ON THE SET OF PRIME NUMBERS

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Abstract. "The octets of the odd numbers" theory categorizes the odd numbers into four categories D1, Q1, D2, Q2. From this categorization we get an algorithm for finding the set of prime numbers of the form D2 and Q2. The algorithm sequentially finds all prime numbers of the form D2 and Q2 in ascending order.

1. Introduction

"The octets of the odd numbers" theory categorizes odd numbers based on the form of the octet they belong to or produce (see [8], equations (5.7), (5.8)):

$$D_1 = 11 + 8m' = 3 + 8m, m \in \mathbb{N} \tag{1}$$

$$Q_1 = 13 + 8m' = 5 + 8m, m \in \mathbb{N} \tag{2}$$

$$D_2 = 7 + 8m, m \in \mathbb{N} \tag{3}$$

$$Q_2 = 9 + 8m' = 1 + 8m, m \in \mathbb{N} . \tag{4}$$

Numbers of the form (1) produce the octets to which the numbers of the form (3) belong (see [8], section 5). Numbers of the form (2) produce the octets to which the numbers of the form (4) belong. Specific subsets of (1), (2), found in a very simple way, produce the set of prime numbers of the form (3) and (4). This algorithm does not have the limitations of the so far known formulas for calculating prime numbers (see [1-7] and [9-16]).

2. The algorithm

As a consequence of the properties of symmetric octets (see [8], section 5), the algorithm derives either from numbers of the form (1) or from numbers of the form (2).

A. The algorithm via equation (1)

Step 1. We choose a power of 2, $2^n, n \in \mathbb{N}^*$.

Step 2. We choose the numbers D_1 of the set S,

$$S = \left\{ D_1 = 5 + 8m \,/\, m \in \mathbb{N}, 2^n \le 8m \le 2^n + \frac{2^{n-2}}{n \ln 2} \right\}. \tag{5}$$

The upper limit of the value of the number m results from the Prime Number Theorem assuming that $\frac{1}{4}$ of them are of the form (1) (see [3], [5], [10]). This value may not be exact, either because of the error of the Prime Number Theorem or because the forms (1), (2), (3), (4) do not give the same percentage of prime numbers.

Step 3. We find the symmetric octets (or quadruple, or pair, see [8], section 6) produced by the numbers D_1 of the set S.

Step 4. From the numbers of the octets we found we ask which are prime numbers P (or powers of prime numbers) or of the form $3^k \times P, k \in \mathbb{N}$, $5^\lambda \times P, \lambda \in \mathbb{N}$. The prime numbers we find are the set of primes of the form (3) and (4) that are less than 2^n.

B. The algorithm via equation (2)

Step 1. We choose a power of 2, $2^n, n \in \mathbb{N}^*$.

Step 2. We choose the numbers Q_1 of the set S,

$$S = \left\{ Q_1 = 5 + 8m \,/\, m \in \mathbb{N}, 2^n \leq 8m \leq 2^n + \frac{2^{n-2}}{n \ln 2} \right\}. \tag{6}$$

The upper limit of the value of the number m results from the Prime Number Theorem assuming that $\frac{1}{4}$ of them are of the form (2).

Step 3. We find the symmetric octets / quadruple / pair produced by the numbers Q_1 of the set S.

Step 4. From the numbers of the octets we found we ask which are prime numbers P (or powers of prime numbers) or of the form $3^k \times P, k \in \mathbb{N}$, $5^\lambda \times P, \lambda \in \mathbb{N}$. The prime numbers we find are the set of primes of the form (3) and (4) that are less than 2^n.

The two algorithms give exactly the same set of prime numbers.

3. The application of the algorithm

We present the application of the algorithm. We apply algorithm A.

Step 1. We choose the power 2^7.

Step 2. The set S is:

$$S = \left\{ D_1 = 3 + 8m \,/\, m \in \mathbb{N}, 2^7 \leq 8m \leq 2^7 + \frac{2^5}{7 \ln 2} \right\} = \{131, 139, 147, 155, 163, 171, 179\}.$$

Step 3. We find the symmetric octets / quadruple / pair produced by the numbers D_1 of the set S:

$131 \to (5,1,1,1,7,7,5,1)$

$139 \to (65,121,71,127)$

$147 \to (33,57,39,63)$

$155 \to (73,89,103,79,119,95,97,113)$.

$163 \to (17,25,23,31)$

$171 \to (81,105,87,111)$

$179 \to (41,55,47,49)$

Step 4. From the numbers of the octets we found we ask which are prime numbers P (or powers of prime numbers) or of the form $3^k \times P, k \in \mathbb{N}$, $5^\lambda \times P, \lambda \in \mathbb{N}$:

$131 \to (5,1,1,1,7,7,5,1)$

$139 \to (65 = 5\times 13, 11^2, 71, 127)$

$147 \to (33 = 3\times 11, 57 = 3\times 19, 39 = 3\times 13, 63 = 3^2 \times 7)$

$155 \to (73, 89, 103, 79, 95 = 5\times 19, 97, 113)$.

$163 \to (17, 25 = 5\times 5, 23, 31)$

$171 \to (81 = 3^4, 105 = 3\times 5\times 7, 87 = 3\times 29, 111 = 3\times 37)$

$179 \to (41, 55 = 5\times 11, 47, 49 = 7^2)$

Thus we get the sets of prime numbers

$131 \to \{5,1,7\}$

$139 \to \{13,11,71,127\}$

$147 \to \{11,19,13,7\}$

$155 \to \{73,89,103,19,97,113\}$

$163 \to \{17,5,23,31\}$

$171 \to \{3,5,7,29,37\}$

$179 \to \{41,11,47,7\}$

and finally the set of prime numbers

$S_P^7 =$ {(1), 3, 5, 7, 11, 13, 17, 19, 23, 31, 37, 41, 47, 71, 73, 79, 89, 97, 103, 107, 113, 139, 127}.

The set S_P^7 contains all prime numbers of the form (2) and (4) from 3 to 127.

The prime numbers 43, 59, 67, 83 which are of the form (1) and 53, 61, 101, 109 which are of the form (2) are absent from the set S_P^7. If we include these numbers we get the set of all primes that are smaller than 2^7.

Running the algorithm for 2^8 we get the set of prime numbers S_P^8,

$S_P^8 =$ {(1), 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 61, 67, 71, 73, 79, 83, 89, 97, 103, 113, 127, 137, 151, 167, 191, 193, 199, 223, 233, 239, 241} .

Prime numbers 59, 107, 131, 139, 163, 179, 211, 227, 251 (are of form (1)) and 101, 109, 149, 157, 173, 181, 197, 229 (are of form (2)) are absent from the set S_P^8. However, the set S_P^8 contains the numbers 43, 61, 67, 83, 53 which are absent from the set S_P^7. As the value of the n increases, prime numbers of the form (1), (2) appear in the products $3\times P$ and $5\times P$ (see [8], Corollary 3.1). The algorithm can give all forms of prime numbers.

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