Sequences of prime numbers

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Abstract. In this article we present sequences - networks of prime numbers.

1. Introduction

In this article we define the sequences W and Θ. Through these sequences we obtain an algorithm for finding prime numbers. This algorithm has a different "logic" from the known mathematical formulas [1-15] for finding prime numbers.

2. The sequences W and Θ

We give the definition of sequences W and Θ:

Definition 1. 1. Definition of sequence W_n:

$$W_n = P_1P_2 - 2n$$
$$P_1, P_2 = \textit{prime numbers}. \qquad (1)$$
$$n = 1, 2, 3, ..., 8$$

2. Definition of sequence Θ_n:

$$\Theta_n = P_1P_2 + 2n$$
$$P_1, P_2 = \textit{prime numbers}. \qquad (2)$$
$$n = 1, 2, 3, ..., 8$$

3. The algorithm

We choose a pair (P_1, P_2) of prime numbers and calculate the terms $W_1, W_2, W_3, ..., W_8$, $\Theta_1, \Theta_2, \Theta_3, ..., \Theta_8$ of the sequences W_n and Θ_n. At least one of these terms is a prime number. In some cases these terms are the product of a large prime number with smaller ones. The algorithm can run with a primality or factorization test.

We present two examples.

Example 1. In this example we run the algorithm starting with a pair of small primes. Then we repeat the algorithm using the prime number we found.

We choose the pair of prime numbers $(3,5)$ and run the algorithm for the sequence Θ:

$$\Theta_1 = 3 \cdot 5 + 2 \cdot 1 = 17$$

$$\Theta_2 = 5 \cdot 17 + 2 \cdot 2 = 89$$

$$\Theta_5 = 17 \cdot 89 + 2 \cdot 5 = 1523$$

$\Theta_6 = 89 \cdot 1523 + 2 \cdot 6$ =135559

$W_3 = 1523 \cdot 135559 - 2 \cdot 3$ =206 456351

$W_3 = 135559 \cdot 206456351 - 2 \cdot 3$ =27 987016 485203

$W_3 = 206456351 \cdot 27987016485203 - 2 \cdot 3$ =5778 097298 911856 874247

$W_2 = 27987016485203 \cdot 5778097298911856874247 - 2 \cdot 2$ =161711 704357 753064 653190 467907 267137

$W_5 = 5778097298911856874247 \cdot 161711704357753064653190467907267137 - 2 \cdot 5$ =3 × 311 461987 383988 579162 878275 956426 373341 917268 333614 906943 (57 digits)

$\Theta_8 = 5778097298911856874247 \cdot 161711704357753064653190467907267137 + 2 \cdot 8$ =5 × 186 877192 430393 147497 726965 573855 824005 150361 000168 944171 (57 digits)

$\Theta_1 = 161711704357753064653190467907267137 \cdot 311461987383988579162878275956426373341917268333614906943 + 2 \cdot 1$

= $3 \times 11^2 \times$ 138752 200612 996627 996041 872068 035897 494157 161533 521691 524024 935808 115036 771788 622214 041411 (90 digits)

$W_8 = 3^2 \times 11 \times$ 436525 617745 715233 828830 837843 505992 677514 423796 486797 201528 615942 658302 489694 959003 456696 222234 959621 653857 376305 384568 694321 369583 393522 357743 (144 digits)

W_6 = 903706 552733 × 67022 740847 670751 354621 823058 752758 175293 036710 860638 249466 163101 656015 217916 141630 595003 931222 155028 676858 263675 828221 251033 221985 319669 489368 669628 388030 412541 802593 422337 974264 543960 755957 492921 525082 145012 240462 763317 (221 digits).

The algorithm gives networks of prime numbers without limitation and with great speed. In

$N_s = 12$

steps,

$N_T = N_s \times 16 = 192$

factorization tests were performed, the algorithm gave

$N_s = 12$

prime numbers from

$P_{\min} = 7$

to

$P_{\max}$ = 67022 740847 670751 354621 823058 752758 175293 036710 860638 249466 163101 656015 217916 141630 595003 931222 155028 676858 263675 828221 251033 221985 319669 489368 669628 388030 412541 802593 422337 974264 543960 755957 492921 525082 145012 240462 763317.

The great speed of the algorithm is due to the fixed number of 16 terms of the sequences W and Θ.

Example 2. In this example we take as P_1 the prime number

P_1 = 1 800000 000143 (127 digits).

As the second number P_2 of the pair of prime numbers of the algorithm we use small primes. We check the difference of digits of the prime numbers given by the algorithm with 127 which is the number of digits of P_1. Also, we apply the algorithm using a P_2 prime number given by the algorithm:

$P_2 = 3$, $W_2 =$ 7 × 173 × 221 374984 626737 178698 807034 805067 027425 900873 201328 249907 760423 072192 842208 830402 164555 405239 207969 499446 562538 433157 053253 (123 digits)

$P_2 = 3$, $\Theta_3 =$ 3^2 × 43 × 4651 162790 697674 418604 651162 790697 674418 604651 162790 697674 418604 651162 790697 674418 604651 162790 697674 418604 651162 790697 674419 (124 digits)

$P_2 = 5$, $\Theta_4 =$ 3 × 5 × 43 × 4651 162790 697674 418604 651162 790697 674418 604651 162790 697674 418604 651162 790697 674418 604651 162790 697674 418604 651162 790697 674419 (124 digits)

$P_2 = 7$, $\Theta_7 =$ 3 × 7 × 43 × 4651 162790 697674 418604 651162 790697 674418 604651 162790 697674 418604 651162 790697 674418 604651 162790 697674 418604 651162 790697 674419 (124 digits)

$P_2 = 19$, $W_1 =$ 11 400000 000929 (128 digits) is prime

$P_2 = 19$, $W_8 =$ 3 × 5 × 760000 000061 (126 digits)

$P_2 = 19$, $\Theta_2 =$ 5 × 7 × 13 × 25054 945057 (125 digits)

$P_2 = 23$, $\Theta_1 =$ 13 800000 001129 (128 digits) is prime

$P_2 = 23$, $\Theta_2 =$ 3^3 × 511111 111153 (126 digits)

$P_2 = 23$, $\Theta_7 =$ 179 × 77094 972067 039106 145251 396648 044692 737430 167597 765363 128491 620111 731843 575418 994413 407821 229050 279329 608938 547486 033519 553079 (125 digits).

The first prime number we found is
221374984626737178698807034805067027425900873201328249907760423072192842208830402164555405239207969499446562538433157053253 (123 digits).

We can implement the algorithm by taking this prime number as P_2. The algorithm gives:

$\Theta_1 =$ 11^2 × 1 097727 196496 217415 035406 784157 357160 789591 106783 445867 311208 709448 890126 655357 366105 233414 409295 716526 181301 843470 200264 064552 457638

402562 989721 004501 697919 705321 232585 015413 919384 134386 202788 836927 542890 132753 841791 084219 053494 888961 844832 195119 (247 digits).

The algorithm maintains a constant pattern, regardless of the difference $d = P_1 - P_2$. However, this difference remains small when the algorithm runs as in example 1 or by choosing suitable pairs (P_1, P_2).

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