# ON SUPERLUMINAL NON-BARYONIC MATTER IN A 3D-BRANE UNIVERSE.

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Abstract. In Einstein's relativity the speed of light is the topmost speed of matter. It can be reached by massless particles like photons and gravitons. The 3D-brane universe model is an alternative theory of gravity. Unlike Einstein's relativity, it is not so restrictive. In the present paper we build the theory of massive matter particles that can be superluminal in a curved 3D-brane universe.

## 1. Introduction.

Within the paradigm of a 3D-brane universe the gravitational field is described by a time-dependent 3D metric with the components

$$g_{ij}=g_{ij}(t,x^1,x^2,x^3),\quad 1\leqslant i,j\leqslant 3. \tag{1.1}$$

Through $t$ in (1.1) we denote the cosmological time (see [1]) and $x^1$, $x^2$, $x^3$ are comoving coordinates (see [2]) in the 3D-brane representing the instantaneous current state of a universe. As it was shown in [3] (see also [4] and [5]), the three-dimensional metric (1.1) obeys the following differential equations:

$$\begin{gathered}\frac{\dot b_{ij}}{c_{\text{gr}}}-\sum_{k=1}^{3}\frac{\dot b^k_k}{c_{\text{gr}}}\,g_{ij}-\sum_{k=1}^{3}(b_{ki}\,b^k_j+b_{kj}\,b^k_i)-\frac{g_{ij}}{2}\sum_{k=1}^{3}\sum_{q=1}^{3}b^k_q\,b^q_k-\\-\frac{g_{ij}}{2}\sum_{k=1}^{3}\sum_{q=1}^{3}b^k_k\,b^q_q+\sum_{k=1}^{3}b^k_k\,b_{ij}+R_{ij}-\frac{R}{2}\,g_{ij}+\Lambda\,g_{ij}=\frac{8\,\pi\,\gamma}{c_{\text{gr}}^4}\,T_{ij},\end{gathered} \tag{1.2}$$

Here $\gamma$ is Newton's gravitational constant (see [6]), $\Lambda$ is the cosmological constant (see [7]), $R_{ij}$ are the components of the three-dimensional Ricci tensor of the metric (1.1), $R$ is the three-dimensional scalar curvature, and $b_{ij}$ are given by the formula

$$b_{ij}=\frac{\dot g_{ij}}{2\,c_{\text{gr}}}=\frac{1}{2\,c_{\text{gr}}}\,\frac{\partial g_{ij}}{\partial t}. \tag{1.3}$$

Through $c_{\text{gr}}$ in (1.2) and (1.3) we denote the speed of gravity. In the standard relativity it coincides with the speed of electromagnetic waves $c_{\text{el}}$, which is the same as the speed of light. In the 3D-brane universe paradigm these two constants

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can be different (see [8]). In the present paper we introduce two more constants $c_{\text{bm}}$ and $c_{\text{nb}}$ of the same sort. They are called the speed of light for baryonic matter and the speed of light for non-baryonic matter. According to experimental data the speed of light for baryonic matter $c_{\text{bm}}$ coincides with the regular speed of light:

$$c_{\text{bm}} = c_{\text{el}}. \tag{1.4}$$

The speed of light for non-baryonic matter can be different from both $c_{\text{gr}}$ and $c_{\text{el}}$.

Initially the equations (1.2) were derived in [1] through 3D reduction of the standard Einstein's equations. Later on the equations (1.2) were rederived within purely three-dimensional Lagrangian and Hamiltonian approaches (see [9], [10], and [11]). In the present paper we study the non-baryonic matter in the form of separate massive particles.

## 2. Lagrangian for a non-baryonic particle.

In [10] the following Lagrangian for the gravitational field was chosen:

$$\mathcal{L}_{\text{gr}} = -\frac{c_{\text{gr}}^4}{16\,\pi\,\gamma}\left(\sum_{k=1}^{3}\sum_{q=1}^{3} b^k_k\,b^q_q - R - \sum_{k=1}^{3}\sum_{q=1}^{3} b^k_q\,b^q_k + 2\,\Lambda\right). \tag{2.1}$$

Let's consider a massive non-baryonic particle with the rest mass $m$. Its motion within the 3D-brane universe is described by the coordinate functions

$$x^i = x^i(t), \qquad i = 1,\,\ldots\,,3. \tag{2.2}$$

Time derivatives of the functions (2.2) are components of its velocity vector $\mathbf{v}$:

$$v^i = \dot{x}^i(t), \qquad i = 1,\,\ldots\,,3. \tag{2.3}$$

The action integral for a non-baryonic particle is given by the formula

$$S_{\text{nb}} = -\int m\,c_{\text{nb}}^2\sqrt{1-\frac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}\,dt. \tag{2.4}$$

The integral (2.4) is consistent with the first integral in the formula (1.7) from § 1 of Chapter IV in [12]. The only difference is that here we use $c_{\text{nb}}$ in place of the baryonic speed of light constant (1.4).

The action integral for the gravitational field uses the Lagrangian (2.1):

$$S_{\text{gr}} = \iint \mathcal{L}_{\text{gr}}\,\sqrt{\det g}\;d^3x\,dt. \tag{2.5}$$

The total action integral $S$ is the sum of the integrals (2.4) and (2.5):

$$S = S_{\text{gr}} + S_{\text{nb}}. \tag{2.6}$$

We use action (2.6) in order to derive the dynamic equations for the non-baryonic matter particle. Applying the stationary action principle (see [13]) to the sum of action integrals (2.6) we get the following Euler-Lagrange equations:

$$-\frac{d}{dt}\left(\frac{\delta\mathcal{L}}{\delta v^i}\right)_{\mathbf{g},\mathbf{b},\mathbf{x}}+\left(\frac{\delta\mathcal{L}}{\delta x^i}\right)_{\mathbf{g},\mathbf{b},\mathbf{v}}=0. \tag{2.7}$$

The Lagrangian $\mathcal{L}$ in (2.7) is given by the formula

$$\mathcal{L}=-m\,c_{\text{nb}}^2\,\sqrt{1-\frac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}+\int\mathcal{L}_{\text{gr}}\,\sqrt{\det g}\;d^3x. \tag{2.8}$$

The second term in the right hand side of (2.8) does not depend on the functions (2.2) and (2.3). Therefore we can easily derive the formulas

$$\left(\frac{\delta\mathcal{L}}{\delta v^i}\right)_{\mathbf{g},\mathbf{b},\mathbf{x}}=\frac{m\,v_i}{\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}},\qquad\left(\frac{\delta\mathcal{L}}{\delta x^i}\right)_{\mathbf{g},\mathbf{b},\mathbf{v}}=\frac{\displaystyle\sum_{r=1}^{3}\sum_{s=1}^{3}\frac{m}{2}\,\frac{\partial g_{rs}}{\partial x^i}\,v^r\,v^s}{\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}. \tag{2.9}$$

It is known that $\nabla_i g_{rs}=0$ and it is known that this covariant derivative is given by the following formula (see § 7 in Chapter III of [14]):

$$\nabla_i g_{rs}=\frac{\partial g_{rs}}{\partial x^i}-\sum_{q=1}^{3}\Gamma^q_{ir}\,g_{qs}-\sum_{q=1}^{3}\Gamma^q_{is}\,g_{rq}. \tag{2.10}$$

From $\nabla_i g_{rs}=0$ and from the formula (2.10) we derive

$$\sum_{r=1}^{3}\sum_{s=1}^{3}\frac{\partial g_{rs}}{\partial x^i}\,v^r\,v^s=\sum_{q=1}^{3}\sum_{s=1}^{3}2\,\Gamma^q_{is}\,v_q\,v^s. \tag{2.11}$$

Due to (2.11) the formulas (2.9) are written as follows:

$$\left(\frac{\delta\mathcal{L}}{\delta v^i}\right)_{\mathbf{g},\mathbf{b},\mathbf{x}}=\frac{m\,v_i}{\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}},\qquad\left(\frac{\delta\mathcal{L}}{\delta x^i}\right)_{\mathbf{g},\mathbf{b},\mathbf{v}}=\frac{\displaystyle\sum_{q=1}^{3}\sum_{s=1}^{3}m\,\Gamma^q_{is}\,v_q\,v^s}{\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}. \tag{2.12}$$

Applying (2.12) to the equations (2.7), we derive

$$\frac{d}{dt}\left(\frac{m\,v_i}{\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}\right)=\frac{\displaystyle\sum_{q=1}^{3}\sum_{s=1}^{3}m\,\Gamma^q_{is}\,v_q\,v^s}{\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}. \tag{2.13}$$

The time derivative in the left hand side of (2.13) is transformed as follows:

$$\frac{d}{dt}\left(\frac{m\,v_i}{\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}\right)=\frac{m\,\dot v_i}{\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}+\frac{m\,v_i}{\left(\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}\right)^3}\,\frac{1}{2}\,\frac{d}{dt}\left(\frac{|\mathbf{v}|^2}{c_{\text{nb}}^2}\right). \tag{2.14}$$

Combining (2.13) and (2.14), we derive the differential equations for $v_i$:

$$\frac{m\,\dot v_i}{\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}+\frac{m\,v_i}{\left(\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}\,\right)^3}\,\frac{1}{2}\,\frac{d}{dt}\!\left(\frac{|\mathbf{v}|^2}{c_{\text{nb}}^2}\right)=\frac{\displaystyle\sum_{q=1}^{3}\sum_{s=1}^{3}m\,\Gamma^q_{is}\,v_q\,v^s}{\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}.\tag{2.15}$$

The second term in the left hand side of (2.15) comprises the time derivative of $|\mathbf{v}|^2$. We calculate this time derivative as follows:

$$\begin{gathered}\frac{d(|\mathbf{v}|^2)}{dt}=\frac{d}{dt}\biggl(\,\sum_{r=1}^{3}\sum_{s=1}^{3}g_{rs}\,v^r\,v^s\biggr)=\sum_{r=1}^{3}\sum_{s=1}^{3}\frac{d(g_{rs}\,v^r)}{dt}\,v^s\,+\\+\sum_{r=1}^{3}\sum_{s=1}^{3}\frac{d(g_{rs}\,v^s)}{dt}\,v^r-\sum_{r=1}^{3}\sum_{s=1}^{3}\frac{dg_{rs}}{dt}\,v^s\,v^r=\sum_{s=1}^{3}2\,\dot v_s\,v^s\,-\\-\sum_{r=1}^{3}\sum_{s=1}^{3}\frac{\partial g_{rs}}{\partial t}\,v^s\,v^r-\sum_{r=1}^{3}\sum_{s=1}^{3}\sum_{i=1}^{3}\frac{\partial g_{rs}}{\partial x^i}\,\dot x^i\;v^s\,v^r.\end{gathered}\tag{2.16}$$

We transform (2.16) using (1.3), (2.3), and (2.11). This yields

$$\begin{gathered}\frac{d(|\mathbf{v}|^2)}{dt}=-\sum_{q=1}^{3}\sum_{s=1}^{3}\sum_{i=1}^{3}2\,\Gamma^q_{is}\,v_q\,v^s\,v^i\,+\\+\sum_{s=1}^{3}2\,\dot v_s\,v^s-\sum_{r=1}^{3}\sum_{s=1}^{3}2\,c_{\text{gr}}\,b_{rs}\,v^s\,v^r.\end{gathered}\tag{2.17}$$

Now we multiply (2.15) by $v^i$ and sum up with respect to $i$ running from 1 to 3:

$$\frac{m\displaystyle\sum_{i=1}^{3}\dot v_i\,v^i}{\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}+\frac{m\,\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}{\left(\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}\,\right)^3}\,\frac{1}{2}\,\frac{d(|\mathbf{v}|^2)}{dt}=\frac{\displaystyle\sum_{q=1}^{3}\sum_{s=1}^{3}\sum_{i=1}^{3}m\,\Gamma^q_{is}\,v_q\,v^s\,v^i}{\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}.\tag{2.18}$$

Then we apply (2.17) for the time derivative of $|\mathbf{v}|^2$ in (2.18). As a result the equality (2.18) simplifies and reduces to the following one:

$$\sum_{i=1}^{3}\dot v_i\,v^i-\sum_{q=1}^{3}\sum_{s=1}^{3}\sum_{i=1}^{3}\Gamma^q_{is}\,v_q\,v^s\,v^i=c_{\text{gr}}\,\frac{|\mathbf{v}|^2}{c_{\text{nb}}^2}\sum_{r=1}^{3}\sum_{s=1}^{3}b_{rs}\,v^s\,v^r.\tag{2.19}$$

Applying (2.19) to (2.17), we derive

$$\frac{d(|\mathbf{v}|^2)}{dt}=-2\biggl(1-\frac{|\mathbf{v}|^2}{c_{\text{nb}}^2}\biggr)\sum_{r=1}^{3}\sum_{s=1}^{3}c_{\text{gr}}\,b_{rs}\,v^s\,v^r.\tag{2.20}$$

Now we substitute (2.20) back into (2.15) and using (2.3) we derive

$$m\Bigl(\dot v_i-\sum_{q=1}^{3}\sum_{s=1}^{3}\Gamma^q_{is}\,v_q\,\dot x^s\Bigr)=m\,\frac{v_i\,c_{\text{gr}}}{c_{\text{nb}}^2}\sum_{r=1}^{3}\sum_{s=1}^{3}b_{rs}\,v^s\,v^r. \tag{2.21}$$

The left hand side of (2.21) fits the definition of the covariant $t$-derivative of a covectorial field along a parametric curve with the parameter $t$, see (8.10) in § 8 of Chapter III in [14]). Therefore we can write (2.3) and (2.21) as a system of ordinary differential equations defining the trajectory of a non-baryonic particle:

$$\begin{cases}\dot x^i=v^i,\\ m\,\nabla_t v_i=m\,\dfrac{v_i\,c_{\text{gr}}}{c_{\text{nb}}^2}\displaystyle\sum_{r=1}^{3}\sum_{s=1}^{3}b_{rs}\,v^s\,v^r.\end{cases} \tag{2.22}$$

Note that we see the same mass constant $m$ in the left and right hand sides of the equations (2.22). This observation can be interpreted as follows.

**Theorem 2.1.** *The inertial and passive gravitational masses of a non-baryonic massive particle are equal to each other.*

The definition of inertial mass and the definitions of active and passive gravitational masses are given in [15].

## 3. Legendre transformation and the energy function of a non-baryonic particle.

The Legendre transformation of a dynamical system is determined by its Lagrangian. In the case of the Lagrangian (2.8) it is given by the formulas

$$p_i=\Bigl(\frac{\delta\mathcal{L}}{\delta v^i}\Bigr)_{\mathbf{g},\mathbf{b},\mathbf{x}}, \qquad\qquad \beta^{ij}=\Bigl(\frac{\delta\mathcal{L}_{\text{gr}}}{\delta b_{ij}}\Bigr)_{\mathbf{g},\mathbf{x},\mathbf{v}} \tag{3.1}$$

The quantities $p_i$ and $\beta^{ij}$ in (3.1) are the generalized momenta associated with the generalized velocities $v^i$ and $b_{ij}$. These quantities are already calculated:

$$p_i=\frac{m\,v_i}{\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}, \qquad\qquad \beta^{ij}=\frac{c_{\text{gr}}^4}{8\pi\gamma}\Bigl(b^{ij}-\sum_{k=1}^{3}b^k_k\,g^{ij}\Bigr). \tag{3.2}$$

see (2.9) and the formula (3.15) in [8]. It is easy to see that $p_i$ in (3.2) are components of a covector $\mathbf{p}$. This covector is called the momentum covector of a non-baryonic particle.

Using the components of the momentum covector $\mathbf{p}$ and taking into account the equations (2.3), we can write the equations (2.13) as

$$\frac{dp_i}{dt}-\sum_{q=1}^{3}\sum_{s=1}^{3}\Gamma^q_{is}\,p_q\,\dot x^s=0. \tag{3.3}$$

The left hand side of the equations (3.3) fits the definition of the covariant $t$-derivative of a covectorial field along a parametric curve with the parameter $t$, see (8.10) in § 8 of Chapter III in [14]). Therefore we can write (2.3) and (3.3) as a system of ordinary differential equations:

$$\begin{cases} \dot{x}^i = v^i, \\ \nabla_t p_i = 0. \end{cases} \tag{3.4}$$

The equations (3.4) are equivalent to the equations (2.22).

The energy function of a non-baryonic particle is written using the components of its momentum covector $\mathbf{p}$ and the components of its velocity vector $\mathbf{v}$:

$$E = \sum_{i=1}^{3} p_i\, v^i + m\, c_{\mathrm{nb}}^2 \sqrt{1 - \frac{|\mathbf{v}|^2}{c_{\mathrm{nb}}^2}}\,. \tag{3.5}$$

Applying the first formula (3.2) to (3.5), we derive

$$E = \frac{m\, c_{\mathrm{nb}}^2}{\sqrt{1 - \dfrac{|\mathbf{v}|^2}{c_{\mathrm{nb}}^2}}}\,. \tag{3.6}$$

The time derivative of the energy function (3.6) is easily calculated using (2.20):

$$\frac{dE}{dt} = -\frac{m\, c_{\mathrm{gr}} \displaystyle\sum_{r=1}^{3} \sum_{s=1}^{3} b_{rs}\, v^s\, v^r}{\sqrt{1 - \dfrac{|\mathbf{v}|^2}{c_{\mathrm{nb}}^2}}}\,. \tag{3.7}$$

The right hand side of the formula (3.7) is a quantitative measure for the action of the gravitational field upon a single non-baryonic particle.

## 4. Conclusions.

Due to the denominator in (3.6) the non-baryonic speed of light constant $c_{\mathrm{nb}}$ is the upper limit for the speed $|\mathbf{v}|$ of a non-baryonic massive particle. If

$$c_{\mathrm{nb}} > c_{\mathrm{el}}, \tag{4.1}$$

then we would deal with a potentially superluminal non-baryonic massive particle. The inequality (4.1) is crucially different from the equality (1.4) for baryonic matter. It opens the prospect for superluminal transportation by converting baryonic matter into non-baryonic one and then, after having moved to a target place, by backward conversion of non-baryonic matter back into regular baryonic matter. This sort of transportation is different from quantum teleportation [16] and from crawling trough wormholes [17]. It is also different from warp engine assisted motion [18].

For a non-baryonic particle at rest, i. e. for $|\mathbf{v}| = 0$, the formula (3.6) turns to $E = m\, c_{\mathrm{nb}}^2$. This formula is an analog of Einstein's formula for baryonic matter $E = m\, c_{\mathrm{el}}^2$. If $c_{\mathrm{nb}} \gg c_{\mathrm{el}}$ in (4.1), if baryonic to non-baryonic matter conversions are possible, and if the energy conservation law is applicable to such conversions, then heavy baryonic particles will be converted into extremely light non-baryonic ones:

$$m_{\text{nb}} = \frac{c_{\text{el}}^2}{c_{\text{nb}}^2}\,m_{\text{bm}} \ll m_{\text{bm}}.$$

Such extremely light non-baryonic particles can be considered as candidates for being constituents of some sorts of dark matter.

## 5. Dedicatory.

This paper is dedicated to my sister Svetlana Abdulovna Sharipova.

## References


1. *Cosmic time*, Wikipedia, Wikimedia Foundation Inc., San Francisco, USA.
2. *Comoving and proper distances*, Wikipedia, Wikimedia Foundation Inc., San Francisco, USA.
3. Sharipov R. A., *A three-dimensional brane universe in a four-dimensional spacetime with a Big Bang*, e-print viXra:2207.0173.
4. Sharipov R. A., *On the dynamics of a 3D universe in a 4D spacetime*, Conference abstracts book "Ufa autumn mathematical school 2022" (Fazullin Z. Yu., ed.), vol. 2, pp. 279–281; DOI: 10.33184/mnkuomsh2t-2022-09-28.104.
5. Sharipov R. A., *The universe as a 3D brane and the equations for it*, Conference abstracts book "Foundamental mathematics and its applications in natural sciences 2022" (Gabdrakhmanova L. A., ed.), p. 37; DOI: 10.33184/fmpve2022-2022-10-19.30.
6. *Gravitational constant*, Wikipedia, Wikimedia Foundation Inc., San Francisco, USA.
7. *Cosmological constant*, Wikipedia, Wikimedia Foundation Inc., San Francisco, USA.
8. Sharipov R. A., *Speed of gravity can be different from the speed of light*, e-print viXra:2304.0225.
9. Sharipov R. A., *Lagrangian approach to deriving the gravity equations for a 3D-brane universe*, e-print viXra:2301.0033.
10. Sharipov R. A., *Hamiltonian approach to deriving the gravity equations for a 3D-brane universe*, e-print viXra:2302.0120.
11. Sharipov R. A., *The universe as a 3D-brane and the gravity field in it*, Conference abstracts book "Complex analysis, mathematical physics, and nonlinear equations", Lake Bannoe conference 2023 (Garifullin R. N., ed.), pp. 129-130.
12. Sharipov R. A., *Classical electrodynamics and theory of relativity*, Bashkir State University, Ufa, 1997; see also arXiv:physics/0311011.
13. *Stationary-action principle*, Wikipedia, Wikimedia Foundation Inc., San Francisco, USA.
14. Sharipov R. A., *Course of differential geometry*, Bashkir State University, Ufa, 1996; see also arXiv:math/0412421.
15. *Mass*, Wikipedia, Wikimedia Foundation Inc., San Francisco, USA.
16. *Quantum teleportation*, Wikipedia, Wikimedia Foundation Inc., San Francisco, USA.
17. *Wormhole*, Wikipedia, Wikimedia Foundation Inc., San Francisco, USA.
18. *Warp drive*, Wikipedia, Wikimedia Foundation Inc., San Francisco, USA.


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