On Energy Numbers

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Abstract

The "energy numbers" of Parker Emmerson are critically examined, and an application of these exotic numbers to particle physics is attempted. Along the way, we establish the **quaternion field identity**, which is an isomorphism between a certain characterization of the abstract structure of a Hermitian space, and the complex Borel algebra of its generators.

Paper

Let $\mathcal{R}_k$ be a ring with identity. Let, for an overring $\widetilde{\boldsymbol{\mathcal{R}}}_k$, that there is a bounded section H_k which subsumes $\mathcal{R}_k$, or in other words, assume that there are fibers $\{k_i\}_{i\in I}$, of the sheaf I, for which there is a natural retract into some 0-truncated element r of $\mathcal{R}_k$ for every vertex in the face map of the realization |I| of I. Synonymously, let there be a shape morphism $\int I: I \to \{r_i\}_{i\in I}$ which corresponds to the right adjoint of the overall retract. Then, if such a property holds for every possible H_k over every subring $\mathcal{R}_k{}^\flat$ of $\mathcal{R}_k$, we call the generic extension $\mathcal{E} \simeq H_k$ the "Prüffer extension" of the original ring.

Prüffer extensions have some nice properties. For starters, we can consider the epimorphism

$$\mathcal{E} \mapsto \mathcal{R}_k{}^\flat$$

as a restriction $\mathcal{E}|_b$, where b is a prime ideal of $\mathcal{R}_k{}^\flat$, and is also the maximal generating ideal. Then, we have a commutative square

$$\begin{array}{ccc} \mathcal{E}\text{:Rings} & \to & \mathcal{E}|_b\text{:Rings} \\ \downarrow & & \downarrow \\ \mathcal{E}\text{:nCats} & \to & \mathcal{E}|_b\text{:sCat} \end{array}$$

with the terminal object being a slice category over the ground field, or equivalently, a quasi-injective refinement of the stack containing the ground field as a base object. Further, all of the images obtained by moving from left to right in the above diagram are integrally closed over a smooth subscheme of the original, once the necessary identification between the formal ring of $\mathcal{E}$ and the scheme over $\mathcal{E}|_b$ is made by identifying b_i as the index for the constructible sheaf (call it $\mathcal{O}_S$) whose values are the $\mathcal{R}_k$-modules of the flat field extension.

Theorem 1.0.1 For any Prüffer extension $\mathcal{E}$, there is a valuation $\mathfrak{v}_{\mathcal{E}}$ and a map

$$\mathfrak{v}_{\mathcal{E}} \twoheadrightarrow \mathfrak{v}_{\mathcal{E}|_b}$$

whose image is strictly coarser than its preimage.

Proof This follows from the fact that $\mathcal{E}$ is an extension of $\mathcal{R}_k$; thus, one has that (excluding the pathological *field with one element*), there is a natural inclusion

$$\mathrm{k} \hookrightarrow \widetilde{k};$$

we make the identification

$$\widetilde{k} = \mathrm{k} \cup \{\mathrm{i}\}$$

and thus, by letting $v(k){:}k \to \widetilde{k}$ be the injection from the spectrum of the base ring to its extension, one concludes that there is an adjoint surjection in the opposite category $\mathrm{sCat}^{\mathbf{op}}$.

Let the index of $\mathscr{O}_S$ vary continuously as a parameter of the function v(k), with k totally real. Then, one obtains the following delooping:

$$\mathcal{O}_{b_i} \twoheadrightarrow \mathrm{Hot}(\mathfrak{o}_{\mathcal{E}|_b}) \sim \boldsymbol{q}$$

and there is an isometry between $\mathfrak{L}_\infty(\mathrm{i})$, the infinite loop group of the Drinfeld center of the the category $\mathcal{Z}(\mathcal{E})$ of the Prüffer extension of our generic ring ,and the homotopy type $\boldsymbol{q}$ of the valuation over the target of the surjection. Thus, there is a blow-up

$$\boldsymbol{b}{:}\,\boldsymbol{p} \to \boldsymbol{q}$$

where $\boldsymbol{p}$ is a prime representative of the structure sheaf over the fiber bundle of $\mathcal{R}$ and its refinements, and $\boldsymbol{q}$ is a representative character of the indigenous bundle $\mathcal{Q}|_{\mathcal{E}}$.

One could, in principle, implement the homologous instantiation in the complex case by assuming $\mathcal{O}_{b_i}$ to be a complex field $\mathbb{C}$ with prime ideal $\boldsymbol{p}$=a+ib and letting $\boldsymbol{p}^{\dagger}=\boldsymbol{q}$ be the real manifold diffeomorphic to the center of $\mathbb{C}$. Interestingly for us, suppose that $\mathbb{H}=\mathrm{rep}(\mathbb{C}\mathrm{P}^2)$ is a manifold of complex dimension 2, and that there is a generative group object $\mathfrak{slp}$. Then, one obtains

$$\boldsymbol{b}_{\mathbb{H}}{:}\ \mathbb{H} \to P^2_n \mathrm{h}^{0,1}$$

where the right-hand-side is a totally real surface of irregularity 0. The morphism is forgetful of the Hodge number of $\mathbb{H}$, and there is a right adjoint $\grave{\boldsymbol{b}}_{\mathbb{H}}{:}\ P^2_n\mathrm{h}^{0,1} \to \mathbb{H}\mathrm{h}^{0,1}$, in which the generator $\mathfrak{slp}$ is representative of the tautological line bundle $\mathcal{l}^1$.

Suppose one wishes for a “complexification” of this morphism, such that the irregularity of the image does not remain fixed at zero. Then, one has a map

$$\mathrm{Comp}{:}\ P^2_n\mathrm{h}^{\mathrm{i},1} \to \mathbb{H}$$

which is a blow-up of the vect-enriched monoidal category P^2_n whose center is bounded by a metric $\mathfrak{o}_n$. This lends a somewhat differential flavor to our computation; observe, we allow i to depend on v(k) as we previously did with b_i, but this time we restrict ourselves to the 1-truncated case, in which the leftmost set of n-places have been removed. We then rewrite our function as

$$\text{Comp}: P_n^2 \mathrm{h}^{i,1} \to P_n^2 \oplus P_{-n+v(k)i}^{-1}$$

and obtain the inclusion

$$\text{Rings} \subset \text{Fields} \subset \text{Rep(Fields)} \simeq \text{Manifolds};$$

specifically, we define the inclusion

$$\mathrm{Re}(\mathbb{H}) \subset \mathbb{H} \oplus \mathrm{h}^{i,j}(\mathcal{Z}(\mathcal{E}))$$

from the reduced total space (real part) of $\mathbb{H}$ to the quaternionic sector. In the special case where the functor

$$\ell_{\mathbb{H}}: \mathbb{H} \to P_n^2 \mathrm{h}^{0,1}$$

is only *trivially forgetful*, i.e. is *effectively faithful*, we have that $\mathbb{H}$ is a K3 surface, and the delooping gives us a correspondence between the Prüffer extension of the ground field $\mathcal{R}^2$ and the preimage of the blow-up of a genus-i stratified vector space consisting of orthonormal slice categories.

We may then proceed to define this correspondence as follows; let $\mathcal{R}^2 \sim \mathrm{rep}(\mathbb{A}^2 \times [0,1] \cup \infty)$ be the affine 2-space, and set $\mathrm{Re}(\mathrm{h}^{0,1}\mathbb{H})$ to be the real part of the canonical quaternionic space. Given that we have already established that there is a flat embedding between the two, we now proceed to define the structure sheaf of $\mathrm{Re}(\mathrm{h}^{0,1}\mathbb{H})$ to be compact within $\mathbb{H} \cup \{\infty\}$. Then,

Definition 1.1.1 The *quaternion-field identity* (QFI) is an fpqc embedding from a subring of R^n consisting of an interval which is open in $\mathbb{R} \cup \{\infty\}$, and the fundamental representation of the group consisting of its centers.

The QFI gives us an atlas $\mathfrak{Q}_{\mathbb{C}}$ which constitutes a symmetric monoidal category. Its objects are real subfields of $\mathbb{R}^n[0,1]$ which represent differential geometric spaces, which we shall call "Q-charts," and its morphisms are transition maps between rigid isometries. The universal property of $\mathfrak{Q}_{\mathbb{C}}$ is that every map

$$\mathfrak{Q}_{\mathbb{C}} \to \mathfrak{Q}_{\mathbb{C}}(\ell)$$

preserves *exodromy*, in the sense that the classifying space $\mathbf{BG}(\mathfrak{Q}_{\mathbb{C}}(\ell))$ preserves *local* (n-truncated) path-connectedness of $\mathbf{BG}(\mathfrak{Q}_{\mathbb{C}})$. This means that the *finite* weight of the vector bundle

$$\mathcal{V}_{\mathrm{FIN}}(\mathfrak{Q}_{\mathbb{C}})$$

over each homotopic copy of $\mathbb{R}^n[0,1]$ is *preserved* by the map

$$\varphi_{\text{proét}}: \mathbb{A}^{-1}\mathbb{R}^n \to \mathbb{A}^2\mathbb{R}^n \sim \mathbb{H}_{\mathrm{perf}}$$

$\dim(\mathbb{H}_{\mathrm{perf}})$ is exactly equal to 2n. This means that the shape monad

$$\int \mathfrak{Q}_{\mathbb{C}} = (\mathfrak{Q}_{\mathbb{C}}, \ell, \varphi)$$

has faithful actions exactly when the supersite of the Drinfeld module is obtained as an involution of the highest weight category $\mathrm{H}_{\overline{W}}$ of $\mathfrak{Q}_{\mathbb{C}}$, which is itself obtained as the sum of valuations of the group subobjects "spanned by" $\mathfrak{slp}$. Here, "spanned by" is in scare quotes to indicate quasi-exactness – that is to say, $H_{\overline{W}}$ is not *spanned by* $\mathfrak{slp}$ per sé, but more properly, it is spanned by its host category. We shall call this category SCat.

SCat is downward closed under disjoint unions, such that if $A \supset \{a,b\}$ and $B \supset \{c,d\}$ are classes of SCat, and if $a \sqcup c$ and $b \sqcup d$ are subsets of distinct objects C and D, then any disjoint unions

$$—\sqcup D \text{ and } —\sqcup C$$

are also contained in SCat. This conveniently turns SCat into a comma category,

$$\{A,B\}\downarrow\{D,C\},$$

which we will exploit because the arrow notation is suggestive of downward closure.

Thus,

Theorem 1.1.2 If $\vec{A}$ and $\vec{B}$ are linearly independent vectors in $\text{Vect} \subset \text{SCat}$, then for $0<\varepsilon<1$, $\varepsilon\vec{A}+\varepsilon'\vec{B}$ are contained within SCat.

Proof Write $\vec{C}=\vec{A}+\varepsilon'\vec{B}$; then, one has that $(\vec{A},\vec{B})\downarrow\vec{C}$ is well inside SCat by downwards closure.

Let I_D be an integral domain, and Rat(Q) be the set of rational numbers in I_D;

Theorem 1.1.3 Maps between Q-charts constitute completions of Rat(Q)

Proof Suppose there is an endomorphism $\varphi_{\text{proét}}$:Q⟶Q, and allow this to be the template for our generic Q-chart. Then, one has

$$\begin{aligned} & f(\mathbb{A}^{-1}\mathbb{R}^n) \to \mathbb{A}^2\mathbb{R}+b \\ = \quad & (\tfrac{1}{\mathbb{A}}\mathbb{R})+b \\ = \quad & \text{Rat}(Q)+b \end{aligned}$$

we allow b to be a number transcendental over Q. Thus, we obtain the completion $Q^\sharp$, which is the image of the proétale functor in $\mathfrak{Q}_{\mathbb{C}}$.

Write $\partial(\mathcal{Q}|_\varepsilon)$ for the boundary of a submanifold whose span is generated by the scalar field embedded within the indigenous bundle over Q. Assume that there is some unipotent character $\boldsymbol{p}$ whose blowup is found within the word $\boldsymbol{q}_w$. Then, there is a conic section

$$\partial(\boldsymbol{q}_w)+i(w_j)^2=k$$

with a distinguished basepoint j about which the infinite loop space $\mathfrak{I}=\Sigma_k\boldsymbol{q}_w\Omega^i_\infty$ is rooted. Assume further that there is a compass $\mathscr{C}(\mathfrak{U}(\mathfrak{I}))$ in which the basepoint is portable, up to unique diffeomorphism. Then, by rigidity of the induced sub-bundle $\varphi(\mathfrak{I})$, there is a unique ∞-topos which is tautologous with the domain of $\mathcal{Z}(\mathfrak{I})$. The one-point compactification

$$\mathcal{Z}(\mathfrak{I})_{\text{red}} \to \{*\}$$

gives the ideal of the codensity monad Q_{cod}, which has as its right adjoint the stabilizer of the compass $\mathscr{C}(\mathfrak{U}(\mathfrak{I}))$, which is tame.

By the tameness of the parent compass, the induced simplicial chain

$$\Delta k_c \to \Delta k_{c+\varepsilon} \to \dots \to \Delta k_{c+\infty\varepsilon}=\Delta k_c$$

yields a surprising well-behavedness that transforms the $\mathfrak{J}$-categorical subobjects into CW complexes. The sequence is right-exact, 0-truncated, spectral, polar, and Abelian up to isogeny with its $\mathfrak{U}$-variety. Thus,

Theorem 1.2.1 For two harmonic functions f($\mathfrak{J}^{-1}$),g($\mathfrak{J}^{-|n|}$), the *principal fiber bundle* $\mathfrak{F}(\mathfrak{J})$ takes germs in $\mathfrak{J}$ and maps them to cokernels in $\mathfrak{J}^{+}$.

Proof Let $\mathbb{T}$ be a hyperbolic triangle encompassed by $\mathscr{C}(\mathfrak{U}(\mathfrak{J}))$, and let the Re($\mathbb{T}^{+}$) be the real representation of $\mathbb{T}$. By isogeny, we can find a doublet {k,Re($\mathbb{T}^{+}$) }, which is comparable under an equivalence relation induced by the universal property of $\mathscr{C}(\mathfrak{U}(\mathfrak{J}))$. That the underlying topology is an infinite loop space means that all path-connected based curves generated by integral domains are homologous with one another. Thus, there is some sheaf, $\mathcal{O}_{\mathfrak{F}}$ whose germs are the pre-images of the functors f and g, and whose underlying topos is locally compatible with the E_{∞}-space of $\mathbf{BG}(\mathfrak{J})$.

Let there be an inclusion

$$\text{Topoi} \subset \text{TopComp}$$

into a vect-enriched tensorable commutative monoidal category. Then, the simplicial subsets of $\int\mathbf{BG}(\mathfrak{J})$ admit graded blow-ups. We then proceed to define the morphism

$$\lambda\colon \mathbf{BG}(\mathfrak{J}) \to \int\mathbf{BG}(\mathfrak{J}) \in \mathscr{C}(\mathfrak{U}(\mathfrak{J})),$$

which takes (2,1)-horns (frames) in the category of Topoi and maps them bijectively to holomorphic products in the fiber-space encompassed by the Grothendieck universe of im($\mathbf{BG}(\mathfrak{J})$). We call the target of this morphism $\mathfrak{J}^{+}$, and write

$$\begin{array}{ccc} \ker(k) & \to & \mathrm{Re}(\mathbb{T}^{+}) \\ \downarrow & & \\ \mathrm{Re}(\mathbb{T}^{+}) & & \end{array}$$

that the diagram is commutative permits us to rewrite Re($\mathbb{T}^{+}$) as coker(im(ker(k))). Thus, we have proven the theorem.

Warning 1.2.2 One could get away with a shorter version of this proof if we were to accept from the get-go that we were working with Riemann surfaces; as this was the setting for the classical version of Gunning's "indigenous bundles," such case easily follows; however, we have generalized the concept here somewhat to be compatible with the less restrictive class of bornological spaces known as "semi-Banach spaces."

We would be remiss not to mention here the possibility for specializing this construction to the case of Euclidean buildings, which is actually quite natural; our classifying space transforms to the classical Bruhat-Tits buildings, and holomorphic functions become localizations from the Coxeter group of the underlying curve complex to the individual apartments, which are more or less synonymous with $\mathrm{Re}(\mathbb{T}^{+})$. In order to achieve this construction, we would like to decompose $\mathbb{T}^{+}$ as follows:

$$\mathbb{T}^{+} = \frac{1}{2}\sum_{n=1}^{3} PSL_{n}$$

Here, each individual n in the equation's right-hand-side gives us a unique "wall" of the apartments, or an "alcove," to use more standard terminology. Remarkably, there is a chain of isogenies

$$\mathbb{T}^{\pm} > \mathbb{T}^{+} > \mathcal{M}_{\mathrm{ell}}(\mathrm{Re}(\mathbb{C}^{2})) > \ldots > \mathcal{Z}(\mathfrak{I}),$$

which gives us the necklace $\mathcal{Nec}(\mathbb{H})$, whose strata are bounded by multipartite graphs which admit isotonic injections into the category of simplicial complexes. In other words, there is a (3g-6)-dimensional surface of a Kähler manifold with complex root which is a bounded, semi-connected, warped product manifold. This surface is K3 when the Hodge diamond of the Grothendieck compass is

$$\begin{matrix} & & 1 & & \\ & 0 & & 0 & \\ 1 & & 20 & & 1 \\ & 0 & & 0 & \\ & & 1; & & \end{matrix}$$

and in such case, there is a bordism $\mathcal{B}_{\mathrm{Emm}}$ which has an Emmerson energy number of

$$\mathbb{E}_{\mathrm{Emm}}=\mathrm{Re}\left(\frac{2i\mathbb{C}+\pi k(\frac{\mathbb{Z}}{p}\mathbb{R})\mathfrak{h}}{\sqrt{2}}\right),$$

and the Q-charts of the Bordism become *oscillatory*, in that they mediate between manifolds of dimension (3g-6) and (3(g-1)-6), giving us an irregular Hamiltonian and condensed ultraviolet spectrum. We now pause to remind the reader of the definition of an *energy number* (liberally interpreted) as provided by Emmerson:

Definition 1.3.1 Let $\mathcal{S}$ be a smooth atlas over a collection of fields of equal or mixed characteristic, and denote by $\mathrm{rep}(\mathrm{Fun}(\mathfrak{U}(\mathbb{F})))=\mathcal{S}_{\mathrm{ger}}$ the set of all *gerbes* over the *frame field* $\mathbb{F}$. Then, write

$$\mathcal{S}(\mathcal{S}_{\mathrm{ger}}) \to \mathrm{Re}(\mathcal{S}(\mathbb{H}))$$

for the functor into $\mathfrak{oct}$, the *presentable category* of octonions and their compasses. Then, an energy number is a *flat embedding* of $\mathfrak{oct}$ into a k-manifold whose ground field is of *less than or equal* characteristic to its left adjoint.

Remark 1.3.2 Much of the potential (pun unavoidable) of energy numbers, as they have been espoused by Emmerson have gone unrealized; their progenitor has instead decided to rely on artificial intelligence to generate a semantically void but linguistically rich theory of "raising the dead." We depart heavily from these eccentricities here. Caution should be used when invoking the energy numbers, in part due to this troubling backstory.

Morally, the energy number of a *mixed state* is a motive which descends from the generative and undisturbed pure state of a parton to its desingularization. We define this descent as a tower

$$\mathbb{T}^{-k}\oplus_k\mathcal{M}_{\tilde{g}}$$

where $\tilde{g}$ is a Chevalley group whose left-codensity monads are all ground fields of positive Euler characteristic. We say that the ground k-fields are *Cauchy* if there is a θ-adherent (read: accumulation) point which is an ideal of the completion of every subfield k_b of $k^\sharp$. A Cauchy k-field is conformal to the ambient space of a CW complex, and the delooping of the parent monad is homologous to renormalization of the winding number of the vector bundle over a particle's path integral. Such renormalization may be written, tersely, as $Re|\ell^p|$, where a suitable ℓ^p norm is chosen so as to induce the property of Baire. Classically, this number is conserved, and is always Lebesgue measurable; as a result, it is an element of the isogeny complex $\tilde{\mathbb{Q}}_p$, and its tangent bundle is coincident with the Finsler geometry of a particle's local 4-momentum. We emphasize here that the 4-momentum is *local* so as to distinguish it from the Regge trajectory of the geodesic formed by the isotropic Yukawa-coupled system bound to the particle's worldline in the relative regime.

Definition 1.3.3 Write $\boldsymbol{w}_0$ for the 4-coordinate of a particle at time t_0. Say that the thermal evolution of the particle is *spectral* if

$$\lambda(\boldsymbol{w}_0) = \frac{1}{\sqrt{\frac{1}{2}dim(W)}}t_\alpha(\boldsymbol{w}_0),$$

where λ is the wavelength of the intermediating boson, and t_α is the Newtonian action of the particle, equal to

$$\frac{(x_n-x_0)+(y_n-y_0)+(z_n-z_0)}{t_n-t_0} = \frac{\partial}{\partial t}.$$

The leading term transforms the spectral time series into a harmonic and quasi-periodic evolution, and in the 2-dimensional case, the wavelength is exactly equal to the Newtonian action. For 4-manifolds, we obtain the traditional quantum picture

$$\frac{1}{\sqrt{2}}\frac{\partial}{\partial t}\hbar,$$

which tells us the probability of observing the particle in either of its eigenstates. We then write the set

$$\dim(W)=k=4$$

and define the energy number of the mixed state of a multipartite entangled system as

$$\mathbb{T}^{-4}\oplus_4\mathcal{M}_{\tilde{g}};$$

subbing in the trace of the matrix

$(i\hbar)^\dagger\chi=\chi$	0

0	$(i\hbar)^{\dagger}\chi = \chi$

for $\mathcal{M}_{\tilde{g}}$, where χ is the Euler characteristic of the Finsler manifold in which $\boldsymbol{w}_0$ is embedded, and subbing in $\mathbb{Z}$ for $\mathbb{T}$, gives:

$$\frac{1}{\mathbb{Z}^4}2\chi$$

$$=$$

$$\frac{2\chi}{\mathbb{Z}^4}$$

as the energy number of a parton field in ordinary Minkowski spacetime.

References

M. Knebush, D. Zhang: *Convexity, Valuations and Prufer Extensions in Real Algebra;* [Documenta Mathematica, 2005]