

ON THE SET OF PRIME NUMBERS

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Abstract. "The Octets of the Odd Numbers" theory categorizes the odd numbers into four categories D_1 , Q_1 , D_2 , Q_2 . We relate the distribution of octets of odd numbers in the set of integers to the distribution of prime numbers and obtain an algorithm for finding the set of prime numbers of the form D_2 and Q_2 . The algorithm sequentially finds all prime numbers of the form D_2 and Q_2 in ascending order.

1. Introduction

"The Octets of the Odd Numbers" theory categorizes odd numbers based on the form of the octet they belong to or produce (see [8], equations (5.7), (5.8)):

$$D_1 = 11 + 8m' = 3 + 8m, m \in \mathbb{N} \quad (1)$$

$$Q_1 = 13 + 8m' = 5 + 8m, m \in \mathbb{N} \quad (2)$$

$$D_2 = 7 + 8m, m \in \mathbb{N} \quad (3)$$

$$Q_2 = 9 + 8m' = 1 + 8m, m \in \mathbb{N}. \quad (4)$$

Numbers D_1 of the form (1) produce the octets to which the numbers D_2 of the form (3) belong (see [8], section 5). Numbers Q_1 of the form (2) produce the octets to which the numbers Q_2 of the form (4) belong. Specific subsets of D_1 and Q_1 , found in a very simple way, produce the set of prime numbers of the form D_2 and Q_2 . This algorithm does not have the limitations of the so far known formulas for calculating prime numbers (see [1-7] and [9-16]).

2. The algorithm

The following lemma gives the form of powers of odd numbers.

Lemma 1. 1. Even powers Π^{2k} of odd numbers Π are of the form Q_2 .

2. Odd powers Π^{2k+1} of an odd number are of the same form (1) - (4) as the odd number Π .

Proof. 1. It suffices to prove that the squares of the odd numbers are of the form A_2 . We do the proof for numbers of the form (1). The proof for the forms of the numbers (2) - (4) is similar. We have:

$$D_1^2 = (3 + 8m)^2 = 64m^2 + 48m + 9 = 64m^2 + 48m + 8 + 1 = 8(8m^2 + 6m + 1) + 1 = 8m' + 1 = Q_2.$$

2. It is easily proven that multiplying an odd number of the form (1) - (4) by Q_2 does not change its form. Taking also 1 of the lemma into account, we get 2. \square

The following lemma gives the form of numbers $3D_1$, $5D_1$, $3Q_1$, $5Q_1$.

Lemma 2. Multiplying by 3 or 5 the equations (1), (2) we get:

$$\begin{aligned}
3D_1 &= Q_2 \\
5D_1 &= D_2 \\
3Q_1 &= D_2 \\
5Q_1 &= Q_2
\end{aligned} \tag{5}$$

Proof. We do the operations and get the equations (6). \square

Whether or not an odd number Π is prime depends on all prime numbers that are less than Π . The octets (or quadruple, or pair, see [8], section 6) produced by the numbers D_1 and Q_1 have a similar property: if $D_1, Q_1 > 2^n$ the numbers of the octets they produce are smaller than 2^n (see [8], Theorem 4.3). We investigate the possibility that the distribution of octets in the set \mathbb{N} is related to the distribution of prime numbers.

$D_1 = 2^n + 3$ produces the symmetric quadruple (7, 5, 1, 1) (see [8], equation (5.13)). Taking into account the Prime Number Theorem we ask for the prime numbers of the symmetric octets produced by the asymmetric odd numbers:

$$S = \left\{ D_1 = 3 + 8m / m \in \mathbb{N}, 2^n \leq 8m \leq 2^n + \frac{2^{n-2}}{n \ln 2} \right\}. \tag{6}$$

The upper limit of the value of the number m results from the Prime Number Theorem assuming that $\frac{1}{4}$ of them are of the form D_1 .

As a consequence of Lemma 1 we also ask for powers of prime numbers.

As a consequence of Lemmas 1 and 2, the prime numbers P of the numbers

$$\begin{aligned}
3^k \times P, k \in \mathbb{N} \\
5^\lambda \times P, \lambda \in \mathbb{N}
\end{aligned} \tag{7}$$

in the symmetric octets produced by the set S are either of the form D_1 or of the form Q_1 . These numbers are easily found from any factorization test. They appear in the symmetric octets produced by the set S and give the asymmetric primes D_1 and Q_1 .

Considering the above we get the following algorithm for finding the set of prime numbers.

Step 1. We choose a power of 2, $2^n, n \in \mathbb{N}^+$ and the set S ,

$$S = \left\{ D_1 = 3 + 8m / m \in \mathbb{N}, 2^n \leq 8m \leq 2^n + \frac{2^{n-2}}{n \ln 2} \right\}.$$

Step 2. We find the symmetric octets / quadruple / pair produced by the numbers D_1 of the set S .

Step 3. From the numbers of the octets we found we ask which are prime numbers P or powers of prime numbers. The prime numbers we find are the set of primes of the form D_2 and Q_2 that are less than 2^n . From the numbers of the octets we found we ask which are of the form (7). The prime numbers in equations (7) give the asymmetric primes D_1 and Q_1 .

3. The application of the algorithm

We present the application of the algorithm.

Step 1. We choose the power 2^7 and the set S ,

$$S = \left\{ D_1 = 3 + 8m / m \in \mathbb{N}, 2^n \leq 8m \leq 2^n + \frac{2^{n-2}}{n \ln 2} \right\} = \{131, 139, 147, 155, 163, 171, 179\}.$$

Step 2. We find the symmetric octets / quadruple / pair produced by the numbers D_1 of the set

S :

$$131 \rightarrow (5, 1, 1, 1, 7, 7, 5, 1)$$

$$139 \rightarrow (65, 121, 71, 127)$$

$$147 \rightarrow (33, 57, 39, 63)$$

$$155 \rightarrow (73, 89, 103, 79, 119, 95, 97, 113).$$

$$163 \rightarrow (17, 25, 23, 31)$$

$$171 \rightarrow (81, 105, 87, 111)$$

$$179 \rightarrow (41, 55, 47, 49)$$

Step 3. From the numbers of the octets / quadruple / pair we found we ask which are prime numbers P or powers of prime numbers or of the form $3^k \times P, k \in \mathbb{N}, 5^\lambda \times P, \lambda \in \mathbb{N}$:

$$131 \rightarrow (5, 1, 1, 1, 7, 7, 5, 1)$$

$$139 \rightarrow (65 = 5 \times 13, 11^2, 71, 127)$$

$$147 \rightarrow (33 = 3 \times 11, 57 = 3 \times 19, 39 = 3 \times 13, 63 = 3^2 \times 7)$$

$$155 \rightarrow (73, 89, 103, 79, 95 = 5 \times 19, 97, 113)$$

$$163 \rightarrow (17, 25 = 5 \times 5, 23, 31)$$

$$171 \rightarrow (81 = 3^4, 105 = 3 \times 5 \times 7, 87 = 3 \times 29, 111 = 3 \times 37)$$

$$179 \rightarrow (41, 55 = 5 \times 11, 47, 49 = 7^2)$$

Thus we get the sets of prime numbers

$$131 \rightarrow \{5, (1), 7\}$$

$$139 \rightarrow \{13, 11, 71, 127\}$$

$$147 \rightarrow \{11, 19, 13, 7\}$$

$$155 \rightarrow \{73, 89, 103, 19, 97, 113\}$$

$$163 \rightarrow \{17, 5, 23, 31\}$$

$$171 \rightarrow \{3, 5, 7, 29, 37\}$$

$$179 \rightarrow \{41, 11, 47, 7\}$$

and finally the set of prime numbers

$$S_p^7 = \{(1), 3, 5, 7, 11, 13, 17, 19, 23, 31, 37, 41, 47, 71, 73, 79, 89, 97, 103, 107, 113, 139, 127\}.$$

The set S_p^7 contains all prime numbers of the form D_2 and Q_2 from 7 to 127.

Prime numbers 43, 59, 67, 83 which are of the form D_1 and 53, 61, 101, 109 which are of the form Q_1 are absent from the set S_p^7 . If we include these numbers we get the set of all primes that are smaller than 2^7 .

Running the algorithm for 2^8 we get the set S ,

$$S = \left\{ D_1 = 3 + 8m / m \in \mathbb{N}, 2^n \leq 8m \leq 2^n + \frac{2^{n-2}}{n \ln 2} \right\} = \{259, 267, 275, 283, 291, 307, 315, 323, 331, 339, 347\}$$

and the set of prime numbers S_p^8 ,

$$S_p^8 = \{(1), 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 61, 67, 71, 73, 79, 83, 89, 97, 103, 113, 127, 137, 151, 167, 191, 193, 199, 223, 233, 239, 241\}.$$

The set S_p^8 contains all prime numbers of the form D_2 and Q_2 from 7 to 241.

Prime numbers 59, 107, 131, 139, 163, 179, 211, 227, 251 which are of form D_1 and 101, 109, 149, 157, 173, 181, 197, 229 which are of form Q_1 are absent from the set S_p^8 . However, the set S_p^8 contains the numbers 43, 61, 67, 83, 53 which are absent from the set S_p^7 . As the value of the n increases, prime numbers of the form D_1, Q_1 appear in the products $3 \times P$ and $5 \times P$.

The set S_p^n contains all prime numbers of the form D_2 and Q_2 that are less than 2^n . Gradually, as the value of n increases, the prime numbers of the form D_1 and Q_1 also appear. This pattern repeats for each value of $n \in \mathbb{N}^*$.

4. Conclusion

Results from the application of the algorithm show a correlation between the distribution of octets and the distribution of primes in the set \mathbb{N} . The complete theoretical proof of the algorithm is an open topic in the theory of octets of the odd numbers.

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