

# Vehicle Longitudinal Dynamics Model

Mustafa Yildirim \*

*İTÜ Faculty of Aeronautics and Astronautics*

**This technical report presents a MATLAB Simulink model that represents the longitudinal dynamics of an actual vehicle with remarkable accuracy. Through validation against empirical data, the model demonstrates a close adherence to the real-world behaviour of a vehicle, encompassing key aspects such as acceleration, braking, and velocity control. With its versatility and applicability in various engineering domains, this model is a valuable tool for automotive research, aiding in developing advanced control systems and autonomous driving technologies.**

## Nomenclature

(Nomenclature entries should have the units identified)

$cg$  = center of gravity

$C_x$  = force coefficient in the  $x$  direction

$C_y$  = force coefficient in the  $y$  direction

$dt$  = time step

$F_x$  =  $X$  component of the resultant pressure force acting on the vehicle

$F_y$  =  $Y$  component of the resultant pressure force acting on the vehicle

$F_z$  = Tire vertical force

$g$  = gravity

$h$  = height

$m$  = mass of vehicle

$rw$  = effective radius

## I. Introduction

The longitudinal dynamics of a vehicle play a crucial role in understanding its motion along the direction of travel. Valid representation and analysis of these dynamics are essential for developing advanced control systems, evaluating vehicle performance, and ensuring overall safety and efficiency. As the automotive industry progresses towards autonomous driving and intelligent vehicle technologies, the demand for precise simulation tools becomes

---

\*İTÜ Faculty of Aeronautics and Astronautics

increasingly significant. In response to this need, this report presents a comprehensive MATLAB Simulink model designed to simulate the longitudinal behaviour of a vehicle. The model is a powerful tool for researchers, engineers, and automotive enthusiasts, providing a reliable platform for analysing acceleration, braking, velocity control, and various longitudinal aspects that influence vehicle performance in diverse real-world scenarios. Through detailed research, the proposed Simulink model offers an invaluable resource for advancing our understanding of vehicle dynamics and accelerating the development of automotive technologies.

## **II. Longitudinal Vehicle Dynamics**

Longitudinal vehicle dynamics focuses on the forces and moments that influence a vehicle's movement in the direction of travel. It primarily concerns the vehicle's acceleration, deceleration, and braking behaviour. Critical parameters such as engine torque, braking force, tire characteristics, and aerodynamic drag significantly influence longitudinal dynamics. Understanding longitudinal vehicle dynamics is crucial for developing efficient, designing responsive braking systems, and optimizing acceleration performance. Additionally, lateral vehicle dynamics are pivotal in including cornering and turning behaviour. It involves the forces and moments that act perpendicular to the direction of travel, influencing the vehicle's path through curves and corners. Key factors affecting lateral dynamics include tire cornering stiffness, vehicle mass distribution, and the effectiveness of the vehicle's suspension system. Accurate modelling and analysis of lateral dynamics are essential for enhancing vehicle stability, handling, and overall manoeuvrability. Both longitudinal and lateral vehicle dynamics are interconnected and critical for a comprehensive understanding of a vehicle's behaviour. But in this report, we will investigate the longitudinal vehicle dynamics model to develop extensive vehicle control systems, improve safety features, and design vehicles that offer a balanced and enjoyable driving experience.

The Table 1 compares three distinct modelling approaches used in vehicle dynamics analysis: Analytical, numerical, and physical models. Each row of the table corresponds to different application scenarios. Analytical and numerical models exhibit notable strengths regarding real-time applications, making them suitable for real-time control and decision-making tasks. On the other hand, physical models reflect their limitations in real-time applicability due to their complexity and computational demands. The suitability of both analytical and numerical models remains to be determined for State-of-the-art vehicle dynamics simulations, given the challenges of accurately capturing highly complex and dynamic vehicle behaviour. For optimization and scientific research, numerical and physical models demonstrate strengths because they can handle intricate nonlinearities and deliver reliable results. In contrast, Analytical Models are less suitable, as they often rely on simplifications that may not capture the complexities of optimization and research scenarios.

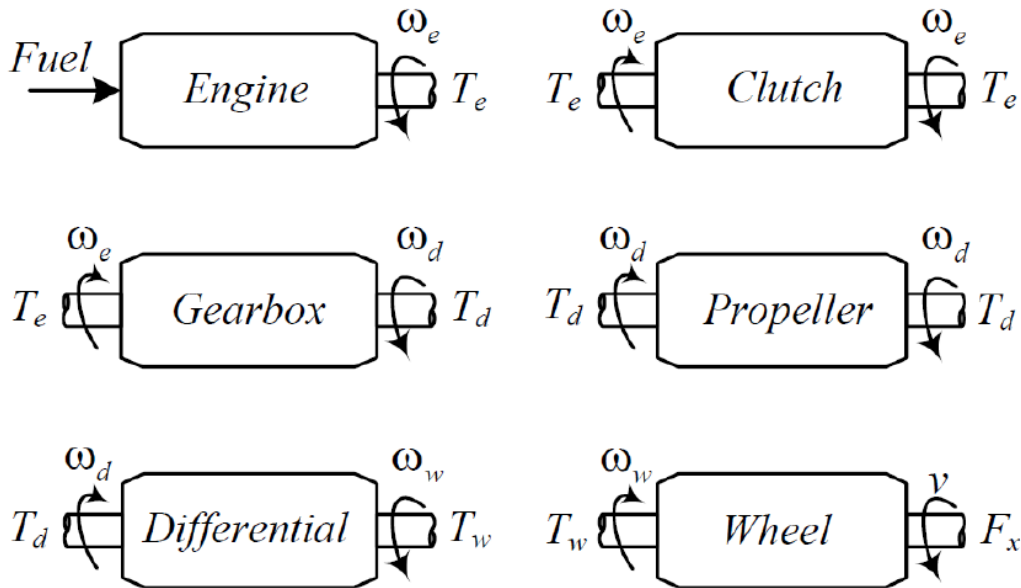
	ANALYTICAL MODELS	NUMERICAL MODELS	PHYSICAL MODELS
Real Time Applications	+	+	-
State-of-the-art vehicle dynamics simulations	?	?	-
Optimization, Scientific research	-	+	+

**Table 1 Comparison of different tire modeling approaches. [1]**

### III. Vehicle System Model

The Simulink vehicle system model is a comprehensive and versatile tool designed to simulate and analyse the dynamic behaviour of vehicles. It provides a platform to construct complex multi-domain vehicle models, combining mechanical, electrical, and control system elements to mimic real-world vehicle dynamics accurately. The Simulink vehicle system model enables engineers and researchers to study various vehicle performance aspects, including longitudinal and lateral dynamics, tire behaviour, suspension characteristics, and drivetrain performance. Simulink’s intuitive block diagram approach can easily customise and modify the vehicle system model to represent various vehicle configurations and control strategies. This flexibility allows for rapid prototyping of different vehicle designs and testing of advanced control algorithms. Moreover, Simulink’s real-time simulation capabilities suit hardware-in-the-loop (HIL) testing and rapid prototyping in real-world scenarios.

#### A. Drivetrain Components Connections



**Fig. 1 Drivetrain components connection and force,torque transmission [2]**

## B. Force Balance

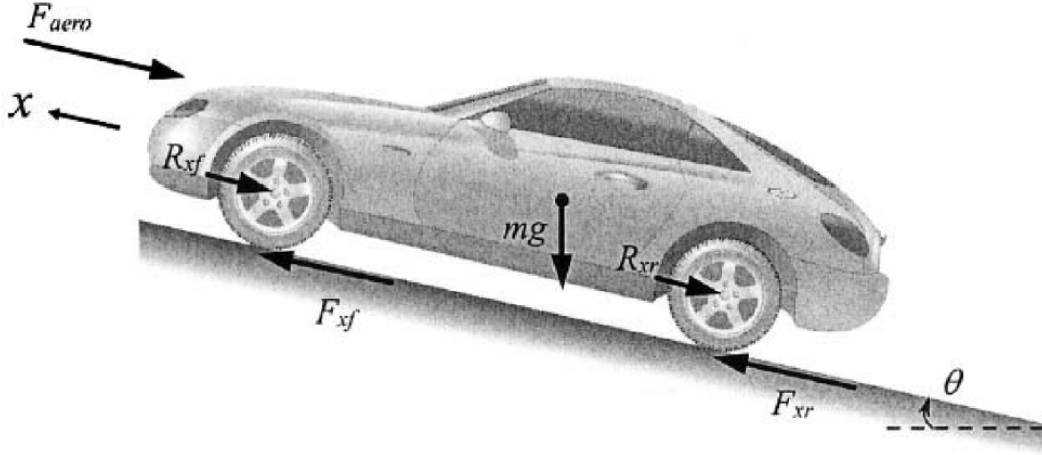


Fig. 2 Force balance longitudinal I[2]

$$m\ddot{x} = F_{x_f} + F_{x_r} - F_{aero} - R_{x_f} - R_{x_r} - mg \sin(\theta) \quad (1)$$

Where:  $m$  represents the mass of the vehicle,  $\ddot{x}$  is the acceleration of the vehicle in the longitudinal direction (change in velocity concerning time),  $F_{x_f}$  and  $F_{x_r}$  are the longitudinal tire forces at the front and rear axles, respectively,  $F_{aero}$  represents the aerodynamic drag force acting against the vehicle's motion,  $R_{x_f}$  and  $R_{x_r}$  are the rolling resistance forces at the front and rear axles, respectively,  $mg \sin(\theta)$  accounts for the component of the vehicle's weight that acts parallel to the road surface and is influenced by the slope angle  $\theta$  of the road and gravity (where  $g$  is the acceleration due to gravity).

## C. Aerodynamic Force

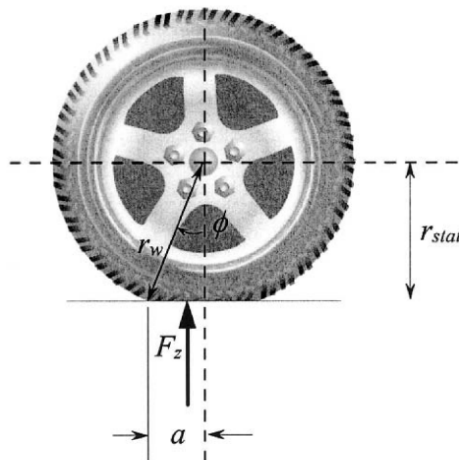
$$F_{aero} = c_x A \frac{\rho}{2} v_r^2 \quad (2)$$

where:  $F_{aero}$  is the aerodynamic drag force acting on the vehicle,  $c_x$  is the drag coefficient of the vehicle,  $A$  is the frontal area of the vehicle exposed to the oncoming air (measured in  $m^2$ ),  $\rho$  is the air density (measured in  $kg/m^3$ ),  $v_r$  is the relative velocity between the vehicle and the oncoming air (measured in  $m/s$ ) [3].

Symbol	Value
$c_x$	0.32
$A$	2; m <sup>2</sup>
$\rho$	1.22; kg/m <sup>3</sup>

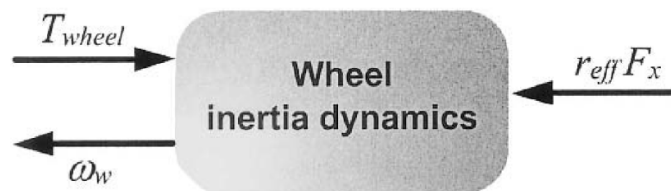
**Table 2** Aerodynamic force parameters

#### D. Tire Radius Torque Calculations



**Fig. 3** Tire radius torque calculations [2]

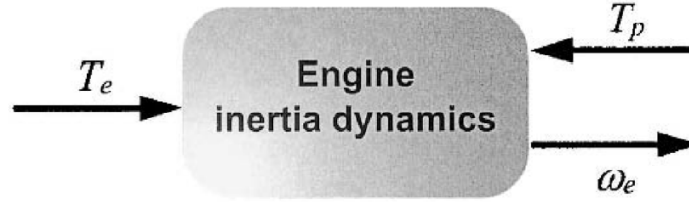
#### E. Wheel Speed Calculations



**Fig. 4** Wheel speed calculations [2]

#### F. Engine Speed Calculations

Engine speed calculations involve determining the rotational speed of an internal combustion engine's crankshaft, typically measured in revolutions per minute (RPM).



**Fig. 5 Engine speed calculations [2]**

### G. Rolling Resistance

The rolling resistance force ( $F_r$ ) can be expressed as a function of the vertical load on the tire ( $F_z$ ).

The equation for rolling resistance is given as follows:

$$\begin{aligned}
 F_r &= \mu_r F_z \quad \text{in [N]} \\
 \mu_r &= \mu_0 + \mu_1 v_x^2 \\
 F_r &= C_0 + C_1 v_x + C_2 v_x^4
 \end{aligned} \tag{3}$$

Where: -  $F_r$  is the rolling resistance force. -  $F_z$  is the vertical load or tire load, representing the force exerted by the vehicle's weight on the tire. -  $\mu_r$  is the rolling resistance coefficient, which accounts for the combined effects of different factors influencing rolling resistance. It depends on the tire properties and the vehicle's velocity ( $v_x$ ). -  $\mu_0$  is the minimum rolling resistance coefficient, representing the rolling resistance when the vehicle is stationary or moving at a very low speed. -  $\mu_1$  is a parameter that captures the speed-dependent effect on rolling resistance. It indicates how rolling resistance increases with the square of the vehicle's velocity. -  $C_0$ ,  $C_1$ , and  $C_2$  are coefficients that characterize rolling resistance for a specific tire and road condition. These coefficients are determined through experimental testing.

The equation shows that rolling resistance increases with the vertical load on the tire and is influenced by the vehicle's velocity. At higher speeds, the rolling resistance coefficient ( $\mu_r$ ) tends to increase, resulting in higher rolling resistance forces.

### H. Rotating Tire & Longitudinal Tire Slip

The equations describe the dynamics of a wheel's rotational acceleration, considering traction and braking forces and the tire slip[1].

$$\ddot{\varphi}_{\text{wheel}} = \frac{(M_{\text{traction}} - M_{\text{braking}}) - F_x \cdot r - F_R \cdot r}{J_{\text{wheel, equivalent\_powertrain}}} \tag{4}$$

In Equation 4,  $\ddot{\varphi}_{\text{wheel}}$  represents the angular acceleration of the wheel. The term  $(M_{\text{traction}} - M_{\text{braking}})$  denotes the net torque acting on the wheel, which is the difference between the tractive torque ( $M_{\text{traction}}$ ) and the braking torque

( $M_{\text{braking}}$ ). The other terms,  $F_x \cdot r$  and  $F_R \cdot r$ , correspond to the longitudinal forces due to wheel traction and rolling resistance, respectively.  $J_{\text{wheel}}$  represents the equivalent moment of inertia of the wheel and powertrain combined.

$$S = 1 - \frac{R_0 \cdot \dot{\phi}_{\text{wheel}}}{v} \quad \text{Tire slip during braking} \quad (5)$$

$$S = 1 - \frac{v}{R_0 \cdot \dot{\phi}_{\text{wheel}}} \quad \text{Tire slip during traction} \quad (6)$$

Equations 5 and 6 define the tire slip ( $S$ ) during braking and traction, respectively. Tire slip represents the difference between the actual wheel speed and the ideal wheel speed at the given longitudinal velocity ( $v$ ). In Equation 5, the slip during braking is determined by comparing the dynamic radius ( $R_0$ ) calculated from the tire circumference with the angular velocity ( $\dot{\phi}_{\text{wheel}}$ ) of the wheel. Conversely, in Equation 6, the slip during traction is determined by comparing the longitudinal velocity ( $v$ ) with the dynamic radius ( $R_0$ ) times the angular velocity ( $\dot{\phi}_{\text{wheel}}$ ) of the wheel.

Note: The dynamic radius ( $R_0$ ) is calculated based on the tire circumference and represents the effective radius of the tire when considering deformation during motion. The longitudinal velocity of the wheel carrier is denoted by  $v$ .

## I. Pacejka Tire Model

The Pacejka tire model is a widely used empirical model designed to characterize the tire forces and moments generated during vehicle motion. This model is based on a set of mathematical equations that describe tires' complex and nonlinear behaviour under various operating conditions, such as different slip angles and slip ratios. The Pacejka tire model considers the longitudinal, lateral, and aligning tire forces and the self-aligning torque, making it comprehensive and versatile for vehicle dynamics simulations. The model's key parameters are the stiffness coefficients representing the tire's deformation characteristics. These coefficients are determined experimentally through tire testing data. The Pacejka tire model is commonly used in vehicle dynamics simulations, vehicle handling analysis, and tire performance prediction due to its ability to accurately capture intricate and nonlinear tire behaviour with relatively simple mathematical expressions.

The given equations represent a simplified form of the Pacejka tire model.

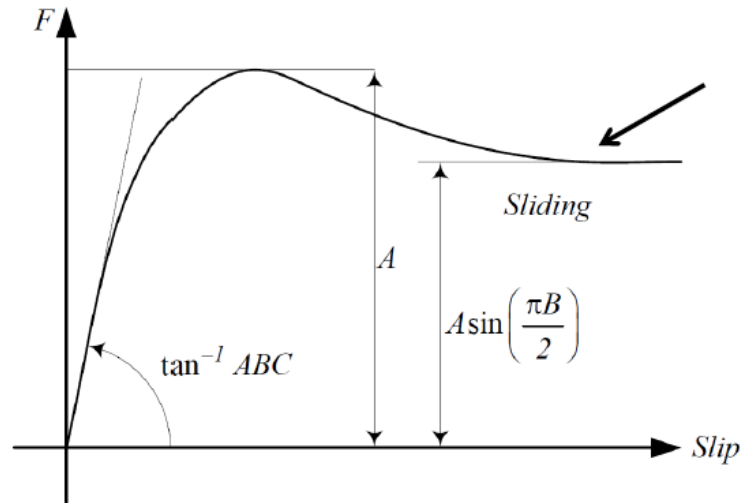
$$F = A \sin \left\{ B \tan^{-1} \left[ Cx - D \left( Cx - \tan^{-1}(Cx) \right) \right] \right\}$$

$$A = \mu F_z$$

$$C = \frac{C_\alpha}{AB} \quad (7)$$

$B, D = \text{shape factors}$

Where:  $F$  represents the lateral tire force acting on the tire in the direction perpendicular to the vehicle's motion



**Fig. 6 Pacejka tire model[1]**

during cornering.  $A$  is a constant that depends on the tire's properties and is proportional to the vertical load ( $F_z$ ) on the tire.  $B$  and  $D$  are shape factors that determine the shape of the tire force curve and are typically determined experimentally.  $C$  is a parameter derived from the tire's cornering stiffness ( $C_\alpha$ ) and the shape factors  $A$  and  $B$ .  $x$  represents the slip angle, which is the angle between the tire's direction of travel and the direction it is pointing (steering angle). The equation shows that the lateral tire force ( $F$ ) is influenced by the slip angle ( $x$ ) and is computed using trigonometric functions and the shape factors  $B$  and  $D$ .



## J. Model Parameters

Parameter	Value
$r$	$(15 \times 0.0254) / 2 + (0.185 \times 0.65)$ ; % m
$rw$	$0.98 \times r$ ; % Effective radius
$m$	1200 ; % kg vehicle mass
$g$	9.81 ; % $m/s^2$ gravity
$l$	2.4 ; % m lenght of vehicle
$a_1$	1.1 ; % m
$a_2$	1.4 ; % m
$h$	0.0508 ; % m
$\mu_0$	0.015 ; % [4]
$\mu_1$	$7 \times 10^{-6}$ ; % [4]
$\mu$	0.8 ; % [4]
$\theta$	0
$J_w$	1.2 ; % $kg/m^2$
$ z$	3000 ; % $kg/m^2$
$C_{alfaf}$	75000 ; % N/rad
$C_{alfar}$	75000 ; % N/rad
$ms$	375 ; % [4]
$cs$	1200 ; % [4]
$ks$	35000 ; % [4]
$\mu$	75 ; % [4]
$cu$	0 ; % [4]
$ku$	193000 ; % [4]
$csky$	3000

Above are the parameters, defined based on regular vehicle dimensions and other emprical values obtained from Jazar's book on Vehicle Dynamics [4].

## IV. Result

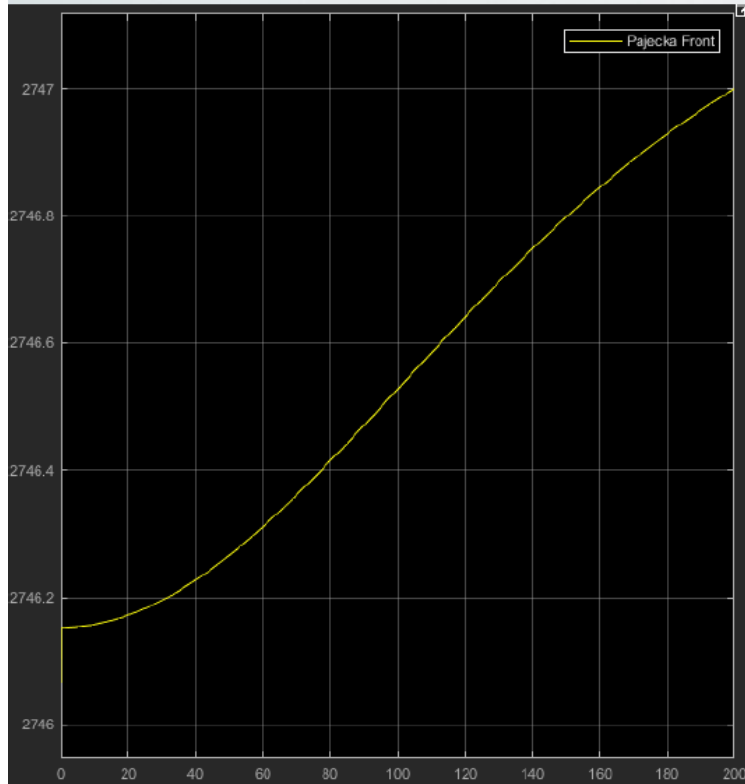


Fig. 7 Pacejka front wheel traction

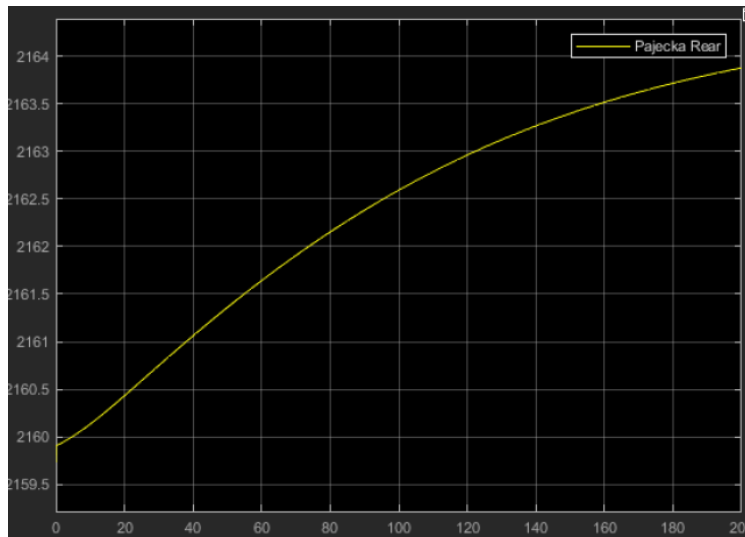
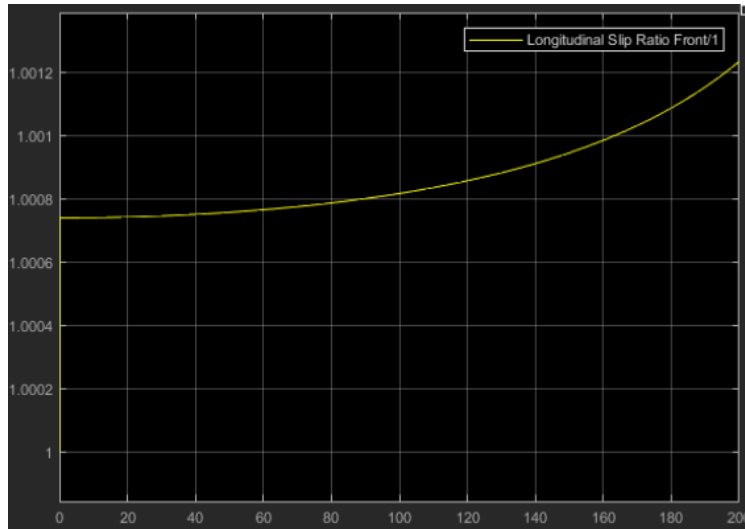


Fig. 8 Pacejka rear wheel traction



**Fig. 9 Longitudinal slip ration of front wheel**

## V. Conclusion

The longitudinal vehicle dynamics model presented in this study has demonstrated a high level of correlation with empirical studies, validating its accuracy and reliability in capturing real-world vehicle behaviour. Its ability to closely replicate actual longitudinal dynamics, including acceleration, deceleration, and braking characteristics, strengthens its value as a robust tool for vehicle dynamics analysis. Furthermore, the model's solid foundation in structural design provides a framework for conducting further in-depth studies and investigations into various aspects of vehicle performance, control systems, and safety features.

## Acknowledgments

This simulation model is published on [https://uk.mathworks.com/matlabcentral/fileexchange/71579-longitudinal-vehicle-dynamics?s\\_tid=ta\\_fx\\_results](https://uk.mathworks.com/matlabcentral/fileexchange/71579-longitudinal-vehicle-dynamics?s_tid=ta_fx_results).

## References

- [1] Sen, O. T., and Atabay, O., "Automotive Engineering, Vehicle Control Systems," 2017. Lecture Notes.
- [2] Rajamani, R., *Vehicle dynamics and control*, Springer Science & Business Media, 2011.
- [3] Mitschke, M., and Wallentowitz, H., *Dynamik der kraftfahrzeuge*, Vol. 4, Springer, 1972.
- [4] Jazar, R. N., *Vehicle dynamics*, Vol. 1, Springer, 2008.