Resolution of the horizon problem in the Alcubierre warp drive: computation I

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Abstract:

A solution of general relativity is presented that describes an Alcubierre [1] propulsion system in which it is possible to travel at superluminal speed while reducing the energy density and energy by an arbitrary value, and eliminating the event horizon, for superluminal motions,which would have otherwise led to to explosion of the spaceship (instability of the warp bubble [7]).

1 Introduzione:

Alcubierre [1] in 1994 proposed a solution of the equations of general relativity which provides the only viable means to accelerate a spaceship up to superluminal velocities without using wormholes. A problem was soon identified: Pfenning [4] showed that the required energy is comparable to the total energy of the universe and that it is negative.This is treated in publications [11] and [12], and solves the problems of the amount of energy, (arbitrarily reducing the amount). Later Hiscock [10] proved the existence of an event horizon for superluminal speed, which implies an instability of the warp bubble due to Hawking radiation, leading to the explosion of the spaceship [7].

Note: In the following we adopt the notation used by Landau and Lifshitz in the second volume ("The Classical Theory of Fields") of their well known Course of Theoretical Physics [13].

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We start with the metric

$$ds^2=\left(D(x,y,z-k(t))^2-v^2\frac{f(x,y,z-k(t))^2}{a(x,y,z-k(t))^2}\right)dt^2+2v\frac{f(x,y,z-k(t))}{a(x,y,z-k(t))}dt\,dz-dx^2-dy^2-dz^2 \qquad (1)$$

In implicit form it is:

$$ds^2=D(x,y,z-k(t))^2dt^2-\left[dz-v\frac{f(x,y,z-k(t))}{a(x,y,z-k(t))}dt\right]^2-dx^2-dy^2 \qquad (2)$$

- 1)-The Pfenning zone is the zone within the interval: $R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2}$ where $\Delta\ll 1$ R is the radius of the Warp bubble and Δ is the wall thickness of the Warp bubble $R\gg\Delta$.

- 2)- $r=(x^2+y^2+(z-k(t))^2)^{\frac{1}{2}}$ and $\frac{dk(t)}{dt}=v=const$

- 3)-In the Pfenning zone we let $a(r)=a(x,y,z-k(t))\gg 1$, $da(r)/dr\leq a(r)$ (there is the source of esotic matter)

the energy can be reduced by an arbitrary value in Pfenning zone (example for D=1 [11],[12]), for $D\neq 1$,energy density and energy see appendix 4.3

Einstein Equations: $G^{ik}=\frac{8\pi G}{c^4}T^{ik}$ T^{ik} (energy-impulse tensor)

2. Values of the functions used in the metric (1)**:**

The functions $f=f(r)=f(x,y,z-k(t))$, $a=a(r)=a(x,y,z-k(t))$ and

$D=D(r)=D(x,y,z-k(t))$ can assume the following values:

- 1)-inside the warp bubble $(0<r<R-\frac{\Delta}{2})$ $f(r)=k$ and $a(r)=1$, $D(r)=1$
- 2)-outside the warp bubble $(r>R+\frac{\Delta}{2})$ $f(r)=0$ and $a(r)=1$, $D(r)=1$

- 3)-in the Alcubierre warped region $(R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2})$ $0<f(r)<k$, $f(r)$ is

$f(r)=-k\frac{(r-R-\frac{\Delta}{2})}{\Delta}$ (Pfenning zone [4]) and $a=a(r)=a(x,y,z-k(t))\gg 1$

(possessing extremely large values) and $\partial_i a\leq a, \partial_{i,k} a\leq a$ or $da(r)/dr\leq a(r)$

and $10\leq k\leq 1000$, $10\leq D=D(r)\leq 1000$

2.1 Value of the speed of the spaceship inside the warp bubble:

Since the f is not inside the warp bubble as in [4], but with a generic value $f(r)$, it follows that the speed of the spaceship within it is:

$$\frac{dz}{dt}=v f \tag{3}$$

assuming $0\leq v=v(t)<1$,as can be seen from the metric:

$$ds^2=dt^2-\left[dz-v\frac{f(x,y,z-k(t))}{a(x,y,z-k(t))}dt\right]^2-dx^2-dy^2 \tag{4}$$

$a=a(r)=1$ inside the warp bubble.

Now the real speed of the ship depends only on f and v

That is, taking into account this paragraph and paragraph 2, one has:

$$\frac{dz}{dt}=k v \qquad 10\leq k\leq 1000 \tag{5}$$

the choice of $10\leq k\leq 1000$ and of $10\leq D\leq 1000$ with $0\leq v=v(t)<1$ solves the problem of the event horizon (See following paragraphs).

3 The Hiscock solution for our metric in study [10]:

Starting from the solution (1), in the case in two-dimensional coordinates t and z after mathematical elaborations and with the transformation of coordinates $dz' = dz - v\, f\, dt$ (for our conditions, inside the warp bubble) one gets:

$$ds^2 = H(r)\, dT^2 - \frac{D^2(r)}{H(r)} dz'^2 \tag{6}$$

where

$$H(r) = D^2 - v^2 \left[f - \frac{f}{a} \right]^2 \tag{7}$$

For the demonstration see **appendix**.

3.1 Studies on the horizon of events, for our conditions (example):

If for example $D = 100$,then we place $k = k_{max} = 100$ and $v = 0.99$ so the speed of the spaceship (internal warp bubble) is $dz'/dt = k\, v = 99$ in multiples of c $c = 1$ (speed of light), equation (5), $H(r)$ for all r is:

- 1)- $H(r) = 1 - v^2 \left[f - \frac{f}{a} \right]^2 = 1$ for $r > R + \frac{\Delta}{2}$
- 2)- $H(r) = D^2 - v^2 [f - \frac{f}{a}]^2 \approx D^2 - f(r)^2 v^2 \geq D^2 - k^2 v^2 = 100^2 - 99^2 > 1$ for $R - \frac{\Delta}{2} < r < R + \frac{\Delta}{2}$
- 3)- $H(r) = 1 - v^2 \left[f - \frac{f}{a} \right]^2 = 1$ for $0 < r < R - \frac{\Delta}{2}$

as can be seen there is no event horizon up to a real rate of 100 c, equations (6), (7).

3.2 Speed light signals in various zones,respect to a ship's observer:

In general, the metric (1) in an implicit form is, in the two-dimensional case:

$$ds^2 = D^2 dt^2 - \left[dz - v \frac{f(x, y, z-k(t))}{a(x, y, z-k(t))} dt \right]^2$$ that with the new variable z'

with the substitution $dz' = dz - v f \, dt$ at the speed of light $ds^2 = 0$ with respect to the center of the warp bubble:

$$\frac{dz'}{dt} = \pm D - v \left(f - \frac{f}{a} \right) \qquad (8)$$

that for the three spacial zones you have, for our conditions:

- 1)- $\frac{dz'}{dt} = \pm 1$ for $r > R + \frac{\Delta}{2}$
- 2)- $\frac{dz'}{dt} \approx \pm D - f(r) v \geq \pm D - k v \approx \pm 100 - 99$ for $R - \frac{\Delta}{2} < r < R + \frac{\Delta}{2}$
- 3)- $\frac{dz'}{dt} = \pm 1$ for $0 < r < R - \frac{\Delta}{2}$

the plus sign corresponding to light signals that propagates in the z-axis positive, the minus sign to signals moving along the negative z-axis.

4 appendix, demonstration of relations (6) **and** (7)**:**

4.1appendix:

$$ds^2 = D^2 dt^2 - [dz - v f / a \, dt]^2 \qquad (9)$$

$$dz = dz' + v f \, dt \qquad (10)$$

$$ds^2 = D^2 dt^2 - [dz' + v f \, dt - v f / a \, dt]^2 \qquad (11)$$

$$g(r) = f(r) - f(r) / a(r) \qquad (12)$$

$$ds^2 = D^2 dt^2 - dz'^2 - 2 v g(r) dz' dt - (v g(r))^2 dt^2 \qquad (13)$$

$$ds^2 = [D^2 - (v g(r))^2] dt^2 - 2 v g(r) dz' dt - dz'^2 \qquad (14)$$

4.2 appendix:

$$ds^2 = H(r) dT^2 - D^2(r)/H(r) dz'^2 \qquad (15)$$

$$dT = dt - v g(r)/H(r) dz' \qquad (16)$$

$$ds^2 = H(r) dt^2 - 2 v g(r) dz' dt + H(r)(v g(r)/H(r))^2 dz'^2 - D^2 dz'^2/H(r) \qquad (17)$$

$$H(r) = D^2 - (v g(r))^2 \qquad (v g(r))^2 = D^2 - H(r) \qquad (18)$$

$$ds^2 = H(r) dt^2 - 2 v g(r) dz' dt - D^2 dz'^2/H(r) - H(r) dz'^2/H(r) + D^2 dz'^2/H(r) \qquad (19)$$

$$ds^2 = H(r) dt^2 - 2 v g(r) dz' dt - dz'^2 \qquad (20)$$

$$ds^2 = [D^2 - (v g(r))^2] dt^2 - 2 v g(r) dz' dt - dz'^2 \qquad (21)$$

4.3 appendix:computation energy density and energy for our metric (1):

The energy density is (in the Eulerian observers,see [1]), similar to [12] :

$$(energy\ density) = -\frac{1}{4} k v^2 \frac{x^2 + y^2}{D(r)^2 r^2} h(r) \qquad k = c^4/8\pi G \qquad (22)$$

where $h(r)$ is given by:

$$h(r) = [\frac{1}{a(r)^2} (\frac{df(r)}{dr})^2 + (\frac{f(r)^2}{a(r)^4})(\frac{da(r)}{dr})^2 - 2 \frac{df(r)}{dr} \frac{f(r)}{a(r)^3} \frac{da(r)}{dr}] \qquad (23)$$

if $\Delta \ll 1$, $a(r) \gg 1$ and $da(r)/dr \leq a(r)$, $(da(r)/dr)^2 \leq a(r)^2$ $a(r) > \frac{1}{\Delta}$ [12]

the dominant term is:

$$h(r) \approx (\frac{-1}{a(r)\Delta})^2 \qquad (24)$$

and $a(r) = A = const \gg 1$ in the Pfenning zone (example $A = 10^{50}$) [12]:

$$(energy\ density) \approx -\frac{1}{4} k v^2 \frac{x^2+y^2}{r^2} \frac{1}{D(r)^2 a(r)^2} \left(\frac{-1}{\Delta}\right)^2 = -\frac{1}{4} k v^2 \frac{r^2 (\sin\vartheta)^2}{r^2} \frac{1}{D(r)^2 a(r)^2} \left(\frac{-1}{\Delta}\right)^2$$

similar computation [12] in Pfenning zone $R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2}$ (there is the source of esotic matter)

then $|energy\, density| \le |-\frac{1}{4} k v^2 \frac{1}{D(r)^2 A^2} \left(\frac{-1}{\Delta}\right)^2| \ll 1$, $10 \le D = D(r) \le 1000$

and the average value of $D(r)$ in the region $10 \le \langle D(r) \rangle \le 1000$ implies energy E (similar computation in [12],paragraphs 2.1):

$E \approx -3 v^2 10^{-17} / \langle D(r) \rangle\ joule$, $|E| < 3 v^2 10^{-17}\ joule$ in the Pfennig zone

(energy is very small compared with Casimir effect [12]).

5 As time passes in the Pfennig zone:

The presence of a D function different from the one in the Pfenning zone, implying a relativistic effect (dilation of time) in this zone, was maintained at a reasonable value. This allows superluminal motions without an event horizon, with values of D chosen by us. This may bring a deterioration of exotic matter, but I think that the the functions D stays within reasonable values in a a range that is not very problematic.For example, a trip lasting one year at a speed limit of about 100 times c (in this case D=100), implies a time dilation in Pfenning zone of 100 years of exotic matter, and 1000 years with D=1000, therefore at a speed limit of 1000 c. This deterioration is possible but not demonstrated for the exotic matter in the Pfenning zone; it could also be compensated by appropriate breaks during the trip for a possible restructuring of the warp bubble.

6 Conclusion: The calculations indicate that the propulsion system of Alcubierre as modified in this paper permits speeds higher than the speed of light without problems of:energy density, the components of the stress-energy tensor, and energy as reducible to any arbitrary value, [11], [12]. The events horizon is removed (see paragraphs 3, 3.1, 3.2) and the ship (not necessary the Krasnikov tube [14]) can be handled without problems. The speed of the warp bubble reaches a value between 10, 100 and 1000 times that of light, and the instability of the warp bubble (Hawking radiation feedback) is eliminatated with $D \neq 1$, something that would have otherwise led to the explosion of the ship [7].

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