An Elementary Proof of Collatz's Conjecture

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Abstract

Any even or odd number can be written as one of 10x + 1, 10x + 3, 10x + 5, 10x + 7, 10x + 9, 10x + 0, 10x + 2, 10x + 4, 10x + 6, or 10x + 8, (x= 1, 2, 3,….n); all 10x + 1, 10x + 3, 10x + 5, 10x + 7, 10x + 9, 10x + 0, 10x + 2, 10x + 4, 10x + 6, or 10x + 8, can be transferred in to 5 x 2y, y = 1, 2, 3, 4, 5, 6, 8,…m by repeating two arithmetic operation (3x + 1 and dividing 2). When y is an odd number, 3 times y plus 1 will always yield one even number, if the even number is not one of 2^n, then the even number divide 2 once or more, a new odd number y' will be yielded, but the new odd number must be different from the original y, 3 times y' + 1 will yield another new even number, if the new even number is not one of 2^n, then the new even number divide 2 once or more, a new odd number y'' will be yielded, so on, every dividing operation will yield one new odd number which is different from previous odd number, every time 3 + 1 will yield a new even number which is different from previous even number, these operations can be going unlimited and infinite different even numbers will be yielded until reach one of 2^n which is less than total even number, but is also infinite, that is: by an infinite number of repeating two arithmetic operation (3x + 1 and dividing 2), one of 2^n must be reach, then 5 x 1 will be reach, final 1 will be reach, this statement must be true, then the Collatz's conjecture will be the Collatz's theorem

Key words: Collatz's conjecture, Collatz's theorem, infinite odd number, infinite even number.

The Collatz's conjecture is one of the most famous unsolved problems in mathematics. The conjecture asks whether repeating two arithmetic operations will eventually transform every positive integer into 1 as follows: if the previous term is even, the next term is one half of the previous term. If the previous term is odd, the next term is 3 times the previous term plus 1.

The proofs of conjectures in number theory are too mathematical and make problems more complex, if we divide complex conjectures into a few of simple conjectures and solve them one by one, the proofs will be much simpler and clearer.

Let us see if the proof of Collatz's conjecture is very simple.

1. Any even number can be wrote as 10x + 0, 10x + 2, 10x + 4, 10x + 6, or 10x + 8, (x= 1, 2, 3, …n);

2. Any old number can be wrote as $10x + 1$, $10x + 3$, $10x + 5$, $10x + 7$, or $10x + 9$, ($x= 1, 2, 3, \ldots n$);

3. 3 times any odd number plus 1 must be an even number, because 3 times any odd number is an odd number and any odd number plus 1 is an even number, so any $10x + 1$, $10x + 3$, $10x + 5$, $10x + 7$, or $10x + 9$ time 3 plus 1 will be changed to one of $10x' + 0$, $10x' + 2$, $10x' + 4$, $10x' + 6$, or $10x' + 8$, for simple, x is used as: $10x + 0$, $10x + 2$, $10x + 4$, $10x + 6$, or $10x + 8$, ($x= 1, 2, 3, ...n$);

4. Any even number $10x + 0$, $10x + 2$, $10x + 4$, $10x + 6$, or $10x + 8$, ($x= 1, 2, 3, \ldots n$) divide 2 will be one of $10x' + 1$, $10x' + 3$, $10x' + 5$, $10x' + 7$, or $10x' + 9$, just for simple, x is used as x', $10x + 1$, $10x + 3$, $10x + 5$, $10x + 7$, or $10x + 9$, ($x= 1, 2, 3, ...n$);

5. Any one of $10x + 1$, $10x + 3$,$10x + 7$, or $10x + 9$ will be changed to $10x + 5$ by a number operation of timing 3 plus 1 and dividing 2:

   5.1 For $10x' + 1$: $(10x' + 1) \times 3 + 1 = 30x' + 3 + 1$, $(30x' + 3 + 1)/2 = 15x' + 2$, if x' is an odd number, then $15x' + 2$ will be $10x + 7$ which will be discussed later, if x' is an even number, then $15x' + 2$ will be $10x'' + 2$ which divide 2 will be a new $10x''' + 1$, $10x''' + 1$ which is different from original $10x' + 1$, and X' is even number which divide 2 must be an odd number (x'''), 3 times $10x''' + 1$, then plus 1 to get $30x''' + 3 +1$, because x''' is an odd number, only $10x +7$ will be the final number, that is, for both x' is odd or even number, only $10x + 7$ will be the result by a few of operations by timing 3 plus 1 and following one or more dividing 2;

   5.2 For $10x' +3$: $(10x' +3) \times 3 + 1 = 30x' + 9 + 1$ $(30x' + 9 + 1)/2= 15x' + 5$, if x' is an odd number, then it can be written as $10x'' + 0$ which divide 2 to be $10x + 5$; for $15x' + 5$, if x' is an even number, then it will be $10x + 5$ directly;

   5.3 For $10x' + 7$: $(10x' +7) \times 3 + 1 = 30x' + 21 + 1$, $(30x' + 22)/2= 15x' + 11$, if x' is an odd number, then it can be written as $10x'' + 6$ which divide 2 to be $10x + 3$ which is back to 5.2 and final to be $10x + 5$, no future discussion is needed; for $15x' + 11$, if x' is an even number, then it will be $10x + 1$ which is back to 5.1, no future discussion is needed;

   5.4 For $10x' + 9$: $(10x' + 9) \times 3 + 1 = 30x' + 27 + 1$, $(30x' + 28)/2= 15x' + 14$, if x' is an even number, then it can be written as $10x'' + 4$ which divide 2 twice or more to be $10x + 1$ which is back to 5.1 no future discussion is needed; for $15x' + 14$, if x' is an old number, then it will be $10x'' + 9$ which is different from $10x' + 9$, $(10x'' + 9) \times 3 + 1 = 30x'' + 27 + 1$, $(30x'' + 28)/2= 15x'' + 14$, here x'' can be only an even number, $15x'' + 14=10x''' + 4$ which divide 2 twice or more to be final $10x + 1$, finally, go back to 5.1, no future discussion is needed;

6. So far, any odd and even number (divide 2 first) can be transferred to 10x + 5. 10y + 0 divide 2 equal to 10x + 5 which is half of 10y + 0 by timing 3 plus 1 and following one or more dividing 2 and repeating the two arithmetic operations, all follow Collatz's conjecture; then, the Collatz's conjecture will be: (10y + 0) divided 2, if the resulted term is 10x + 0, the next term is one half of the previous term; if the previous term is 10x + 5, the next term is 3 times the previous term plus 1(3x +1), then the next term is one half of the previous term, The conjecture is that these sequences always reach 1.

7. 10y + 0 can be written as 5 x 2y, y = 1, 2, 3, 4, 5, 6, 8,…m,

   7.1 when m = 2^n, such as 2, 4, 8, 16, 32, 64…, then 2y divide by 2 for y times, 1 will be reach to yield 5 x 1 = 5, 5 time 3 + 1 = 16 which is divide 2 for 4 times, 1 will be reach, so the proof of the Collatz's conjecture will be simplified to whether repeating two simple arithmetic operations will eventually transform any y into 2^n;

   7.2 when m is an even number, but not 2^n, the even number divide 2 once or more, an odd number will reach, so for 5 x 2y, only y = 3, 5, 7, 9…m' are needed to be included in the analysis;

   7.3 when y is any odd number, 3 times y plus 1 will always yield one new even number, if the new even number is not one of 2^n, then the new even number divide 2 once or more, a new odd number y' will be yielded, but the new odd number must be different from the original y, 3 times y' + 1 will yield another new even number, if the new even number is not one of 2^n, then the new even number divide 2 once or more, another new odd number y'' will be yielded, so on, every dividing operation will yield one new odd number which is different from previous odd number, every time 3 + 1 will yield a new even number which is different from previous even number, these operations can be going unlimited and infinite different even numbers will be yielded until reach one of 2^n which is less than total even number, but is also infinite, that is: by an infinite number of repeating two arithmetic operation (3x + 1 and dividing 2), one of 2^n must be reach, then 5 x 1 will be reach, final 1 will be reach, this statement must be true, then the Collatz's conjecture will be the Collatz's theorem.