

The Proof: Goldbach's Conjecture

Aniket Bhattacharjee

May 30, 2023

Abstract

In this paper, I want to present the proof to one of the most famous conjecture - The Goldbach's Conjecture.

1 The Goldbach's Conjecture

Conjecture:- *Every even natural number greater than 2 is the sum of two prime numbers.*

Proof. **Case I:- (Even Prime)**

- Considering the even prime number i.e. 2, the sum of two even primes is always 4.

Case II:- (Odd Prime)

- *Any odd prime is of the form $4k \pm 1$. ($k \in N$)*

Therefore, when two odd primes are added,

$$4k \pm 1 + 4k' \pm 1 (k, k' \in N) \tag{1}$$

- *Thus, 4 cases can arise. ($m \in N$)*

Case A:-

$$= 4k + 1 + 4k' + 1 = 2(2k + 2k' + 1) = 2m \tag{2}$$

Case B:-

$$= 4k - 1 + 4k' - 1 = 2(2k + 2k' - 1) = 2m \tag{3}$$

Case C:-

$$= 4k + 1 + 4k' - 1 = 2(2k + 2k') = 2m \tag{4}$$

Case D:-

$$= 4k - 1 + 4k' + 1 = 2(2k + 2k') = 2m \tag{5}$$

- In all the 4 cases, the outcome is even and it is always true as addition of 2 odd (prime) numbers is always even.

Case III: (Odd Prime + Even Prime)

- The sum of odd prime and even prime number is not possible as it gives an odd number which does not follow the conjecture which is based on even natural numbers.

Thus, the above mentioned conjecture (Goldbach's conjecture) is always true. (Proved) \square

1.1 Acknowledgement

I would be thankful to my parents, my teachers and the Almighty without whose blessings this paper would not be successful. I hope my paper will serve as an aid in the field of Number Theory for the future generation.