The Velocity-Wavelength Right Triangles Unit Circles
Standard Clock and the Twin Paradox

Using energy and momentum definitions:
E=(m_0)c²/sqrt(1-v²/c²) and p=(m_0)v/sqrt(1-v²/c²)
their frequencies and wavelengths may be shown as reciprocal transformations:
f=c/λ_{c} , λ=c/f_{c} yielding the generally referred to:
(λ_{c})(f_{c})=v=v_{g} group velocity ; and λf=(c/f_{c})(c/λ_{c})=c²/((λ_{c})(f_{c}))=c²/v=v_{p} phase velocity
and yields the right triangles:
((((m_0)c)/h)λ_{c})²+(v/c)²=1 and [((((m_0)v)/h))λ]²+(v/c)²=1
represent wavelength-velocity right triangles with unit hypotenuses (radii of unit circles)
As unit circles, the time rate of change of the angle of thsse wavelength-velocity right triangles may be considered a standard clock of a relativistic reference frame.
Analysis may be made for any planets in a solar system; and similarly for binary star/planet/moon system(s) and also for bodies within rotating galaxies.
Usng this standard relativistic clock the effects of acceleration concerning the twin paradox may be calculated.

$$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = hf \Rightarrow (m_0c)^2 = \left(\frac{hf}{c}\right)^2\left(1-\left(\frac{v}{c}\right)^2\right) \Rightarrow \left(\frac{m_0c^2}{hf}\right)^2 = 1-\left(\frac{v}{c}\right)^2$$

$$p = \frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{h}{\lambda} \Rightarrow (m_0c)^2\left(\frac{v}{c}\right)^2 = \left(\frac{h}{\lambda}\right)^2\left(1-\left(\frac{v}{c}\right)^2\right)$$

$$\Rightarrow \frac{p}{E} = \left(\frac{v}{c^2}\right) = \frac{\left(\frac{h}{\lambda}\right)}{(hf)} \Rightarrow \frac{1}{f\lambda} = \frac{v}{c^2} \Rightarrow \left(\frac{v}{c}\right) = \frac{c}{f\lambda} \Rightarrow \frac{v}{c^2}\lambda = \frac{1}{f} \Rightarrow \boxed{\lambda f = \frac{c^2}{v}} \Rightarrow v = \frac{c^2}{\lambda f}$$

$$\Rightarrow \left(\frac{m_0c^2}{h}\right)^2\left(\frac{1}{f}\right)^2 = 1-\left(\frac{v}{c}\right)^2 \Rightarrow \left(\frac{v}{c}\right)^2 = 1-\left(\frac{m_0c^2}{h}\right)^2\left(\frac{1}{f}\right)^2$$

$$\Rightarrow \frac{v}{c} = \sqrt{1-\left(\frac{m_0c^2}{h}\right)^2\left(\frac{1}{f}\right)^2}$$

$$\Rightarrow \left(\frac{m_0c}{h}\right)^2\lambda^2\left(\frac{v}{c}\right)^2 = 1-\left(\frac{v}{c}\right)^2 \Rightarrow \left(1+\left(\frac{m_0c}{h}\right)^2\lambda^2\right)\left(\frac{v}{c}\right)^2 = 1$$

$$\Rightarrow \frac{v}{c} = \frac{1}{\sqrt{1+\left(\frac{m_0c}{h}\right)^2\lambda^2}}$$

$$\Rightarrow 1-\left(\frac{m_0c^2}{h}\right)^2\left(\frac{1}{f}\right)^2 = \frac{1}{1+\left(\frac{m_0c}{h}\right)^2\lambda^2}$$

$$\Rightarrow \left[1-\left(\frac{m_0c^2}{h}\right)^2\left(\frac{1}{f}\right)^2\right]\left[1+\left(\frac{m_0c}{h}\right)^2\lambda^2\right] = 1$$

$$\Rightarrow 1-\left(\frac{m_0c^2}{h}\right)^2\left(\frac{1}{f}\right)^2+\left(\frac{m_0c}{h}\right)^2\lambda^2-\left(\frac{m_0c^2}{h}\right)^2\left(\frac{1}{f}\right)^2\left(\frac{m_0c}{h}\right)^2\lambda^2 = 1$$

$$\Rightarrow -\left(\frac{m_0c^2}{h}\right)^2\left(\frac{1}{f}\right)^2+\left(\frac{m_0c}{h}\right)^2\lambda^2-\left(\frac{m_0c^2}{h}\right)^2\left(\frac{1}{f}\right)^2\left(\frac{m_0c}{h}\right)^2\lambda^2 = 0$$

$$\Rightarrow -\left(\frac{m_0c^2}{h}\right)^2\left(\frac{v}{c^2}\lambda\right)^2+\left(\frac{m_0c}{h}\right)^2\lambda^2-\left(\frac{m_0c^2}{h}\right)^2\left(\frac{v}{c^2}\lambda\right)^2\left(\frac{m_0c}{h}\right)^2\lambda^2 = 0$$

$$\Rightarrow -\left(\frac{m_0c}{h}\right)^2\lambda^2 = \left(\frac{m_0c^2}{h}\right)^2\left(\frac{v}{c^2}\lambda\right)^2\left(\frac{m_0c}{h}\right)^2\lambda^2-\left(\frac{m_0c^2}{h}\right)^2\left(\frac{v}{c^2}\lambda\right)^2$$

$$\Rightarrow -\left(\frac{m_0c}{h}\right)^2\lambda^2 = \left[\left(\frac{m_0c^2}{h}\right)^2\left(\frac{m_0c}{h}\right)^2\lambda^2+\left(\frac{m_0c^2}{h}\right)^2\right]\left(\frac{v}{c^2}\lambda\right)^2$$

$$\Rightarrow -\left(\frac{m_0c}{h}\right)^2 = \left[\left(\frac{m_0c^2}{h}\right)^2\left(\frac{m_0c}{h}\right)^2\lambda^2+\left(\frac{m_0c}{h}\right)^2c^2\right]\left(\frac{v}{c^2}\right)^2$$

$$\Rightarrow 1 = \left[\left(\frac{m_0c^2}{h}\right)^2\lambda^2+c^2\right]\left(\frac{v}{c^2}\right)^2$$

$$\Rightarrow \left(\frac{v}{c}\right)^2 = \frac{1}{\left[\left(\frac{m_0c}{h}\right)^2\lambda^2+1\right]}$$

$$\Rightarrow \left(\frac{v}{c}\right) = \frac{1}{\sqrt{1+\left(\frac{m_0c}{h}\right)^2\lambda^2}} \quad \checkmark$$

$$\Rightarrow \left\{\begin{array}{|l|l|}\hline \frac{v}{c} = \sqrt{1-\left(\frac{m_0c^2}{h}\right)^2\left(\frac{1}{f}\right)^2} & f = \frac{\left(\frac{m_0c^2}{h}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \\ \hline \frac{v}{c} = \frac{1}{\sqrt{1+\left(\frac{m_0c}{h}\right)^2\lambda^2}} & \lambda = \left(\frac{h}{m_0}\right)\left(\frac{1}{v}\right)\sqrt{1-\left(\frac{v}{c}\right)^2} \\ \hline \end{array}\right.$$

Note:

$$v = \frac{c^2}{\lambda f} = \left(\frac{c}{\lambda}\right)\left(\frac{c}{f}\right)$$

$$\Rightarrow \left\{\begin{array}{|c|c|}\hline f_c \equiv \frac{c}{\lambda} & \lambda = \frac{c}{f_c} \\ \hline \lambda_c \equiv \frac{c}{f} & f = \frac{c}{\lambda_c} \\ \hline \end{array}\right\} \Rightarrow \boxed{\lambda_c f_c = v} \Rightarrow \frac{c^2}{\lambda_c f_c} = \boxed{\lambda f = \left(\frac{c}{\lambda_c}\right)\left(\frac{c}{f_c}\right) \Leftrightarrow \lambda_c f_c = \left(\frac{c}{\lambda}\right)\left(\frac{c}{f}\right)}$$

NOTE:
The λ_c , f_c are Compton wavelengths and frequencies corresponding to the usual wavelength/frequencies as recipricals (in natural units) whose product is the speed ; and as such is merely a useful and convenient reciprocal tranformation.

$$\Delta t = \frac{1}{f} \ \ \& \ \ \Delta\ell = \lambda_c$$

thus, establish the energy of the intermediate envelope via the velocity.
(Clearly the minimum energy is ($v = 0$) m_0c^2 and wavelength $\frac{h}{m_0c}$)
($v_p = \frac{c^2}{v}$ is known as the phase velocity ; $v_g = v$ is known as the group velocity)

$$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = hf = \frac{hc}{\lambda_c}$$

$$p = \frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}} = \left(\frac{h}{c}\right)f_c = \frac{h}{c}\frac{v}{\lambda_c} = \left(\frac{h}{c\lambda_c}\right)v \Rightarrow \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \left(\frac{h}{c\lambda_c}\right)$$

$$\Rightarrow \frac{pc^2}{E} = \frac{\frac{h}{c}f_c}{\frac{c}{\lambda_c}}c^2 = \lambda_c f_c = v \Rightarrow pc^2 = Ev$$

$$\Rightarrow p = \frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{h}{\lambda} = \frac{h}{\frac{c}{f_c}} = \left(\frac{h}{c}\right)f_c = \frac{h}{c}\frac{v}{\lambda_c} = \frac{\left(\frac{h}{c}v\right)}{\lambda_c} = \left(\frac{h}{c\lambda_c}\right)v$$

$$\Rightarrow \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \left(\frac{h}{c\lambda_c}\right) \Rightarrow \lambda_c = \left(\frac{h}{m_0c}\right)\sqrt{1-\frac{v^2}{c^2}} = \frac{c}{f} \quad \checkmark$$

$$\left(\frac{h}{m_0}\right)\left(\frac{1}{v}\right)\sqrt{1-\left(\frac{v}{c}\right)^2} = \lambda = \frac{c}{f_c} \Rightarrow f_c = \left(\frac{\left(\frac{m_0c}{h}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)v$$

$\Rightarrow$

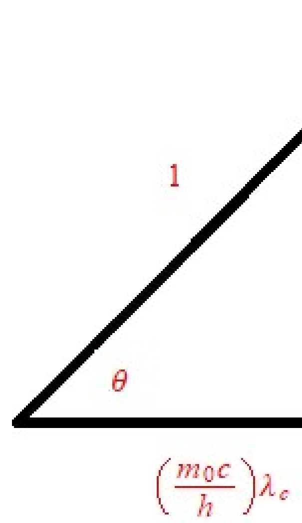
$\frac{v}{c} = \sqrt{1-\left(\frac{m_0c^2}{h}\right)^2\left(\frac{1}{f}\right)^2}$	$f = \frac{\left(\frac{m_0c^2}{h}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$	$\lambda_c \equiv \frac{c}{f}$	$\lambda_c = \left(\frac{h}{m_0c}\right)\sqrt{1-\frac{v^2}{c^2}}$
$\frac{v}{c} = \frac{1}{\sqrt{1+\left(\frac{m_0c}{h}\right)^2\lambda^2}}$	$\lambda = \left(\frac{h}{m_0}\right)\left(\frac{1}{v}\right)\sqrt{1-\left(\frac{v}{c}\right)^2}$	$f_c \equiv \frac{c}{\lambda}$	$f_c = \left(\frac{\left(\frac{m_0c}{h}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)v$
$E = hf$	$p = \frac{h}{\lambda}$	$p = \left(\frac{h}{c}\right)f_c$	$E = \frac{(hc)}{\lambda_c}$

$$\frac{c}{f} = \lambda_c = \left(\frac{h}{m_0c}\right)\sqrt{1-\frac{v^2}{c^2}}$$

$$\Rightarrow \left\{\begin{array}{|c|}\hline \left(\left(\frac{m_0c}{h}\right)\lambda_c\right)^2 + \left(\frac{v}{c}\right)^2 = 1 \\ \hline \lambda_c^2 + \left(\frac{h}{m_0c^2}\right)^2 v^2 = \left(\frac{h}{m_0c}\right)^2 \\ \hline (m_0\lambda_c)^2 + \left(\frac{h}{c^2}v\right)^2 = \left(\frac{h}{c}\right)^2 \\ \hline \end{array}\right\} \Rightarrow \left\{\boxed{\left(\left(\frac{m_0c}{h}\right)\lambda_c\right)^2 + \left(\left(\frac{1}{c}\right)v\right)^2 = 1}\right\}$$

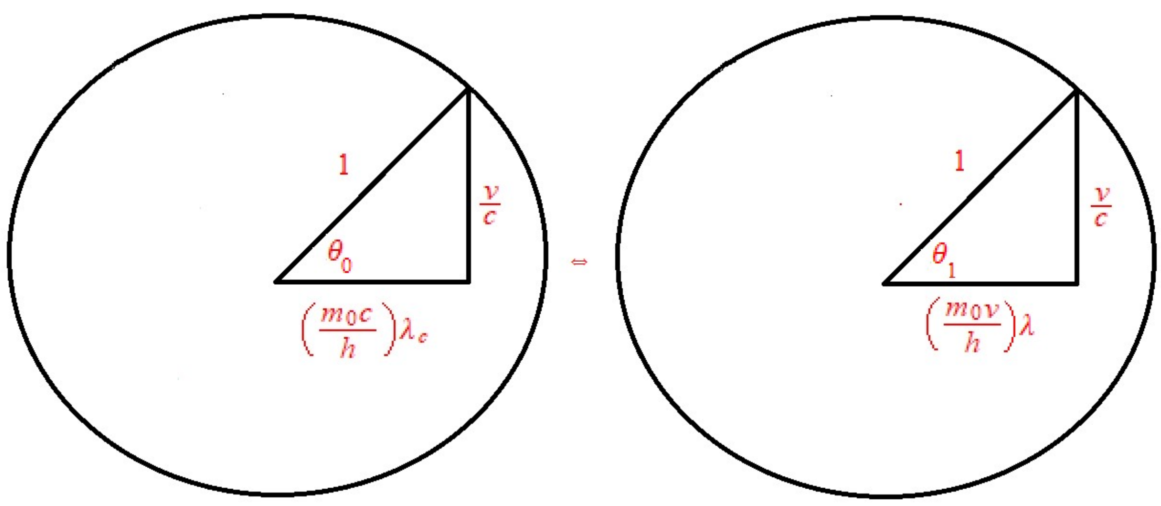
is the wavelength-velocity right triangle
So, let:

$$\Rightarrow \left\{\boxed{\lambda_c = \left(\frac{h}{m_0c}\right)\cos\theta\ ,\ v = c\sin\theta}\right\}$$



(similarly: $(\frac{c}{v})^2 = \left[\left(\frac{m_0 c}{h}\right)\lambda\right]^2 + 1 \Leftrightarrow 1 = \left[\left(\frac{m_0 v}{h}\right)\lambda\right]^2 + (\frac{v}{c})^2$)

Now, considering the wavelength-velocity right triangles as unit circles:



Note: $\lambda_c f_c = v \Rightarrow \lambda_c = \frac{v}{f_c} = \frac{v}{\left(\frac{c}{\lambda}\right)} = \left(\frac{v}{c}\right)\lambda \Rightarrow \theta_0 = \theta_1 = \theta$

$\Rightarrow$ these triangles are actually the same

$\sin\theta = \frac{v}{c} \Rightarrow \cos\theta = \sqrt{1-\left(\frac{v}{c}\right)^2} = \frac{m_0 v}{h}\lambda = \frac{m_0 v}{hf}\lambda f = \frac{m_0 v}{hf}\frac{c^2}{v} = \frac{m_0 c^2}{hf}$

I don’t know why this hasn′t been noticed before, because it’s so simple and clearly obvious, now, but so:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{v}{c}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{c}\frac{v}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{m_0 c}\frac{m_0 v}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{m_0 c}p$$

$\Rightarrow p = (m_0 c)\tan\theta$

and: $Ev = pc^2 \Rightarrow E = \left(\frac{c^2}{v}\right)p \Rightarrow \frac{E}{p} = \frac{c^2}{v} = \lambda f \Rightarrow h = \frac{E}{f} = p\lambda = h$

$\Rightarrow E = \left(\frac{c^2}{v}\right)(m_0 c)\tan\theta$

and, of course:

$$m = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{m_0}{\cos\theta} \Rightarrow m_0 = m\cos\theta \quad \& \quad \frac{m_0}{m} = \cos\theta$$

Thus, a standard clock may be envisioned as follows:

$$p = (m_0 c)\tan\theta$$
$$\Rightarrow p' = (m_0 c)(\tan\theta)'$$
$$= (m_0 c)\left(\frac{1}{\cos^2\theta}\right)\theta' = (m_0 c)\left(\frac{hf}{m_0 c^2}\right)^2\theta'$$
$$\Rightarrow \theta' = \left(\frac{1}{m_0 c}\right)\left(\frac{m_0 c^2}{hf}\right)^2 p' = (m_0 c)\left(\frac{c}{hf}\right)^2 p' = (m_0 c)\left(\frac{c}{E}\right)^2 p'$$

NOTE:

For: $v \ll c$:

$p \approx m_0 v \quad , \quad E \approx \frac{1}{2}m_0 v^2$

$$\Rightarrow \theta' \approx (m_0 c)\left(\frac{c}{\frac{1}{2}m_0 v^2}\right)^2 m_0 v' = (4c^3)\frac{v'}{v^4}$$

this may be considered a **standard clock of a non-relativistic reference frame**.

For a planet orbiting about it's star: $v' = v^2/R_{planet}$

$$\Rightarrow \boxed{\theta' \approx (4c^3)\frac{v'}{v^4} = (4c^3)\frac{v^2/R_{planet}}{v^4} = \frac{4c^3}{v^2 R_{planet}}}$$

For the Earth, $v \approx 29.72\ km/\sec$, $v' = v^2/r = v^2/R_{Earth} \approx \left(29.72\ km/\sec\right)^2/149.6 \times 10^6\ km$

$$\Rightarrow \theta' \approx \frac{4c^3}{v^2 R_{Earth}} \approx \frac{4(3 \times 10^5 km/s)^3}{\left(29.78\ km/s\right)^2\left(149.6 \times 10^6\ km\right)} = \frac{108 \times \times 10^{15} km^3/s^3}{1.3267 \times 10^{11} km^3/s^2} \approx 8.140 \times 10^4/\sec$$

$$\Rightarrow w = \frac{1}{2\pi}\theta' \approx 12956\ revolutions/second$$

Clearly, the same analysis may be made for any planets in a solar system; and similarly for binary star/planet/moon system(s) and also for bodies within rotating galaxies.

For a planet orbiting a star:

$$F_g = G\frac{mM}{R^2} = ma = m\frac{v^2}{R} = F_{cf}$$

(G=gravitational constant , M=mass of star , R=obital radius from star)

($\Rightarrow v^2 = \frac{GM}{R} \Rightarrow v = \sqrt{\frac{GM}{R}}$)

(G=$6.674 \times 10^{-11} m^3 kg^{-1} s^{-2}$, M=$1.99 \times 10^{30} kg$)

($\Rightarrow v^2 R = GM \Rightarrow \theta' \approx \frac{4c^3}{GM}$

$$\approx \frac{4(3 \times 10^5 km/s)^3}{(6.673 \times 10^{-11} m^3 kg^{-1} s^{-2})(1.988 \times 10^{30} kg)}$$

$$\approx \frac{108 \times \times 10^{15} km^3/s^3}{1.326 \times 10^{11} km^3/s^2}$$

$$\approx 8.141 \times 10^4\ radians/\sec$$

$$\Rightarrow w = \frac{1}{2\pi}\theta' \approx 12957\ revolutions/\sec \quad \approx 6^4 \times 10 = 6^2 \times 360\ revolutions/\sec$$

($(12957 - 12956)/12957 \approx 0.000008$)

(This calculation is planet orbital radius and speed independent.)

(This standard clock is a star system constant, only dependent on the mass of the star the planets orbit.)

(so it truly is a **standard clock for an ordinary solar system**)

$$\Rightarrow w = \frac{1}{2\pi}\theta' \approx 12957\ revolutions/\sec$$

$$12960 = 6^4 \times 10 \Rightarrow w = \frac{1}{2\pi}\theta' = 60^4 \times 10^{-3} = 6^2 \times 360 = \frac{1}{2\pi}\frac{4c^3}{GM}$$

$$\Rightarrow G = \frac{1}{2\pi}\frac{4(3 \times 10^5 km/s)^3}{\left(6^4 \times 10\ /\sec\right)((1.988 \times 10^{30} kg))} = \frac{108 \times \times 10^{15} km^3/s^3}{161883 \times 10^{49} kg} \approx 6.6715 \times 10^{-11} m^3/kgs^2$$

planets orbits are slightly elliptical

(essentially due to the star center of mass differing slightly from the two body center of mass)

but average orbital distance, velocity, eccentricity, etc. may be found at:

https://nssdc.gsfc.nasa.gov/planetary/factsheet/

Planet	Orbital Distance $(R)(10^6 km)$	Orbital Velocity $(v)(km/s)$	$(v^2R)(10^{11} km^3/s^2)$	$(v^2R)/GM$	$GM/v^2(10^6 km)$
Mercury	57.9	47.4	1.301	0.981	59.0
Venus	108.2	35.0	1.326	1.000	108.2
Earth	149.6	29.8	1.329	0.998	149.3
Mars	228.0	24.1	1.324	0.998	228.3
Jupiter	778.5	13.1	1.336	1.008	772.7
Saturn	1432.0	9.7	1.347	1.016	1409.3
Uranus	2867.0	6.8	1.326	1.000	2867.6
Neptune	4515.0	5.4	1.317	0.993	4547.3
Pluto	5906.4	4.7	1.305	0.984	6002.7

($GM = 1.326 \times 10^{11} km^3/s^2$)

Also: $v' = v^2/R = G\frac{M}{R^2} \Rightarrow v = \sqrt{\frac{GM}{R}} \Rightarrow v_2 = v_1\sqrt{\frac{R_1}{R_2}}$

ordinary clocks operate at different time zones, but tick at the same rate.

ordinary clocks second hands rotate at: $1\ revolution/\min = 1/60\ revolution/\sec$

so the ratio of this standard clock to the ordinary clock is: $60^5 \times 10^{-3}$

Since this standard clock is common within planets in this star system conversions between clocks would be similar to time zone conversions concerning planet orbital period, rotation period, day length, local time zones, etc.

Using this solar system standard clock: $\theta' \approx \frac{4c^3}{GM}$:

the black hole Sagittarius A* (containing over 99% of the mass at the center of the Milky Way)

it's mass $\approx 8.26 \times 10^{36}\ kg$:

$$\Rightarrow \theta' \approx \frac{4c^3}{GM} \approx \frac{108 \times \times 10^{15} km^3/s^3}{(6.674 \times 10^{-11} m^3 kg^{-1} s^{-2})(8.26 \times 10^{36}\ kg)} \approx 1.96 \times 10^{-2}\ radians/\sec$$

$$\Rightarrow w = \frac{1}{2\pi}\theta' \approx 0.312\ revolutions/\sec$$

so the ratio of this Milky Way standard clock to the ordinary clock is: 1.872

(i.e.: this Milky Way standard clock runs almost twice as fast as the ordinary clock)

Local star system clocks may superceed non-local star system clocks, or correspondence maintained. (solar system-galactic, galactic-universal)

' Usng this standard star system clock the effects of acceleration concerning the twin paradox may also be rather simply calculated.

Note first that v' is an accellration, i.e.: a g-force.

Astronauts normally experience a maximum g-force of around $3g'$s during a rocket launch.

Fighter pilots can manage up to about $9g'$s for a second or two.

But sustained g-forces of even $6g'$s would be fatal.

g is a constant acceleration, so: $v' = 3.4g'$s $\Rightarrow v = \int v'dt = \int 3.4gdt = 3.4g\triangle t$

$$\Rightarrow \triangle t = \frac{v}{3.4g}$$

so, for $v = \frac{1}{10}c$: $\triangle t = \dfrac{\frac{1}{10} \times (3 \times 10^5 km/s)}{3.4 \times (0.00981 km/s^2)} = \dfrac{30 \times 10^3 km/s}{3.4 \times 0.981 km/s^2} = 8994\sec \approx 2.5\ hours$

and: $\dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-\left(\frac{1}{10}\right)^2}} = 1.005$ (the time and mass are virtually unchanged)

and, for $v = \frac{5}{10}c$: $\triangle t = \dfrac{\frac{5}{10} \times (3 \times 10^5 km/s)}{3.4 \times (0.00981 km/s^2)} = \dfrac{15 \times 10^4 km/s}{3.4 \times 0.981 km/s^2} = 44970\sec \approx 12.5\ hours$

and: $\dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}} =.1.155$ (the time and mass are only increased 15%)

and, for $v = \frac{9}{10}c$: $\triangle t = \dfrac{\frac{9}{10} \times (3 \times 10^5 km/s)}{3.4 \times (0.00981 km/s^2)} = \dfrac{27 \times 10^4 km/s}{3.4 \times 0.981 km/s^2} = 80946\sec \approx 22.5\ hours$

and: $\dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-\left(\frac{9}{10}\right)^2}} =.2.29$ (the time and mass are more than doubled)

$$\Rightarrow \theta' = (m_0 c)\left(\frac{c}{m_0 c^2}\right)^2 m_0 \left(\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}\right)' = \left(\frac{\left(\frac{v}{c}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)'$$

$$= \left(\frac{\left(\frac{v}{c}\right)'}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)\left[1+\left(\frac{v}{c}\right)^2\right] = \begin{cases} \dfrac{\left(\frac{3.4 \times 0.981 km/s^2}{(3 \times 10^5 km/s)}\right)}{\sqrt{1-\left(\frac{9}{10}\right)^2}}\left[1+\left(\frac{9}{10}\right)^2\right] & , v' = 3.4g \\ 0 & , v' = 0 \end{cases}$$

$$= \begin{cases} \dfrac{\left(\frac{3.3354}{3}\right) \times 10^{-5}}{(.4359)}[1.81] & , v' = 3.4g & \text{, at end of acceleration} \\ ? & , v' = ? & \text{, after acceleration} \end{cases}$$

$$= \begin{cases} 2.55 \times 10^{-5}[1.81] & , v' = 3.4g & \text{, at end of acceleration} \\ ? & , v' = ? & \text{, after acceleration} \end{cases}$$

$$= \begin{cases} 4.62 \times 10^{-5} & , v' = 3.4g & \text{, at end of acceleration} \\ ? & , v' = ? & \text{, after acceleration} \end{cases}$$

$$\Rightarrow w = \frac{1}{2\pi}\theta' = \begin{cases} 7.35 \times 10^{-6} & , v' = 3.4g & \text{, at end of acceleration} \\ ? & , v' = ? & \text{, after acceleration} \end{cases}$$

rocket launch speed $\approx 7.8 km/s$

Earth escape velocity $\approx 11 km/s$

Fastest spacecraft speed $\approx 163 km/s$ (Parker Solar Probe , 11/20/2021)

Speed of light: $3 \times 10^3 km/s$

9/10 the speed of light: $27 \times 10^4 km/s$

$$\frac{7.81 km/s}{3.23 \times 0.981 km/s^2} = \frac{7.81}{3.17}s = 2.465s$$

$$\frac{27 \times 10^4 km/s}{3.23 \times 0.981 km/s^2} = \frac{27 \times 10^4 km/s}{3.17 km/s^2} = 8.5174 \times 10^4 s = 85174\sec$$

$$= 1419.56\min = 23.659 hrs \approx 0.9858 day$$

So, it only requires acceleration at $3.23g's$ for about 1 day to achieve 9/10-th the speed of light.

However, at current = the fuel costs would be substantial, as would the spacecraft size, weight, etc.
But with simple timer and transmitter the time dilation effect could be conclusively demonstrated.

There’s a great difference between acceleration and gravity.
Gravity accelerates mass up to it;s center, while motive acceleration is up to the speed of light.

After the acceleration the star system standard clock stops ticking, so a correspondence to Earth’s star system standard clock must be maintained to compare time differences between the referrence frames.