Proof that the Real Part of All Non trivial Zeros of Riemann Zeta Functions is 1/2

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Abstract

Riemann hypothesis is that the real part of all nontrivial zeros of Riemann zeta functions is 1/2.Mr. Riemann formed Riemann zeta function by Analytic continuation of Euler zeta function,There are trivial and non trivial zeros in the Riemannian zeta function that make its value zero.Standard By analyzing the Trigonometric functions relationship in the equivalent Algebraic expression of Riemannian zeta function, it is concluded that the real part of all nontrivial zeros is 1/2.

Keywords:

Trivial zero/nontrivial zero/Analytic continuation

Riemann first starts from a prime number relationship proposed by Euler：

$$\prod \frac{1}{1-\frac{1}{\mathrm{p}^{s}}} = \sum \frac{1}{\mathrm{n}^{s}} \qquad \left(\mathrm{Re} > 1, n \in N^{+}\right)$$

In the above equation, p runs through all prime numbers， And n runs through all natural numbers， Then the following Riemannian $\zeta(\mathrm{s})$ functions are constructed through Analytic continuation:

$$2\sin s\pi \cdot \Gamma(s-1) \cdot \zeta(\mathrm{s}) = i\int_{-\infty}^{+\infty} \frac{(-x)^{s-1}dx}{e^{x}-1} \qquad (s_{\mathrm{Re}} \neq 1) \qquad (1)$$

This integral expression is analyzed on the whole Complex plane except that it is meaningless at s=1.According to the integral expression (1) above, it can be proven that $\zeta(\mathrm{s})$ satisfy algebraic relationship:

$$\zeta(\mathrm{s}) = 2^{s}\pi^{s-1}\sin\frac{s\pi}{2}\Gamma(1-s)\zeta(1-\mathrm{s}) \qquad (2)$$

Note: In $\sin(\frac{s\pi}{2})$ of the above equation:When the value of s is an even number such as 2, 4, 6, etc., the value of $\zeta(\mathrm{s})$ is zero，At this point, s is the trivial zero of the $\zeta(\mathrm{s})$.

Furthermore, it was found that in (2), the factors that cause the $\zeta(\mathrm{s})$ to be zero are more than $\sin(\frac{s\pi}{2})$, In ζ（1-s), if s in $\sin\frac{(1-s)\ \pi}{2}$ is equal to an odd number such as 3, 5, etc., it can also make the value of the ζ（s） equal to zero,This is a new discovery about the trivial zero point.

Next, we will analyze the situation when s is a complex number， On the right side of equation (2)， It can be seen that the product of $\sin\frac{s\pi}{2}$ in ζ（s） and $\sin\frac{(1-s)\ \pi}{2}$ in ζ（1-s） determines whether the value of the ζ（s） is zero ， If the product of $\sin\frac{s\pi}{2}\times\sin\frac{(1-s)\ \pi}{2}$ is zero, then the value of ζ（s） will also be zero accordingly， The product values given below are:

$$(\sin\frac{s\pi}{2})\times(\sin\frac{(1-s)\ \pi}{2})=\frac{1}{2}\sin(s\pi) \qquad (3)$$

Next, starting from this $(\sin\frac{s\pi}{2})\times(\sin\frac{(1-s)\ \pi}{2})=\frac{1}{2}\sin(s\pi)$, we will search for non trivial zeros.

The method is as follows:

Let $s=c+ib$ on the Complex plane be the nontrivial zero point of ζ（s）， s_{Re} is the real part of s, $s_{Re}=c$ （It has been proven that 0< c < 1）， ib is the imaginary part of s

In equation (3)， as long as $\sin(s\pi)=0$, then ζ（s）=0,Now expand $\sin(s\pi)$:

$$\sin(s\pi)=\sin(c\pi+ib\pi)=\sin c\pi\cdot\cos ib\pi+\cos c\pi\cdot\sin ib\pi \qquad (4)$$

Because $\sin c\pi\neq 0(0<c<1)$,So equation (4) can be transformed as follows:

$$\sin c\pi\cdot\cos ib\pi+\cos c\pi\cdot\sin ib\pi=\frac{\cos ib\pi+\cot c\pi\cdot\sin ib\pi}{\frac{1}{\sin c\pi}} \qquad (5)$$

Because: the value of ζ（s） at $s=c+ib$ is zero

So: the molecules in equation (5) must be zero,that is:

$$\cos ib\pi+\cot c\pi\cdot\sin ib\pi=0 \qquad (6)$$

Next, analyze equation (6) as follows:

If $\sin ib\pi=0$, then $\cos ib\pi\neq 0$, so $\cos ib\pi+\cot c\pi\cdot\sin ib\pi\neq 0$,

So ζ（s）$\neq 0$, obviously contradictory.

So it can be inferred that: $\sin ib\pi\neq 0$

So equation (6) can be transformed as follows:

$$\cot c\pi = -\frac{\cos ib\pi}{\sin ib\pi} = -\cot ib\pi \qquad (7)$$

It can be seen that the symbols on the left and right sides of equation (7) are opposite, and the left Trigonometric functions has no imaginary part, and the right Trigonometric functions has no real part, then immediately we can get:

$$\cot c\pi = -\cot ib\pi = 0$$

At this point: $\frac{1}{2}\sin(s\pi) = 0$, which is: $\zeta(s)$=0

Below is an analysis of the value of c when $\cot c\pi = 0$:

When $\cot c\pi = 0$, then: $c\pi = (1/2 + n)\pi \ (n \in N)$

We know that the range of values for the real part c is $0 < c < 1$, We can directly obtain: $c = \frac{1}{2}$, That is, the real part of the non trivial zero of $\zeta(s)$ is:

$$s_{Re} = c = \frac{1}{2}$$

So the non trivial zero of $\zeta(s)$: $s = \frac{1}{2} + ib$

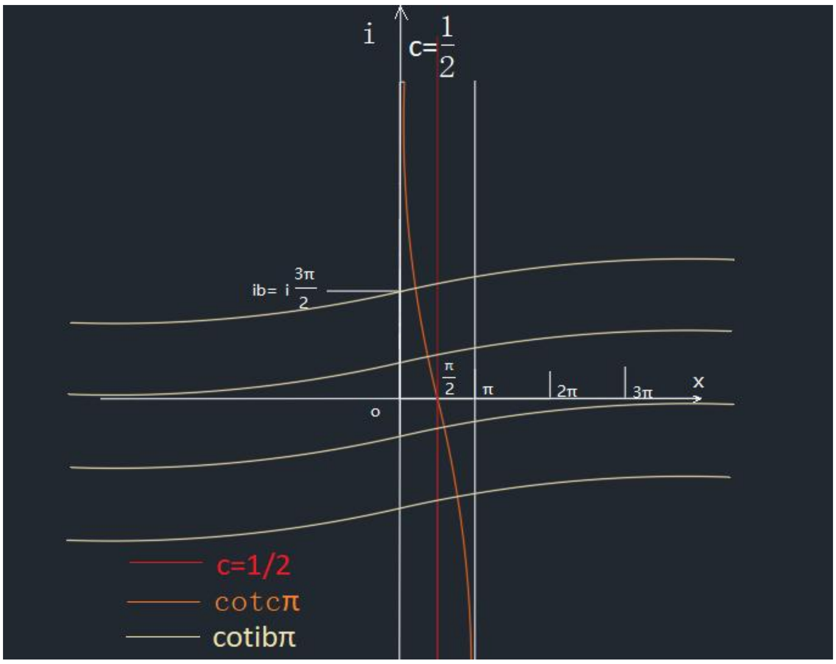
Now we know the real parts of all nontrivial zeros on the Complex plane: $s_{Re} = \frac{1}{2}$

When the value of $\zeta(s)$ is zero:

$$\cot ib\pi = 0$$

So the value of the imaginary part ib should be:: $ib\pi = i\,(1/2 + n)\,\pi$

At this time, the function image of $\cot c\pi = -\cot ib\pi = 0$ on the Complex plane is:



(a)

The value of imaginary part ib is：$ib\pi = i（1/2+\mathrm{n}）\pi$

Example:

When n=1，$ib\pi = i\frac{3}{2}\pi$， Below is a comparison table for non trivial zero values:

非平凡零点序列号	现代计算值c+ib	本文c+ib=(1/2+n) π i	n	c	本文ib=(1/2+n) π i	误差值：
		1/2+4. 7123889i	1	1/2	4. 7123889i	
		1/2+7. 8539815i	2	1/2	7. 8539815i	
		1/2+10. 9955741i	3	1/2	10. 9955741i	
1	1/2+14. 1347251i	1/2+14. 1371667i	4	1/2	14. 1371667i	-0. 0024416i
		1/2+17. 2787593i	5	1/2	17. 2787593i	
2	1/2+21. 0220396i	1/2+20. 4203519i	6	1/2	20. 4203519i	0. 6016877i
		1/2+23. 5619445i	7	1/2	23. 5619445i	
3	1/2+25. 0108575i	1/2+26. 7035371i	8	1/2	26. 7035371i	-1. 6926796i
4	1/2+30. 4248761i	1/2+29. 8451297i	9	1/2	29. 8451297i	0. 5797464i
5	1/2+32. 9350615i	1/2+32. 9867223i	10	1/2	32. 9867223i	-0. 0516608i
6	1/2+37. 5861781i	1/2+36. 1283149i	11	1/2	36. 1283149i	1. 4578632i
7	1/2+40. 9187190i	1/2+39. 2699075i	12	1/2	39. 2699075i	1. 6488115i
8	1/2+43. 3270732i	1/2+42. 4115001i	13	1/2	42. 4115001i	0. 9155731i
		1/2+45. 5530927i	14	1/2	45. 5530927i	
9	1/2+48. 0051508i	1/2+48. 6946853i	15	1/2	48. 6946853i	-0. 6895345i
10	1/2+49. 7738324i	1/2+51. 8362779i	16	1/2	51. 8362779i	-2. 0624455i
11	1/2+52. 9703214i	1/2+54. 9778705i	17	1/2	54. 9778705i	-2. 0075491i
12	1/2+56. 4462476i	1/2+58. 1194631i	18	1/2	58. 1194631i	-1. 6732155i
13	1/2+60. 83177785i	1/2+61. 2610557i	19	1/2	61. 2610557i	-0. 4292772i
		1/2+64. 4026483i	20	1/2	64. 4026483i	
注： π=3. 1415926						

（b）

From Figure (b), it can be seen that the error between the non trivial zero values calculated in this paper and the 13 non trivial zero values currently calculated is relatively small(The value marked in red is the value of the non-trivial zero calculated using the method in this paper)

The reason for the error is that modern calculation values are Π (x) The results calculated by the Li (x) function, while the value of non trivial zeros in this paper is calculated using the exact value of $c+ib=\frac{1}{2}+i（1/2+\mathrm{n}）\pi$.

Conclusion: The real part of all nontrivial zeros of Riemann zeta function is 1/2, and Riemann hypothesis is true!

References

[1] Riemann， On the number of primes less than a given value, Riemann's complete works, Vol. 1, Berlin Academy of Sciences monthly, 1859.11