) + \sigma(17) = \sigma(8) + \sigma(8)$$

$$31 = \sigma(1) + \sigma(29) = \sigma(2) + \sigma(12) = \sigma(4) + \sigma(14)$$

$$32 = \sigma(1) + \sigma(25) = \sigma(3) + \sigma(12) = \sigma(6) + \sigma(19)$$

$$33 = \sigma(1) + \sigma(31) = \sigma(2) + \sigma(29) = \sigma(9) + \sigma(19)$$

$$34 = \sigma(2) + \sigma(25) = \sigma(3) + \sigma(29) = \sigma(5) + \sigma(13)$$

$$35 = \sigma(2) + \sigma(31) = \sigma(3) + \sigma(25) = \sigma(4) + \sigma(12)$$

$$36 = \sigma(3) + \sigma(31) = \sigma(5) + \sigma(29) = \sigma(7) + \sigma(12)$$

$$37 = \sigma(1) + \sigma(22) = \sigma(5) + \sigma(25) = \sigma(4) + \sigma(29)$$

$$38 = \sigma(5) + \sigma(31) = \sigma(4) + \sigma(25) = \sigma(10) + \sigma(19)$$

$$39 = \sigma(1) + \sigma(37) = \sigma(4) + \sigma(31) = \sigma(7) + \sigma(16)$$

$$40 = \sigma(1) + \sigma(18) = \sigma(3) + \sigma(22) = \sigma(7) + \sigma(31)$$

$$41 = \sigma(1) + \sigma(28) = \sigma(2) + \sigma(37) = \sigma(9) + \sigma(12)$$

$$42 = \sigma(2) + \sigma(18) = \sigma(3) + \sigma(37) = \sigma(12) + \sigma(13)$$

$$43 = \sigma(1) + \sigma(20) = \sigma(2) + \sigma(28) = \sigma(4) + \sigma(22)$$

$44 = \sigma(3) + \sigma(27) = \sigma(5) + \sigma(37) = \sigma(6) + \sigma(31)$

$45 = \sigma(1) + \sigma(43) = \sigma(2) + \sigma(20) = \sigma(5) + \sigma(18)$

$46 = \sigma(3) + \sigma(20) = \sigma(5) + \sigma(27) = \sigma(4) + \sigma(18)$

$47 = \sigma(2) + \sigma(43) = \sigma(4) + \sigma(27) = \sigma(7) + \sigma(18)$

$48 = \sigma(3) + \sigma(43) = \sigma(5) + \sigma(20) = \sigma(7) + \sigma(27)$

$49 = \sigma(1) + \sigma(47) = \sigma(4) + \sigma(20) = \sigma(9) + \sigma(22)$

$50 = \sigma(5) + \sigma(43) = \sigma(6) + \sigma(37) = \sigma(7) + \sigma(22)$

$51 = \sigma(2) + \sigma(47) = \sigma(4) + \sigma(43) = \sigma(6) + \sigma(18)$

$52 = \sigma(3) + \sigma(35) = \sigma(7) + \sigma(43) = \sigma(6) + \sigma(27)$

$53 = \sigma(9) + \sigma(27) = \sigma(13) + \sigma(18) = \sigma(8) + \sigma(37)$

$54 = \sigma(5) + \sigma(33) = \sigma(6) + \sigma(20) = \sigma(8) + \sigma(18)$

$55 = \sigma(1) + \sigma(34) = \sigma(4) + \sigma(47) = \sigma(9) + \sigma(41)$

$56 = \sigma(7) + \sigma(35) = \sigma(6) + \sigma(43) = \sigma(13) + \sigma(20)$

$57 = \sigma(1) + \sigma(28) = \sigma(2) + \sigma(34) = \sigma(8) + \sigma(41)$

$58 = \sigma(1) + \sigma(49) = \sigma(3) + \sigma(53) = \sigma(13) + \sigma(43)$

$59 = \sigma(2) + \sigma(28) = \sigma(8) + \sigma(43) = \sigma(18) + \sigma(19)$

$60 = \sigma(2) + \sigma(49) = \sigma(3) + \sigma(39) = \sigma(5) + \sigma(53)$

$61 = \sigma(1) + \sigma(59) = \sigma(3) + \sigma(49) = \sigma(4) + \sigma(53)$

$62 = \sigma(5) + \sigma(28) = \sigma(7) + \sigma(53) = \sigma(13) + \sigma(35)$

$63 = \sigma(1) + \sigma(61) = \sigma(2) + \sigma(59) = \sigma(5) + \sigma(49)$

$64 = \sigma(1) + \sigma(32) = \sigma(3) + \sigma(59) = \sigma(4) + \sigma(49)$

$65 = \sigma(2) + \sigma(61) = \sigma(7) + \sigma(49)$

$66 = \sigma(2) + \sigma(32) = \sigma(3) + \sigma(61) = \sigma(6) + \sigma(53)$

$67 = \sigma(3) + \sigma(32) = \sigma(4) + \sigma(59) = \sigma(9) + \sigma(53)$

$68 = \sigma(7) + \sigma(59) = \sigma(12) + \sigma(27) = \sigma(13) + \sigma(53)$

$69 = \sigma(1) + \sigma(67) = \sigma(5) + \sigma(32) = \sigma(6) + \sigma(49)$

$70 = \sigma(4) + \sigma(32) = \sigma(7) + \sigma(61) = \sigma(9) + \sigma(49)$

$71 = \sigma(2) + \sigma(67) = \sigma(7) + \sigma(32) = \sigma(8) + \sigma(28)$

$72 = \sigma(3) + \sigma(67) = \sigma(6) + \sigma(59) = \sigma(8) + \sigma(49)$

$73 = \sigma(1) + \sigma(71) = \sigma(9) + \sigma(59) = \sigma(16) + \sigma(20)$

$74 = \sigma(5) + \sigma(67) = \sigma(6) + \sigma(61) = \sigma(10) + \sigma(28)$

$75 = \sigma(1) + \sigma(73) = \sigma(2) + \sigma(71) = \sigma(4) + \sigma(67)$

$76 = \sigma(3) + \sigma(71) = \sigma(7) + \sigma(67) = \sigma(9) + \sigma(32)$

$77 = \sigma(2) + \sigma(73) = \sigma(8) + \sigma(61) = \sigma(13) + \sigma(32)$

$78 = \sigma(3) + \sigma(73) = \sigma(8) + \sigma(32) = \sigma(10) + \sigma(59)$

$79 = \sigma(1) + \sigma(45) = \sigma(4) + \sigma(30) = \sigma(16) + \sigma(33)$

$$80 = \sigma(5) + \sigma(73) = \sigma(6) + \sigma(67) = \sigma(14) + \sigma(28)$$

$$81 = \sigma(1) + \sigma(57) = \sigma(2) + \sigma(45) = \sigma(18) + \sigma(20)$$

$$82 = \sigma(3) + \sigma(45) = \sigma(7) + \sigma(73) = \sigma(20) + \sigma(27)$$

$$83 = \sigma(2) + \sigma(79) = \sigma(8) + \sigma(67) = \sigma(18) + \sigma(43)$$

$$84 = \sigma(3) + \sigma(57) = \sigma(6) + \sigma(30) = \sigma(11) + \sigma(55)$$

$$85 = \sigma(1) + \sigma(44) = \sigma(4) + \sigma(45) = \sigma(9) + \sigma(46)$$

$$86 = \sigma(5) + \sigma(57) = \sigma(6) + \sigma(73) = \sigma(7) + \sigma(45)$$

$$87 = \sigma(2) + \sigma(44) = \sigma(4) + \sigma(57) = \sigma(8) + \sigma(30)$$

$$88 = \sigma(3) + \sigma(65) = \sigma(7) + \sigma(79) = \sigma(12) + \sigma(24)$$

$$89 = \sigma(8) + \sigma(73) = \sigma(21) + \sigma(49) = \sigma(31) + \sigma(49)$$

$$90 = \sigma(5) + \sigma(44) = \sigma(6) + \sigma(45) = \sigma(10) + \sigma(46)$$

$$91 = \sigma(1) + \sigma(40) = \sigma(4) + \sigma(44) = \sigma(24) + \sigma(25)$$

$$92 = \sigma(1) + \sigma(36) = \sigma(6) + \sigma(57) = \sigma(7) + \sigma(44)$$

$$93 = \sigma(2) + \sigma(40) = \sigma(8) + \sigma(45) = \sigma(9) + \sigma(57)$$

$$94 = \sigma(1) + \sigma(50) = \sigma(2) + \sigma(36) = \sigma(3) + \sigma(58)$$

$$95 = \sigma(3) + \sigma(36) = \sigma(18) + \sigma(28) = \sigma(31) + \sigma(32)$$

$$96 = \sigma(2) + \sigma(50) = \sigma(5) + \sigma(58) = \sigma(6) + \sigma(65)$$

$$97 = \sigma(1) + \sigma(42) = \sigma(4) + \sigma(40) = \sigma(5) + \sigma(36)$$

$98 = \sigma(4) + \sigma(36) = \sigma(7) + \sigma(40) = \sigma(26) + \sigma(28)$

$99 = \sigma(1) + \sigma(97) = \sigma(2) + \sigma(62) = \sigma(41) + \sigma(49)$

$100 = \sigma(3) + \sigma(69) = \sigma(4) + \sigma(50) = \sigma(12) + \sigma(30)$

Here we just show numbers[1] up to $100$;for finding such pair of $(a, b) \in \mathbb{Z}^+ \times \mathbb{Z}^+$ such that

$$n = \sigma(a) + \sigma(b)$$

for $n > 3$; we can use this code:

```
#include <iostream>
#include <vector>
#include <algorithm>
using namespace std;
int sum_of_divisors(int num) {
    int sum = 0;
    for (int i = 1; i <= num; ++i) {
        if (num % i == 0) {
            sum += i;
        }
    }
    return sum;
}
int main() {
    vector<pair<int, int>> result;
    int K;
    cout << "Enter the value of K: ";
    cin >> K;
    for (int m = 1; m <= K;++m) {
        for (int n = 1; n <= m;++n) {
            if (sum_of_divisors(m) + sum_of_divisors(n) == K) {
                result.emplace_back(m, n);
            }
        }
    }
    cout << "Pairs (n,m) such that sigma(n)+sigma(m)=K:\n";
    for (const auto& pair : result) {
        cout << "(" << pair.first << ", " << pair.second << ")\n";
    }
   return 0;
}
```

[1]Just for fun let us take large numbers:
$1000000000063 = \sigma(1)+\sigma(1000000000061) = \sigma(400)+\sigma(999999999101) = \sigma(2025)+\sigma(999999996311)$
$123456789101112 = \sigma(6) + \sigma(123456789101099) = \sigma(11) + \sigma(123456789101099) = \sigma(2028) + \sigma(123456789095987)$

So there is an observation that every positive integer $n > 3$ can be expressed as a sum of two sigma numbers

*i.e*;

$$n = \sigma(a) + \sigma(b)$$

for some $a\,, b \in \mathbb{Z}^{+}$

## 3 Conjecture

Every positive integer $n > 3$ is expressible as a sum of two sigma numbers in atleast one way.

In other words: For every postive integer $n > 3$ there exists atleast one pair $(a, b) \in \mathbb{Z}^{+} \times \mathbb{Z}^{+}$ such that

$$n = \sigma(a) + \sigma(b)$$

## 4 Remark 1

Goldbach's conjecture states [2] that every even natural number greater than 2 is the sum of two prime numbers. If Goldbach's Conjecture is true then we can easily show that every even positive integer $m^{*} > 4$ can be written as sum of two sigma numbers.

*Proof.* Let $m > 2$ be any even positive integer then it can be expressed as a sum of two primes say $p_1$ and $p_2$ *i.e*;

$$m = p_1 + p_2$$

$$\implies m + 2 = (p_1 + 1) + (p_2 + 1)$$

$$\implies m + 2 = \sigma(p_1) + \sigma(p_2)$$

$$\implies m^{*} = \sigma(p_1) + \sigma(p_2)$$

for some even positive integer $m^{*} > 4$

Now since, $4 = \sigma(1) + \sigma(2)$

So it is Proven for even positive integers $m > 3$ .

Although there will exist some $a$ and $b$ (both not prime together) such that $\sigma(2n) = \sigma(a) + \sigma(b)$ for $n > 1$

## 5 Remark 2

Now for the odd positive integers, it is very difficult to show that there always exists atleast one pair of $(a, b) \in \mathbb{Z}^+ \times \mathbb{Z}^+$ such that

$$2n + 1 = \sigma(a) + \sigma(b)$$

for $n \geq 1$ , Moreover we have no idea how to prove this statement.

## 6 Acknowledgement

The author thanks the anonymous referee for carefully reading the paper.

### References

[1] Paul J McCarthy. *Introduction to arithmetical functions.* Springer Science & Business Media, 2012.

[2] Eric W Weisstein. Goldbach conjecture. *https://mathworld. wolfram. com/*, 2002.