The method by the definition of sets to compare the relative number of elements in infinite sets

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Abstract In mathematics, the derivation from the definition is reliable. Therefore, the method proposed in this paper to compare the relative number of elements of infinite sets from the definition of sets is highly reliable and accurate, far more reliable than the traditional one-to-one correspondence method, and will not cause any paradox. This paper also eliminates the paradoxes of Galileo, the infinite hotel, the whole equals the part and so on.

Key words infinite set; Definition method; One-to-one correspondence; Galileo's paradox; Infinite Hotel Paradox; the whole equals parts paradox

Mathematicians are intelligent, and mathematics itself has always been known for its rigor and has a high reputation in academia.

But in the infinite field, this is a different story. It is not only full of paradoxes such as Galileo's paradox, which is not allowed by any serious science, but also people seem to have no way to deal with them, and have to "adapt" them, live with them, and even use them for learning.

This situation has been going on for at least a few hundred years and it has aroused criticism from many people. For example, literature [1] argues that set theory is no more than a religious creed and lacks scientific significance; Literature [2] holds that the logic of axiomatic set theory is incoherent and infinite quantifiers have no meaning; Literature [3-4] discusses the consistency of set theory, while literature [5] attempts to reconstruct a set theory without paradox.

In the paradoxical set theory, how to judge the size of infinite sets is not only a basic problem in set theory, but also the fundamental source of all kinds of paradoxes.

For finite sets, the sizes of two sets can be compared precisely by comparing the number of elements in the sets or establishing one-to-one correspondence between the sets. However, for infinite sets, people thought that it was difficult to compare the number of elements, so Cantor extended the one-to-one correspondence to infinite sets, thus forming the so-called cardinal theory: if two infinite sets can establish a one-to-one correspondence, their cardinal numbers are said to be equal, otherwise their are not equal, and the size of the infinite set is measured by the cardinal number.

However, finite sets are fundamentally different from infinite sets. It is necessary to prove whether the effective one-to-one correspondence for finite sets can be generalized to infinite sets.

Unfortunately, no one has ever been able to prove this, and contrary examples abound.

One of the simplest counterexamples is the one-to-one correspondence between the set of rational numbers and the set of natural numbers.

It should be admitted that Cantor did a masterful job of establishing a one-to-one correspondence between the two, which any set theory textbook would seem to be proud of.

However, is the set of rational numbers and the set of natural numbers really the same size?

The answer is obviously no: the set of rational numbers contains the set of natural numbers, how can two sets be the same size?

In mathematics, derivations by definition are reliable. Any derivation that deviates from or conflicts with the definition, or even arbitrarily introduces unproven assumptions, is either unreliable or wrong, and has no scientific significance.

The same is true of the derivation of sets.

The number of elements in a set is the result of counting the elements of the set. Since the number of elements in an infinite set is infinite, the result of counting must be greater than any natural number, so we cannot give the result of counting the number of elements in any infinite set. But this does not mean that the relative number of elements in sets cannot be compared. In fact, mathematics starts with definitions, for example, when we define a proper subset of the set A, since the proper subset is just a set of some of the elements of A, and anyone with normal thinking ability knows that it must have fewer elements than A.

From this, we can reliably obtain their relative values according to the definition of infinite sets. This is because, when we define another set in terms of the elements of one or more sets, the relative relation between the number of elements of these sets is already determined, so the relative relation between the number of elements of these sets can be determined directly from the definition.

For example, the set of rational numbers can be defined as:

Set of rational numbers ={0}U{positive integer}U{negative integer}U{positive fraction with denominator not equal to 1}U{negative fraction with denominator not equal to 1}

Since the intersection of the sets to the right of the above equation is empty, the number of elements of the set of rational numbers is the sum of the number of elements of the above sets. This strictly proves that the number of elements of rational set is greater than that of natural numbers.

Thus, it is not possible to compare the sizes of infinite sets with unproven one-to-one correspondence.

Since all sets are defined, so long as the definition is clear, the relative number of elements between them can be determined.

For the sake of discussion, the method to compare the relative number of elements of the sets by their definition is called the definition method.

Because one-to-one correspondence can only compare the relative size of infinite sets at most, and its results are not reliable, and the definition method is directly from the definition, so it has a high degree of accuracy and reliability, which is incomparable to the so-called one-to-one correspondence method.

Since the definition method is highly reliable, it is impossible to produce any unreliable results when use definition method. This has been seen in the discussion above.

The following discussion is used to resolve Galileo's paradox.

Galileo's paradox is so influential that it can be said to be the fundamental paradox of set theory that it is unlikely that any teacher of set theory will recount the history of set theory without mentioning the paradox.

This paradox has been going on for nearly 400 years, and no one had really solved it before this writer, and Hilbert and Cantor had no idea what to do about it.

A misreading of this paradox also leads to the fundamental error that the so-called infinite set can correspond to its proper subset one by one.

Galileo's paradox means that even numbers (or square numbers) are only part of the natural numbers:

$$1,2,3,4,5,6....... \quad (1)$$

Therefore, the number of even numbers should be less than the natural numbers.

But you can actually establish the following one-to-one correspondence between the two by multiplying each natural number by 2:

$$1\rightarrow 2,\ 2\rightarrow 4,\ 3\rightarrow 6,...... \quad (2)$$

That is, each natural number corresponds to and only corresponds to one even number, which seems to indicate that there are as many even numbers as there are natural numbers, thus creating a paradox.

Although there was no concept of set in Galileo's time, it is still possible to analyze his ideas from the perspective of set theory.

When using the definition method to solve Galileo's paradox, since the method starts from the definition of sets, we must first give the clear definition of each related set.

Let the set of natural numbers be N={1,2,3...... }, the paradox refers to an even number that is only part of the natural numbers, i.e., the even number in eq. (1) is actually an even proper subset of N, defined as:

$$E=\{x|x \bmod 2=0, x\in N\}=\{2,4,6,......\} \quad (3)$$

Since every two adjacent elements of N can define only one element of the set E, as Galileo said, E has fewer elements than N, and is actually only half elements of N.

This is the relationship of the number of elements betwee sets N and E according to the reliable definition method.

The definition of the set of even numbers shown in (2) that can correspond to N one by one is:

$$E' = \{y \mid x, y = 2x \in a.\} = \{2,4,6......\}, \quad (4)$$

From the point of view of the definition method, since each element of N is defined only one element of E', the number of elements of E' and N are exactly the same, indeed forming a strict one-to-one correspondence.

With the above definitions and breaks, it is easy to dispel the so-called Galileo paradox.

Galileo was the first to make the astute observation that E' and E have different numbers of elements, but since both can be expressed as {2,4,6...... }, Therefore, as almost all mathematicians do, they mistakenly believe that they are the same set, how can the number of elements of the same set be different? That's why we think there's a paradox.

But from the definition point of view, the set is determined by the definition. Since the sets E’ and E are defined differently, how can they be considered to be the same set without good reason?

Besides, the two sets have different numbers of elements, so they can't be the same set at all.

Therefore, as long as the two sets are not the same set, the so-called paradox no longer exists: of course, the number of elements of different sets can be different, how can there be a paradox?

In fact, although E and E’ can both be expressed as {2,4,6....}, but for infinite sets, just because the elements seem to be the same doesn't mean they must be the same set. This is because only two sets with exactly the same elements are the same set. E’ and E have different numbers of elements, which is a fact, how can the elements be the same if the number of elements is different? Therefore, it is obvious that E’ and E are two different sets of even numbers. Since it is not the same set, the paradox is resolved.

Moreover, since the number of elements of E’ and N is strictly equal, and any proper subset of N is composed of only some elements of N, and of course has fewer elements than N, E’ having the same number of elements as N cannot be a proper subset of N.

Thus, the mistake of E’ as proper subset of N was made by Galileo, Hilbert, and Cantor, as well as by most mathematicians, and is the reason why the so-called Galileo paradox has not been resolved for about 400 years.

Cantor obviously went further on this wrong path: not only did he mistake E’ as proper subset of N, but he simply enlarged and "legitimized" this mistake: he assumed that any infinite set could correspond to its proper subset one by one, thus forming the astonishing paradox of "the part equals the whole."

In fact, there is no one-to-one correspondence between N and its proper subset E : E can only form an injective form with half of the elements in N, so this injective form is not surjective. Therefore, the assertion that "any infinite set can correspond to its proper subset" due to a misinterpretation of Galileo's paradox is also completely wrong.

In fact, not only is the set of even numbers not unique, but also the set of natural numbers is not unique although the set of natural numbers can all be represented as {1,2,3.......}.

For example, if two machines A and B are never stopping to produce parts, and A is twice as fast as B, let the natural numbers be used to number the parts produced by two machines A and B respectively, then after infinite time, two different sets of natural numbers are obtained. Let n represent the number of parts produced by machine B, then the number of parts produced by machine A can be represented by 2n. Because Lim[(2n)/n]=2 when $n\to\infty$, the set of natural numbers corresponding to A has twice as many elements as the set corresponding to B.

For another example, if a school consists of two infinite classes A and B, in which B has twice as many students as A, then the class numbers of the two classes can be represented by two different sets of natural numbers, and the school numbers can be represented by another set of natural numbers, which are all different.

Even, assuming there are infinite number of RMB, can the set of natural numbers in units of cent, yuan, billion yuan and trillion yuan {1,2,3,.....} be the same natural numbers set?

Thus, without any strict and reliable proof, to assert that the set of natural numbers is unique is too much to take for granted and too unrigorous.

As part of the set of natural numbers, the even or odd set is certainly not unique.

Further research, for example, which elements belong to E’ but not to E, will be discussed in the future.

There are other paradoxes in set theory that are not difficult to resolve by definition. For example, let N’={0}UN, obviously N’ has one more element 0 than N, according to the definition method, the corresponding number of elements is of course 1 more than N, , so there will be no infinite hotel, $\infty+1=\infty$ and other paradoxes obtained from one-to-one correspondence.

In fact, all paradoxes can ultimately be attributed to the confusion of some concept, or the introduction of some unproven hypothesis. For example, the so-called Galileo paradox, caused by the confusion of E’ and E, is essentially a simple error. Therefore, as long as enough care is taken to distinguish between various concepts that are not actually identical, and no unproven and not obviously valid assumptions are introduced, no paradox should exist.

The community of set theory should rectify its style of study and learn from other fields of mathematics with an open mind. It should no longer discuss sets apart from the definition of sets, nor introduce unproven hypotheses at will, nor build the community of set theory into a religious or even cult organization that does not allow discussion and questioning and only depends on faith because of problems that cannot be solved for the time being or in order to maintain authority [1]. Otherwise, even the most obvious and absurd errors, such as the equal number of natural and rational numbers, will be covered up by the religious atmosphere and become unquestionable "truths." However, the paper can not cover fire, mistakes must be exposed. Not every mathematician is willing to be blinded by foolish doctrines, or even defend them at all costs, and eventually become the joke of history.

It is suggested that the education department should suspend the infuse of infinite set theory, especially the theory of one-to-one corresponding and cardinal number, in middle school and undergraduate education, so as not to mislead young people.

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