Positive gravitational mass defect

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Abstract. We show that the gravitational mass defect is positive, in contrast to the electromagnetic or nuclear mass defects, which are negative.

Keywords: gravitational energy; gravitational collapse; curvature of space

1. Introduction

It is generally accepted that the gravitational mass defect is negative in the same way as the electromagnetic or nuclear mass defect. Misner, Thorne and Wheeler write [1]: "The mass-energy of the Earth-moon system is less than the mass-energy that the system would have if the two objects were at infinite separation. The mass-energy of a neutron star is less than the mass-energy of the same number of baryons at infinite separation".

But please be careful. When the apple, which Newton watched, fell on ground, it acquired kinetic energy $mv^2/2 = mgh$. So the apple acquired the mass $mgh/c^2 = \Delta m$, because $E = mc^2$. Thus, the law of mass conservation of the closed Earth-apple system was violated. The place where the apple fell absorbed this additional mass Δm in the form of the mass-energy of heating after the fall of the apple.

Similarly, during a gravitational compression of a dust cloud, the cloud heats up with a corresponding increase in the mass and, accordingly, with a violation of the mass conservation law of a closed system.

On the contrary, if the velocity of revolving of the Moon around the Earth is directed away from the Earth, the Moon will move away from the Earth, and its speed, and, accordingly, the mass will decrease compared to the initial values, and not increase, as the authors claim.

The law of conservation of mass-energy is usually violated during gravitational interaction. Einstein geometrized the Newtonian gravitational field. "Gravitational field" does not exist within the framework of general relativity. Weyl writes about "leading" (Führung) [2]. All gravitational phenomena are explained by the curvature of space-time. At the same time, no energy or mass is attributed to space itself. That's why the mutual gravitational attraction of masses is fundamentally different from the mutual electrical attraction of electric charges of different signs. To compare electrical and gravitational attraction, it is convenient to consider an isolated centrally symmetric system.

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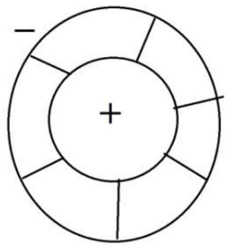

Fig. 1

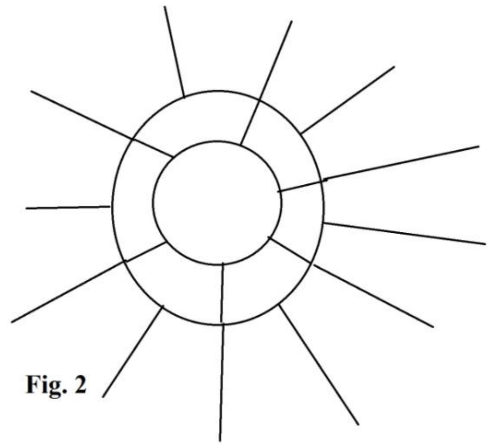

Fig. 2

In the case of electricity, this is an oppositely charged ball and a shell located at some distance from the ball (Fig. 1). Between them, and only between them, there is an electric field $\mathbf{E}$ with its mass-energy density [3, p. 41] $w = \varepsilon_0 |\mathbf{E}|^2 /2$. The field is eliminated when the shell falls on the ball, and the field energy goes first into kinetic energy, and then into heat. So the mass-energy of the system remains unchanged during the fall.

In the gravitational case (Fig. 2), this is a massive ball and a massive shell (we will assume that the masses are the same). Unlike the electrical case, the so-called “gravitational field” exists not only between the ball and the shell. The field of double strength exists outside the shell. The field is not eliminated when the shell falls on the ball. On the contrary, the field doubles in this interval. But the entire gravitational field exists in empty space, to which no mass is attributed. However, the kinetic mass-energy, and then the thermal mass, arise in the same way as in the electrical case. So in the gravitational case, the mass of the system increases! The gravitational mass defect is positive. It is an increase in mass. An example is constructed for the creation of matter in the form of a compact body in a gravitational field [4]. But since the gravitational field does not exist within the framework of general relativity, this mass increase comes from nothing from the modern point of view. Or you need to assign a negative sign to the energy density of the gravitational field, since the field increases simultaneously with the increase in mass. However Weyl writes: "Einstein left to the mercy of fate the principle of unique localization for gravitational energy. This negative sign was the main stumbling block" [2].

2. Space-time curvature

Einstein's great discovery is that matter turns out to bend space (space-time). The curvature of space depends both on the mass-energy density of the substance and on the pressure in the substance. This curvature of space occurs both inside the substance and in adjacent empty regions of space. In particular, the three-dimensional space around the Sun is curved. This curvature in the literal sense is observed as an additional curvature of the flight trajectory of photons attracted to the Sun with respect to the trajectory in Euclidean space [5-7].

Mathematically, Einstein's discovery is expressed by the fact that the Einstein tensor G^{α}_{β}, which describes the local geometric properties of space-time, is proportional to the matter energy-momentum tensor T^{α}_{β}, which locally describes the mass-energy and momentum density of matter:

$$G_\beta^\alpha \equiv R_\beta^\alpha - R_\mu^\mu \delta_\beta^\alpha / 2 = 8\pi\gamma T_\beta^\alpha, \quad \gamma = 7.4 \cdot 10^{-28} \text{ m/kg}. \tag{1}$$

The nature of the curvature of the space surrounding a star was studied, for example, in [8 § 100, problem 3]. In any case, the curvature of the centrally symmetric space is described by the linear element

$$dl^2 = g_{rr} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\varphi^2) . \tag{2}$$

The value of the metric coefficient g_{rr} was found by Schwarzschild in two cases [9 § 96]. For the empty space surrounding a star,

$$g_{rr} = \frac{1}{1 - r_g / r} ; \tag{3}$$

for the interior of a star of constant density,

$$g_{rr} = \frac{1}{1 - r^2 / R^2} , \tag{4}$$

moreover, if the radial coordinate of the star's surface is r_1, then

$$r_g = r_1^3 / R^2 . \tag{5}$$

This is the condition that the coefficients g_{rr} (3) and (4) coincide on the surface of the star.

In both cases $g_{rr} > 1$. This means that the curvature of space by a star is as follows. The distance between two spherical surfaces, the equators of which have lengths $2\pi r$ and $2\pi(r + dr)$, is not equal to dr, as it would be in Euclidean space, but more, namely $\sqrt{g_{rr}}dr$. Therefore, the volume of space enclosed between these spherical surfaces is equal to

$$dV = 4\pi r^2 \sqrt{g_{rr}} dr . \tag{6}$$

This means that the volume of space inside a sphere with an equator of length $2\pi r$ is greater than the Euclidean value $4\pi r^3 / 3$. This volume is obtained by integrating

$$V = \int_0^r 4\pi r^2 \sqrt{g_{rr}} dr . \tag{7}$$

And if this volume is filled with a substance with density ρ kg/m3, then the mass of this substance in the volume is obtained by integrating

$$M = \int_0^r \rho 4\pi r^2 \sqrt{g_{rr}} dr . \tag{8}$$

Such an integral was considered by Tolman. He called it "the total *proper* energy (of the liquid sphere)" [9 (97.4), (97.10)]:

$$\int \rho dV_0 , \quad dV_0 = \sqrt{-g_{xx} g_{yy} g_{zz}} dxdydz . \tag{9}$$

The calculation of the integral (8) for the metric coefficient (4) is presented in [10-12]. We repeat it here for the convenience of readers.

3. Mass of a ball of constant density ρ

According to Einstein's equation (1), $G_t^t = 8\pi\gamma\rho$. The component G_t^t is presented in [9 (96.7)] $G_t^t = 3 / R^2$. So

$$\rho = \frac{3}{8\pi\gamma R^2} = \frac{3 r_g}{8\pi\gamma r_1^3} . \tag{10}$$

Calculation of the proper mass of the ball using formula (8) or (9) gives the expression

$$M=\int_0^{r_1}\rho\sqrt{g_{rr}}\,4\pi r^2dr=\frac{3}{2\gamma R}\int_0^{r_1}\frac{r^2dr}{\sqrt{R^2-r^2}}=\frac{3r_1}{4\gamma}\left(\frac{\arcsin\xi}{\xi}-\sqrt{1-\xi^2}\right),\quad \xi^2=\frac{r_g}{r_1} \tag{11}$$

If the star is slightly compressed, then $\xi^2=\dfrac{r_g}{r_1}\ll 1$. Then

$$M\approx\frac{3r_1}{4\gamma}\left(\frac{2}{3}\xi^2+\frac{1}{5}\xi^4\right)=\frac{r_g}{2\gamma}+\frac{3r_g^2}{20\gamma r_1}=m+\frac{3r_g}{10r_1}m. \tag{12}$$

It can be seen that as the compression increases, the mass of the star increases due to the second term. We denoted

$$m=\frac{r_g}{2\gamma} \tag{13}$$

the initial mass of the initially rarefied cloud when $r_1\to\infty$ and the curvature of space is absent. So m is obtained by integration in flat space, without $\sqrt{g_{rr}}$ [8 (100.24)],

$$m=\int_0^{r_1}\rho 4\pi r^2dr=\int_0^{r_1}\frac{3r_g}{8\pi\gamma r_1^3}4\pi r^2dr=\frac{r_g}{2\gamma} \tag{14}$$

The increase in mass when compressing is easy to explain. According to (10), as the radius of the star r_1 decreases, the matter density increases $\rho\sim 1/r_1^3$. At the same time, the volume of the star decreases more slowly than $4\pi r_1^3/3$, due to the curvature of space. This is taken into account by the metric coefficient $g_{rr}>1$.

The increase in the mass-energy of a star during compression is beyond doubt, since the star heats up during gravitational collapse, which corresponds to an increase in the mass of iron during heating. Thus, both the calculation and the observation show that the *gravitational mass defect is positive*. This result was published [12].

4. Conclusion

We considered the gravitational compression of a star, during which the matter of the star was heated, and its mass was grown. Consider now the removal of the baryons of a neutron star to infinity. This removal requires mass-energy. But there is nowhere to take this mass from, except from the star itself. So when you remove baryons, you have to reduce the mass of the star. The mass has to be taken away from the baryons and introduced into the curved Schwarzschild space, spending this mass on straightening the space. Therefore, the total mass of baryons after their removal turns out to be less than the mass of the neutron star, contrary to the assertion of the respected authors.

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Notes

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