A DARK MATTER THEORY BY GRAVITATION GOT IN M31 AND MILKY WAY –V2

Author Manuel Abarca Hernandez   **email** mabarcaher1@gmail.com

1.   ABSTRACT

In this paper is develop carefully an original theory of dark matter, whose main hypothesis is that dark matter is generated by the own gravitational field. In this work is introduced the best version of the theory physical and mathematically, previously published since 2014.

The hypothesis of dark matter by gravitation has two main consequences: the first one is that the law of dark matter generation has to be the same for all the galaxies and the second one is that the haloes are unlimited so the total dark matter goes up without limit. Both properties are crucial for the success of this theory. Apparently the hypothesis of unlimited haloes would seem impossible but further cosmological studies may demonstrate that dark energy may cancel the growing of dark matter and this balance happens at cluster scale.

This paper is similar to one previous paper published in 2019, although there is an important newness, which is the new rotation curve of Milky Way published by Sofue,Y.in 2020. Results got with this new data in Milky Way back strongly the universality of dark matter generation law.

Briefly will be explained method followed to develop this work. Rotation curve data come from Sofue,Y.2015 for M31 galaxy and from Sofue 2020 for Milky Way. Thanks these remarkable rotation curves, both regression curves at halo region are fitted with power regression functions whose exponent are the same for both galaxies

By the fitted function is possible to calculate a dark matter density function depending on radius which is transformed into a dark matter density depending on gravitational field. This change is the core of the theory because at such moment it is possible to study the formula of dark matter density by the Buckingham theorem in order to change the statistical calculus by physical formulas which depend on the universal constants G, h and c.

From now on the statistical theory becomes a perfect physical theory that despite it is based on the Newtonian framework allows to get new formulas such as DM density and total DM mass.

In chapters 10 , 11 and 12 are calculated masses for M31, Milky Way and M33 with excellent results.

But it is in the chapter 13 where are published the most brilliant results got in this work.
The total mass of the Local group of galaxies calculated is 4.8E+12 Msun which match perfectly, if it is considered the range of error, with the dynamical mass currently accepted by the Local Group 5E+12Msun.

The chapter 14 is dedicated to show how dark energy may counter balance DM mass at cluster scale. Namely is calculated that total mass DE+DM is zero at 2,2 Mpc in Local Group whereas in Coma cluster total mass is zero at 16,2 Mpc. In other word the law of growing of DM is counter balanced by the law of growing of DE at cluster scale.

2. INTRODUCTION

Since 2013 up to 2019 I have published several papers studying DM in galactic halos, especially in M31 and Milky Way although also I have published some papers studying DM in Coma cluster [3] Abarca,M.2019. and other galaxies.

As reader knows M31 is the twin galaxy of Milky Way in Local Group of galaxies. According [5] Sofue, Y. 2015. Its baryonic masses are $M_{BARYONIC-M31} = 1,61 \cdot 10^{11} M_{SUN}$ and $M_{BARYONIC-MILKY WAY} = 1,4 \cdot 10^{11} M_{SUN}$

The DM theory introduced in [1] Abarca, M.2014. *Dark matter model by quantum vacuum* considers that DM is generated by the own gravitational field. Therefore, in order to study purely the phenomenon it is needed to consider a radius dominion where it is supposed that baryonic matter is negligible. i.e. radius bigger than 30 kpc for MW and 40 kpc for M31.

During these years I have developed an original theory based on the hypothesis that it is the gravitational field according an unknown quantum mechanism that generates the dark matter. This hypothesis has two main consequences: the first one is that the law of dark matter generation has to be the same for all the galaxies and the second one is that the haloes are unlimited so the total dark matter goes up without limit.

In the chapter 9, is demonstrated mathematically that total mass by Direct mass formula goes up proportionally to the root square of distance, so this property may explain how the ratio of dark matter versus baryonic matter at cluster scale is bigger that at galactic scale.

By other side the growing of the total mass is so slow that Dark energy phenomenon dominates when it is considered radius measured in mega parsecs. Precisely, this fact may explain the size of galactic clusters. This phenomenon will be studied in 14 chapter.

The first consequence before mentioned, dark matter generated by a Universal law, has been studied by all my papers, especially inside M31 and Milky Way thanks the papers [5] Sofue, Y.2015 and [6] Sofue, Y.2020.

In fact I could develop rigorously my theory because the rotation curve of M31 at halo region decreased with a power regression fitted curve whose exponent is -1/4. However with data published for Milky Way at the same paper (2015) it was not possible to fit rigorously the rotation curve with such exponent.

Fortunately, in a new paper [6] Sofue,Y.2020, the author gives a new rotation curve data for Milky Way at halo region whose fitted curve has an exponent -1/4. Such result it is the confirmation of my theory, because it is needed a same law for all the galaxies. So in this paper it is firstly developed all the theory carefully with M31 galaxy up to chapter 10 and chapter 11 is dedicated to apply the theory to Milky Way with magnificent results.

In the chapter 12 is estimated the total DM for M33, which is a newness regarding the first version of this paper. This estimation is made by Direct mass formula because is more simple than Bernoulli mass formula, and both give results very close.

The chapter 13 is dedicated to estimate the total mass of Local Group. The total mass calculated for Milky Way, M31 and M33 is $4,8*10^{12}$ Msun. As it is known the dynamical mass for Local group is estimated to be $5*10^{12}$ Msun. So it is possible to state that both results are equivalents when it is considered the range of errors.

The importance of these findings is high because there is not any other theory able to explain such amount of mass. In fact, the current theories of dark matter gives a maximum of dark matter for the Local Group up to $3*10^{12}$ Msun. Therefore, the theory not only may explain the rotation curves in halo region for M31 and MW but also it is the only one that may explain the total mass of the Local group of galaxies. i.e. the theory works perfectly at cluster scale.

In fact in the paper [3] Abarca,M.2019, it is studied the Coma cluster with the theory and it is shown how works perfectly inside this cluster, which is one of the most massifs between known clusters.

Thanks the possibility to study the gravitational effect of dark matter pure, in halo regions of M31 and MW, it have been possible to develop a theory mathematically simple. When baryonic mass is mixed with dark matter as it happens inside the galactic disc the mathematical treatment is by far more complex.

Taking into account that the only ones giant galaxies quite close to be able to study with accuracy the rotation curve at halo region are Milky Way and M31, the coincidence of the same exponent to the fitted function for the rotation curves for both galaxies is crucial in order to state that dark matter is generated according an universal law. In other words the two only ones galaxies where it is possible to develop this theory with simple mathematical methods are M31 and Milky Way.

3.   OBSERVATIONAL DATA FOR M31 GALAXY FROM SOFUE. 2015 DATA

Graphic come from [5] Sofue,Y. 2015. The axis for radius has logarithmic scale. Although Sofue rotation curve extend from 0,1 kpc up to 352 kpc the range of dominion considered for this work is only the halo region where ratio baryonic matter is negligible. In chapter 6, will be shown that this happens for radius bigger than 40 kpc, despite the fact that disc radius for M31 is accepted to be 35 kpc.
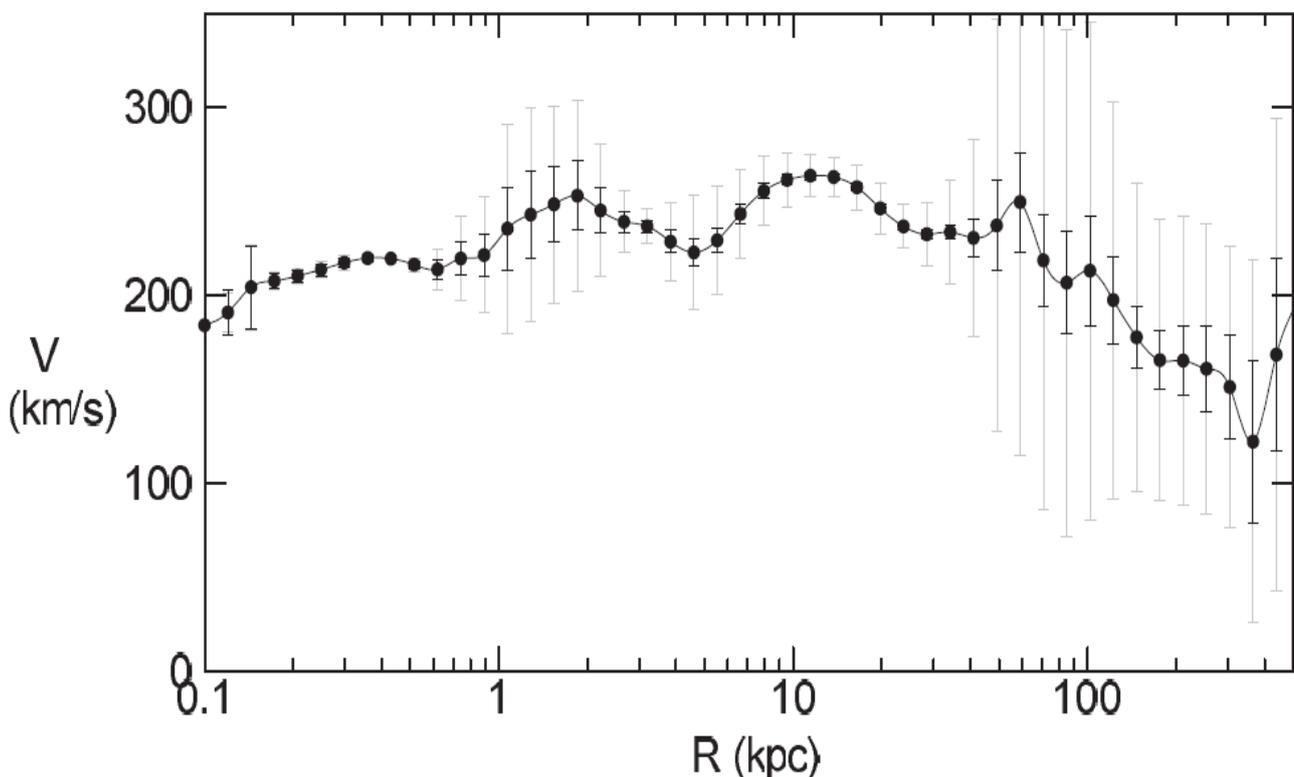
kpc	km/s
40,5	229,9
49,1	237,4
58,4	250,5
70,1	219,2
84,2	206,9
101,1	213,5
121,4	197,8
145,7	178,8
175	165,6
210,1	165,6
252,3	160,7
302,9	150,8

The last measure at 352 kpc has been rejected because has a velocity too high, so does not match at all with the other measures.

3.1 POWER REGRESSION TO ROTATION CURVE
The measures of rotation curve have a very good fitted curve by power regression.

In particular coefficients of $v = a \cdot r^b$ are in table below. Units are into I.S.

Power regression for M31 rot. curve	
V=a*r^b	
a	$4,32928*10^{10}$
b	-0.24822645
Correlation coeff.	0,96

Data fitted are in grey columns below.
In fifth column is shown results of fitted velocity and sixth column shows relative difference between measures and fitted results.

Radius kpc	Vel. km/s	Radius m	Vel. m/s	Vel. fitted	Relative Diff.
40,5	229,9	1,250E+21	2,299E+05	2,510E+05	8,397E-02
49,1	237,4	1,515E+21	2,374E+05	2,393E+05	7,777E-03
58,4	250,5	1,802E+21	2,505E+05	2,292E+05	-9,304E-02
70,1	219,2	2,163E+21	2,192E+05	2,190E+05	-8,154E-04
84,2	206,9	2,598E+21	2,069E+05	2,093E+05	1,138E-02
101,1	213,5	3,120E+21	2,135E+05	2,000E+05	-6,755E-02
121,4	197,8	3,746E+21	1,978E+05	1,911E+05	-3,500E-02
145,7	178,8	4,496E+21	1,788E+05	1,826E+05	2,107E-02
175	165,6	5,400E+21	1,656E+05	1,745E+05	5,115E-02
210,1	165,6	6,483E+21	1,656E+05	1,668E+05	7,100E-03
252,3	160,7	7,785E+21	1,607E+05	1,594E+05	-8,307E-03
302,9	150,8	9,347E+21	1,508E+05	1,523E+05	9,891E-03

Below is shown a graphic with measures data and power regression function.
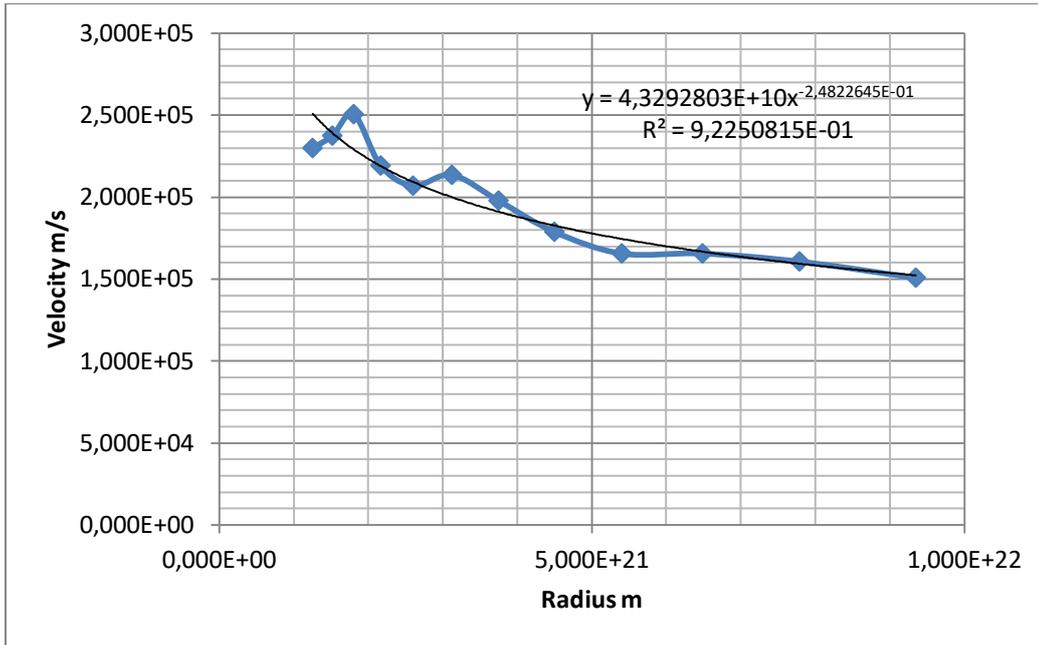
Correlation coefficient equal to 0,96 whis is a superb result especially when dominion measures is up to 303 kpc. There is not any other galaxy to measure a rotation curve so magnificent. According theory of DM generated by field, galaxy haloes are unlimited although up to a half of distance i.e. 375 kpc toward Milky Way direction is dominated by M31 field whereas the other half distance is dominated by Milky Way.

Furthermore, it has been calculated regression curve with another data placed at 363 kpc, but power regression is -0.28 and correlation coefficient is 0.954. This result shows that such data is not trustworthy because according dimensional analysis power has to be -0.25. As 363 kpc is placed in the border of M31 halo it is possible that such data might be influenced by a different field. Therefore it is better to study data only up to 303 kpc.

4. DIRECT FORMULA FOR DM DENSITY ON M31 HALO GOT FROM ROTATION CURVE

4.1 THEORETICAL DEVELOPPMENT FOR GALACTIC HALOS

Outside disk region, rotation curve it is fitted by power regression with a high correlation coefficient according

formula $v = a \cdot r^b$. As $M(<r) = \dfrac{v^2 \cdot R}{G}$ represents total mass enclosed by a sphere with radius r, by substitution of

velocity results $M = \dfrac{v^2 \cdot R}{G} = \dfrac{a^2 \cdot r^{2b+1}}{G}$. Hereafter this formula will be called Direct Mass $M_{DIRECT}(<r) = \dfrac{a^2 \cdot r^{2b+1}}{G}$

If it is considered outside region of disk where baryonic matter is negligible regarding dark matter it is possible to calculate DM density by a simple derivative. In next chapter will be show that for r > 40 kpc baryonic matter is negligible.

As density of D.M. is $D_{DM} = \dfrac{dm}{dV}$ where $dm = \dfrac{a^2 \cdot (2b+1) \cdot r^{2b} \, dr}{G}$ and $dV = 4\pi r^2 dr$ results

$$D_{DM} = \dfrac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$$

Writing $L = \dfrac{a^2 \cdot (2b+1)}{4\pi G} \cdot$ results $D_{DM}(r) = L \cdot r^{2b-2}$. In case b = -1/2 DM density is cero which is Keplerian rotation.

4.2 DIRECT DM DENSITY FOR M31 HALO

Parameters a & b from power regression of M31 rotation curve allow calculate easily direct DM density

Direct DM density for M31 halo $40 < r < 300$ kpc
$D_{DM}(r) = L \cdot r^{2b-2}$ kg/m^3

5. DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

As independent variable for this function is E, gravitational field, previously will be studied formula for E in the following paragraph.

5.1 GRAVITATIONAL FIELD E BY VIRIAL THEOREM

As it is known total gravitational field may be calculated through Virial theorem, formula $E = v^2/R$ whose I.S. unit is m/s^2 is well known. Hereafter, virial gravitational field, E, got through this formula will be called E.

By substitution of $v = a \cdot r^b$ in formula $E = \dfrac{v^2}{r}$ it is right to get $E = \dfrac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ briefly $E = a^2 \cdot r^{2b-1}$

5.2 DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

According hypothesis dark matter by quantum vacuum $D_{DM} = A \cdot E^B$. Where A & B are parameters to be calculated. This hypothesis has been widely studied by the author in previous papers. [1] Abarca,M. [2] Abarca,M.

[8] Abarca,M. [9] Abarca,M. [10] Abarca,M.

As it is known direct DM density $D_{DM} = \dfrac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ depend on a & b parameters which come from power

regression formula for velocity. In previous paragraph has been shown formula for gravitational field

$E = \dfrac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ which depend on a & b as well. Through a simple mathematical treatment it is possible to get

A & B to find function of DM density depending on E. Specifically formulas are $A = \dfrac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \dfrac{2b-2}{2b-1}$.

According parameters a & b got in previous chapter, A& B parameters are:

M31 galaxy	$D_{DM} = A \cdot E^B$
A	$3{,}6559956 \cdot 10^{-6}$
B	$1{,}6682469$

As conclusion, in this chapter has been demonstrated that a power law for velocity

$v = a \cdot r^b$ *is mathematically equivalent to a power law for DM density depending on E.* $D_{DM} = A \cdot E^B$

5.3 THE IMPORTANCE OF $D_{DM} = A \cdot E^B$

This formula is vital for theory of dark matter generated by gravitational field because it is supposed that DM is generated locally according an unknown quantum gravity mechanism. In other words, the propagation of gravitational field has this additional effect on the space as the gravitational wave goes by.

The formulas $D_{DM} = \dfrac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ and $E = a^2 \cdot r^{2b-1}$ have been got rightly from rotation curve. Therefore it can

be considered more specific for each galaxy. However the formula $D_{DM} = A \cdot E^B$ is much more essential.

The basis of this theory is that such formula is right for different gravitational systems. Therefore A & B parameters have to be similar for different galaxies on condition that they were similar. In further chapters will be got that power B is exactly the same for M31 and Milky Way although A will be a bit different.

However, there is an important fact to highlight. It is clear that A depend on a and b, both parameters are global parameters. As the gravitational interaction time between masses is proportional to distance, it is right to think that DM generated by a gravitational field has a bigger proportion as the system increase its size. For example inside the Solar system it is clear that Newton and General Relativity Theory is able to explain with total accuracy every gravitational phenomenon without DM hypothesis. Therefore it is right to conclude that DM arises when gravitational interaction takes a longer time to link the matter. Namely, for galaxy scale or bigger systems.

6. RATIO BARYONIC MASS VERSUS DARK MATTER MASS DEPENDING ON RADIUS FOR M31

In this paragraph will be estimated radius which is needed to consider negligible baryonic density regarding DM density in M31 galaxy. [5] According Sofue, Y. data for M31 disk are

M31 Galaxy	Baryonic Mass at disk	a_d	Σ_0
	$M_d = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$		
	$M_d = 1{,}26 \cdot 10^{11}$ Msun	5,28 kpc	1,5 kg/m^2

Where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ represents superficial density at disk. Total mass disk is given by integration of

superficial density from cero to infinite. $M_d = \displaystyle\int_0^\infty 2\pi \cdot r \Sigma(r) \cdot dr = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$

In order to compare baryonic density and DM density it is considered differential baryonic mass and differential DM masses depending on radius.

$dM_{DISK} = 2\pi r \Sigma(r) dr$ where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ and

$dM_{DM} = 4\pi r^2 D_{DM}(r) dr$ where $D_{DM}(r) = \dfrac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$

It is defined ratio function as quotient of both differential quantities $Ratio = \dfrac{dM_{DISK}}{dM_{DM}} = \dfrac{\Sigma(r)}{2 \cdot r \cdot D_{DM}(r)}$

Radius	Radius	Ratio (r)	$\Sigma(r)$	Direct DM
Kpc	m	Ratio	kg/m^2	kg/m^3

36	1,110852E+21	2,310614E-02	1,64056151250E-03	3,1957946476E-23
38	1,172566E+21	1,715255E-02	1,12327743139E-03	2,7924857817E-23
40	1,234280E+21	1,268028E-02	7,69097762116E-04	2,4570213865E-23
42	1,295994E+21	9,339073E-03	5,26594188719E-04	2,1754010061E-23
44	1,357708E+21	6,854954E-03	3,60554214629E-04	1,9370002366E-23

For a radius 40 kpc ratio baryonic matter versus DM is only 1,2 % therefore is a good approximation to consider negligible baryonic mass density regarding DM density when radius is bigger than 40 kpc.
This is the reason why in this work dominion for radius begin at 40 kpc.

7. A DIFFERENTIAL EQUATION FOR A GRAVITATIONAL FIELD

7.1 INTRODUCTION

This formula $D_{DM} = \dfrac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ is a local formula because it has been got by differentiation. However E,

which represents a local magnitude $E = \dfrac{G \cdot M(<r)}{r^2} = \dfrac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ has been got through $v = a \cdot r^b$ whose

parameters a & b were got by a regression process on the whole dominion of rotation speed curve. Therefore, D_{DM} formula has a character more local than E formula because the former was got by a differentiation process whereas the later involves M(<r) which is the mass enclosed by the sphere of radius r.

In other words, the process of getting D_{DM} involves a derivative whereas the process to get E(r) involves M(r) which is a global magnitude. This is a not suitable situation because the formula $D_{DM} = A \cdot E^B$ involves two local magnitudes. Therefore it is needed to develop a new process with a more local nature or character.

It is clear that a differential equation for E is the best method to study locally such magnitude.

7.2 A DIFFERENTIAL BERNOULLI EQUATION FOR GRAVITATIONAL FIELD IN A GALACTIC HALO

As it is known in this formula $\vec{E} = -G\dfrac{M(r)}{r^2}\hat{r}$, M(r) represents mass enclosed by a sphere with radius r. If it is considered a region where does not exit any baryonic matter, such as any galactic halo, then the derivative of M(r) depend on dark matter density essentially and therefore $M'(r) = 4\pi r^2 \varphi_{DM}(r)$.

If $E = G\dfrac{M(r)}{r^2}$, vector modulus, is differentiated then it is got $E'(r) = G\dfrac{M'(r) \cdot r^2 - 2rM(r)}{r^4}$

If $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ is replaced above then it is got $E'(r) = 4\pi G \varphi_{DM}(r) - 2G\dfrac{M(r)}{r^3}$ As

$\varphi_{DM}(r) = A \cdot E^B(r)$ it is right to get $E'(r) = 4\pi \cdot G \cdot A \cdot E^B(r) - 2\dfrac{E(r)}{r}$ which is a Bernoulli differential equation.

$E'(r) = K \cdot E^B(r) - 2\dfrac{E(r)}{r}$ being $K = 4\pi \cdot G \cdot A$

Calling **y** to E, the differential equation is written in this simple way $y` = K \cdot y^B - \dfrac{2 \cdot y}{r}$

9

Bernoulli family equations $y`= K \cdot y^B - \dfrac{2 \cdot y}{r}$ may be converted into a differential linear equation with this variable

change $u = y^{1-B}$. Which is $\dfrac{u`}{1-B} + \dfrac{2u}{r} = K$

The homogenous equation is $\dfrac{u`}{1-B} + \dfrac{2u}{r} = 0$ Whose general solution is $u = C \cdot r^{2B-2}$ being C the integration constant.

If it is searched a particular solution for the complete differential equation with a simple linear function u=z*r then it

is got that $z = \dfrac{K \cdot (1-B)}{3-2B}$. Therefore the general solution for u- equation is $u = C \cdot r^{2B-2} + z \cdot r$

When it is inverted the variable change it is got the general solution for field E.

General solution is $E(r) = \left(Cr^{2B-2} + \dfrac{Kr(1-B)}{3-2B} \right)^{\frac{1}{1-B}}$ with $B \neq 1$ and $B \neq 3/2$ where C is the parameter

of initial condition of gravitational field at a specific radius.

Calling $\alpha = 2B-2$ $\beta = \dfrac{1}{1-B}$ and $D = \left(\dfrac{K(1-B)}{3-2B} \right)$ formula may be written as $E(r) = \left(Cr^\alpha + Dr \right)^\beta$

Calculus of parameter C through initial conditions R_0 and E_0

Suppose R_0 and E_0 are the specific initial conditions for radius and gravitational field, then $C = \dfrac{E_0^{1/\beta} - D \cdot R_0}{R_0^\alpha}$

Final comment

It is clear that the Bernoulli solution contains implicitly the fact that it is supposed there is not any baryonic matter inside the radius dominion and the only DM matter is added by $\varphi_{DM}(r) = A \cdot E^B(r)$. Therefore this solution for field works only in the halo region and R_0 and E_0 could be the border radius of galactic disk where it is supposed begins the halo region and the baryonic density is negligible.

8. DIMENSIONAL ANALYSIS FOR D.M. DENSITY AS POWER OF E FORMULA

8.1 POWER OF E BY BUCKINGHAM THEOREM

As it is supposed that DM density as power of E come from a quantum gravity theory, it is right to think that constant Plank h should be considered and universal constant of gravitation G as well.
So the elements for dimensional analysis are D, density of DM whose units are Kg/m^3, E gravitational field whose units are m/s^2, G and finally h.
In table below are developed dimensional expression for these four elements D, E, G and h.

	G	h	E	D
M	-1	1	0	1
L	3	2	1	-3
T	-2	-1	-2	0

According Buckingham theorem it is got the following formula for Density

$$D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$$ being K a dimensionless number which may be understood as a coupling constant between field

E and DM density.

As it is shown in previous epigraph, parameters for M31 is B = 1,6682469

In this case relative difference between B = 1,6682469 and 10/7 is 16,7 %. A 17% of error in cosmology could be acceptable. However by the end of the chapter it will be found a better solution.

8.2 POWER E FORMULA FOR DM DENSITY WITH TWO PI MONOMIALS

As this formula come from a quantum gravitation theory it is right to consider that Universal constants involved are G, h and c. So elements to make dimensional analysis are D, E, G,h and c =$2.99792458 \cdot 10^8$ m/s.

	G	h	E	D	c
M	-1	1	0	1	0
L	3	2	1	-3	1
T	-2	-1	-2	0	-1

According Buckingham theorem, as matrix rank is three, there are two pi monomials. The first one was calculated in previous paragraph and the second one involves G, h, E and c.

These are both pi monomials $\pi_1 = D \cdot \sqrt[7]{G^9 \cdot h^2} \cdot E^{-\frac{10}{7}}$ and $\pi_2 = \frac{c}{\sqrt[7]{G \cdot h}} E^{-\frac{2}{7}}$. So formula for DM density through

two pi monomials will be $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ being J a dimensionless number and $f(\pi_2)$ an unknown

function, which can not be calculated by dimensional analysis theory.

8.3 MATHEMATICAL ANALYSIS TO DISCARD FORMULA WITH ONLY ONE PI MONOMIAL

As it was shown in paragraph **5.2** $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ and $B = \frac{2b-2}{2b-1}$. Being a, b parameters got to fit rotation

curve of velocities $v = a \cdot r^b$

Conversely, it is right to clear up parameters a and b from above formulas.

Therefore $b = \frac{B-2}{2B-2}$ and $a = \left[\frac{4\pi GA(B-1)}{2B-3} \right]^{\frac{2b-1}{2}}$ being $B \neq 1 \; and \; B \neq 3/2$.

As A is a positive quantity then $2b+1 > 0$. As $2b+1 = \frac{2B-3}{B-1} > 0$ Therefore $B \in (-\infty, 1) \cup (3/2, \infty)$.

If B=3/2 then 2b+1=0 and A=0 so dark matter density is cero which is Keplerian rotation curve.

In every galactic rotation curve studied, B parameter has been bigger than 3/2. See Abarca papers quoted in Bibliographic references. Therefore experimental data got in several galaxies fit perfectly with mathematical findings made in this paragraph especially for $B \in (3/2, \infty)$.

The main consequence this mathematical analysis is that formula $D = \dfrac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ got with only a pi monomial is

wrong because B=10/7 = 1.428. Therefore formula $D = \dfrac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ got thorough dimensional analysis by

two pi monomials it is more suitable formula.

This formula is physically more acceptable because it is got considering G, h and c as universal constant involved in formula of density. As according my theory, DM is generated through a quantum gravitation mechanism it is right to consider not only **G** and **h** but also **c** as well.

8.4 LOOKING FOR A D.M. DENSITY FUNCTION COHERENT WITH DIMENSIONAL ANALYSIS

It is right to think that $f(\pi_2)$ should be a power of π_2, because it is supposed that density of D.M. is a power of E.

M31 galaxy	$D_{DM} = A \cdot E^B$
A	$3,6559956 \cdot 10^{-6}$
B	1,6682469

Taking in consideration A &B parameters on the left, power for π_2 must be -5/6. This way, power of E in formula

$D_{DM} = A \cdot E^B$ will be $5/3 = 1.666666$, which is the best approximation to B= 1.6682469.

Finally $D = \dfrac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ becomes $D = \dfrac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}} E^{\frac{5}{3}}$ being M a dimensionless number.

CALCULUS OF DIMENSIONLESS NUMBER INCLUDED IN FORMULA OF DARK MATTER DENSITY

By equation of $D = \dfrac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}} E^{\frac{5}{3}}$ and D=A*E^B

It is right that $A = \dfrac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}}$ and then $M = A \cdot \sqrt[6]{G^7 \cdot c^5 \cdot h}$

9. RECALCULATING FORMULAS IN M31 HALO WITH B = 5/3

Findings got through Buckingham theorem are crucial. It is clear that a physic formula has to be dimensionally coherent .Therefore it is a magnificent support to the theory of DM generated by gravitational field that statistical value got by regression analysis in M31, differs less than 2 thousandth regarding value got by Buckingham theorem.

Now it is needed to rewrite all the formulas considering B=5/3. Furthermore, with B= 5/3, a lot of parameters of the theory become simple fraction numbers. In other words, theory gains simplicity and credibility.

In chapter 5 was shown the relation between a&b parameters and A&B parameters. Now considering B= 5/3

as $A = \dfrac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \dfrac{2b-2}{2b-1}$. It is right to get $b = \dfrac{B-2}{2B-2} = -\dfrac{1}{4}$ and $A = \dfrac{a^{\frac{-4}{3}}}{8\pi G}$

Therefore, the central formula of theory becomes $\quad D_{DM} = A \cdot E^{\frac{5}{3}} = \dfrac{a^{\frac{-4}{3}}}{8 \cdot \pi \cdot G} \cdot E^{\frac{5}{3}}$

9.1 RECALCULATING THE PARAMETER a IN M31 HALO

Table below comes from chapter 3 and represents regression curve of velocity depending on radius.

Regression for M31 dominion 40-303 kpc	
V=a*r^b	
a	$4,32928*10^{10}$
b	-0.24822645
Correlation coeff.	0,96

Due to Buckingham theorem it is needed that b= -1/4 . Therefore it is needed to recalculate parameter **a** in order to find a new couple of values a&b that fit perfectly to experimental measures of rotation curve in M31 halo.

RECALCULATING a WITH MINUMUN SQUARE METHOD

When it is searched the parameter a, a method widely used is called the minimum squared method. So it is searched a new parameter **a** for the formula V= a*r $^{-0.25}$ on condition that $\sum_e (v-v_e)^2$ has a minimum value. Where v represents the value fitted for velocity formula and v_e represents each measure of velocity. It is right to calculate the formula for a.

$$a = \frac{\sum\limits_e Ve \cdot r_e^{-0.25}}{\sum\limits_e r_e^{-0.5}} = 4.727513 \cdot 10^{10}$$ Where r_e represents each radius measure and v_e represents its velocity associated. Consequently the new A= 3.488152*10^{-6}

New parameters a&b and A&B	
B	5/3
$b = \dfrac{B-2}{2B-2}$	b = -1/4
a new	$4,727513*10^{10}$
$A = \dfrac{a^{\frac{-4}{3}}}{8\pi G}$	New parameter A 3.488152*10^{-6}

9.2 FORMULAS OF DIRECT D.M.

With these new parameters recalculated it is going to get the direct formulas got at the beginning of paper.

Function of Density DM depending on radius.

$$D_{DM}(r) = L \cdot r^{2b-2} = L \cdot r^{\frac{-5}{2}} \text{ being } L = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot = \frac{a^2}{8 \cdot \pi \cdot G} = 1,3326*10^{30}$$

Function of E depending on radius $\quad E = a^2 \cdot r^{2b-1} = a^2 \cdot r^{\frac{-3}{2}} \quad$ being a^2 = 2,235*10^{21}

Mass enclosed by a sphere of radius r, known as dynamical mass because it is calculated with velocity.

$$M_{DYN}(<r) = \frac{v^2 \cdot R}{G} \quad \text{When velocity is replaced by its fitted function it is got } \quad M_{DIRECT}(<r) = \frac{a^2 \cdot r^{2b+1}}{G}$$

being $\dfrac{a^2}{G} = 3{,}349 \times 10^{31}$

9.3 BERNOULLI SOLUTION FOR E IN M31 HALO

In chapter 7 was got the solution for field in the halo region, now thanks dimensional analysis it is possible to get formulas far simple because some parameters are simple fractions.

$$E(r) = \left(Cr^{\alpha} + Dr\right)^{\beta} \quad \text{being } \alpha = 2B - 2 = \frac{4}{3} \quad \text{being } \beta = \frac{1}{1-B} = \frac{-3}{2} \quad \text{then, the initial condition } C$$

$$C = \frac{E_0^{1/\beta} - D \cdot R_0}{R_0^{\alpha}} \quad \text{becomes} \quad C = \frac{E_0^{-\frac{2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}} \quad \text{and} \quad D = \left(\frac{4 \cdot \pi \cdot G \cdot A(1-B)}{3 - 2B}\right) = 8 \cdot \pi \cdot G \cdot A = a^{-\frac{4}{3}} \quad \text{so} \quad D = a^{-\frac{4}{3}} = 5{,}85 \times 10^{-15}$$

Therefore $E(r) = \left(Cr^{\frac{4}{3}} + Dr\right)^{-\frac{3}{2}}$ being C the initial condition of differential equation solution for E and $D = a^{-\frac{4}{3}}$ is

a parameter closely related to the global rotation curve at halo region, being parameter **a** $=4{,}7275 \times 10^{10}$

9.4 DARK MATTER AT A SPHERICAL CORONA BY BERNOULLI SOLUTION

Formula below express the dark matter contained inside a spherical corona defined by R_1 and R_2 belonging at halo.

$$M_{DM} = \int_{R1}^{R2} 4\pi \cdot r^2 \cdot \rho(r) dr = \int_{R1}^{R2} 4\pi \cdot r^2 A E^B dr = 4\pi A \int_{R1}^{R2} r^2 \left[C \cdot r^{4/3} + D \cdot r\right]^{-\frac{5}{2}} \cdot dr$$

The indefinite integral $I = 4\pi A \cdot \int \dfrac{r^2}{\left(C \cdot r^{4/3} + D \cdot r\right)^{2.5}} = \dfrac{8\pi A \sqrt{r}}{D \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}} = \dfrac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$

As $\dfrac{8\pi A}{D} = \dfrac{1}{G}$. Calling $M(r) = \dfrac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ and by the Barrow`s rule, it is got

$M_{DM}\Big|_{R1}^{R2}$ = M(R2)-M(R1) that provided the DM contained inside the spherical corona defined by R2 and R1.

9.5 CALCULUS OF PARAMETER C

$$C_O = \frac{E_0^{-\frac{2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$$

In this formula, E_O is the gravitational field at R_O radius which is V^2_0 / R_O i.e. the centripetal acceleration.

As experimental measures have errors, it will be calculated Co for every data in order to calculate function dark matter mass for all of them.

A DARK MATTER THEORY BY GRAVITATION GOT IN M31 AND MILKY WAY –V2

	Radius	Radius	Velocity	Field Eo	Eo^(-2/3)	Eo^(-2/3)-D*R	Parameters
	kpc	m	m/s	m/s²			Co
1	40,5	1,250E+21	2,299E+05	4,23E-11	8239256,95	9,27E+05	6,8830E-23
2	49,1	1,515E+21	2,374E+05	3,72E-11	8973615,749	1,11E+05	6,3753E-24
3	58,4	1,802E+21	2,505E+05	3,48E-11	9377625,482	-1,16E+06	-5,3082E-23
4	70,1	2,163E+21	2,192E+05	2,22E-11	12654654,13	1,20E+03	4,2804E-26
5	84,2	2,598E+21	2,069E+05	1,65E-11	15443489,73	2,45E+05	6,8683E-24
6	101,1	3,120E+21	2,135E+05	1,46E-11	16732951,68	-1,52E+06	-3,3317E-23
7	121,4	3,746E+21	1,978E+05	1,04E-11	20928758,45	-9,85E+05	-1,6934E-23
8	145,7	4,496E+21	1,788E+05	7,11E-12	27043372,43	7,42E+05	9,9988E-24
9	175	5,400E+21	1,656E+05	5,08E-12	33846627,1	2,26E+06	2,3822E-23
10	210,1	6,483E+21	1,656E+05	4,23E-12	38232870,38	3,08E+05	2,5446E-24
11	252,3	7,785E+21	1,607E+05	3,32E-12	44959131,06	-5,83E+05	-3,7771E-24
12	302,9	9,347E+21	1,508E+05	2,43E-12	55281485,72	6,02E+05	3,0572E-24

Values in yellow are negatives because these points are above the fitted curve . See graph below. In addition, the more close to curve the point is, the smaller, in absolute value, the parameter C is. The cyan value is the smaller.

$$y = 4{,}3292803E{+}10x^{-2{,}4822645E{-}01}$$
$$R^2 = 9{,}2250815E{-}01$$
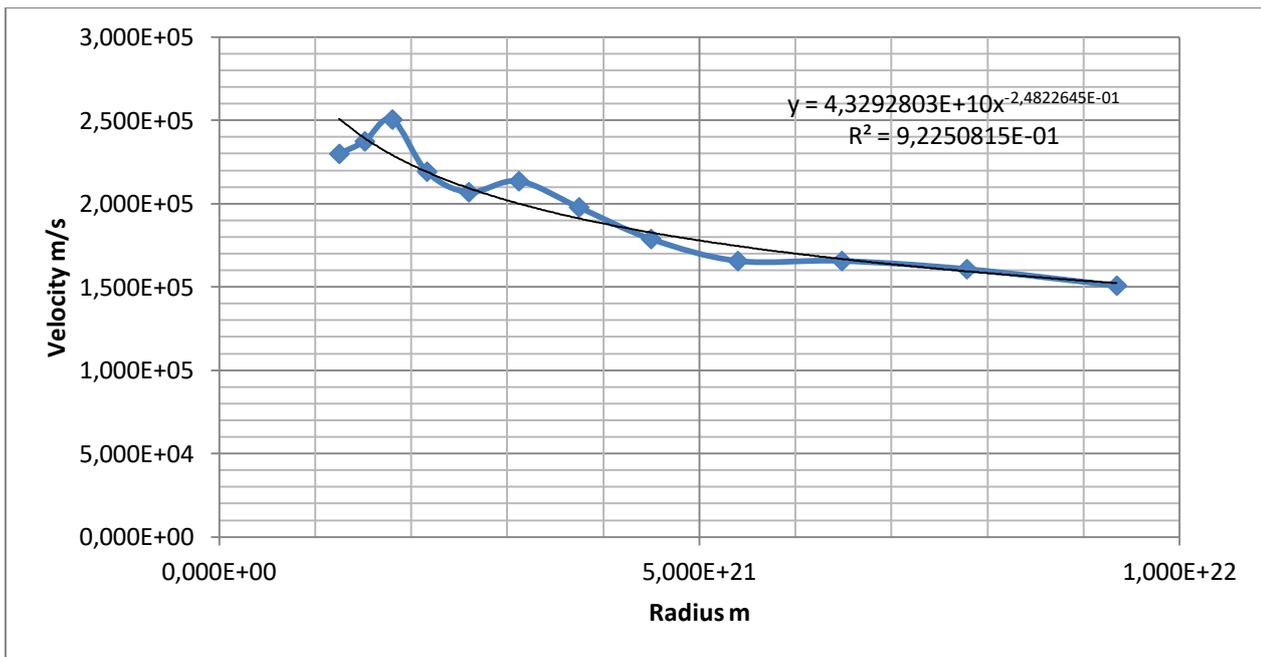
FUNCTION DARK MATTER INSIDE SPHERICAL CORONA FOR DIFFERENT PARAMETERS Co

Function of dark matter is $M(r) = \dfrac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ being $D = a^{\frac{-4}{3}} = 5{,}85 * 10^{-15}$ and parameters C

calculated in previous paragrapah.

The row 1 gives the value M(R1) in Msun units. See table below, the grey row.

The row 2 and the followings one give the dark matter   inside the spherical corona with radius Rn and R1, where n

ranges from 2 to 14. For example, $M_{DM}{}^{R2}_{R1} = M(R2)-M(R1)$ that provided the DM contained inside the spherical

corona defined by R2 and R1.

The columns 1 to 12 give the 12 dark matter functions inside spherical corona Rn –R1 ,where n ranges from 2 to 12, associated at different values of parameters C showed in previous table.

A good option to minimize error is to considerate the average function mass to try to adopt the most suitable parameter C for M31.

The last column, gives de Average dark matter function considering the 12 functions.

	kpc	1 C at 40kpc	2 C at 49 kpc	3 C at 58 kpc	4 C at 70 kpc	5 C at 84kpc	6 C at 101kpc
1	40,5	4,9743E+11	5,8461E+11	6,9418E+11	5,9486E+11	5,8382E+11	6,542E+11
		Dark matter inside spherical corona between Rn and R1 with the formulas M(Rn)-M(R1)					
		Dark matter inside spherical corona between Rn and R1 with the formulas M(Rn)-M(R1)					
2	49,1	4,4135E+10	5,8251E+10	7,8343E+10	6,0022E+10	5,8116E+10	7,0714E+10
3	58,4	8,6985E+10	1,1573E+11	1,5717E+11	1,1936E+11	1,1545E+11	1,4137E+11
4	70,1	1,3538E+11	1,8174E+11	2,4957E+11	1,8763E+11	1,8129E+11	2,2357E+11
5	84,2	1,8753E+11	2,5417E+11	3,5325E+11	2,627E+11	2,5352E+11	3,1506E+11
6	101,1	2,4344E+11	3,333E+11	4,6928E+11	3,449E+11	3,3242E+11	4,1656E+11
7	121,4	3,0327E+11	4,1969E+11	5,9926E+11	4,3487E+11	4,1854E+11	5,2918E+11
8	145,7	3,6717E+11	5,1392E+11	7,45E+11	5,3324E+11	5,1246E+11	6,5417E+11
9	175	4,3574E+11	6,173E+11	9,0967E+11	6,4144E+11	6,1548E+11	7,9385E+11
10	210,1	5,0878E+11	7,3E+11	1,095E+12	7,5974E+11	7,2775E+11	9,4917E+11
11	252,3	5,8674E+11	8,5327E+11	1,3047E+12	8,8953E+11	8,5053E+11	1,1227E+12
12	302,9	6,6967E+11	9,8779E+11	1,5422E+12	1,0316E+12	9,8449E+11	1,3164E+12
13	770	1,178E+12	1,8919E+12	3,3875E+12	1,9981E+12	1,884E+12	2,7375E+12
14	1000	1,3479E+12	2,226E+12	4,1909E+12	2,36E+12	2,216E+12	3,3166E+12

kpc	7 C at 121 kpc	8 C at 145 kpc	9 C at 175 kpc	10 C at 210 kpc	11 C at 252kpc	12 C at 302 kpc	Average DM mass Msun
40,5	6,2388E+11	5,7887E+11	5,5783E+11	5,9077E+11	6,0119E+11	5,8994E+11	
	Dark matter inside spherical corona between Rn and R1 with formulas M(Rn)-M(R1)						
49,1	6,5156E+10	5,7271E+10	5,3736E+10	5,9313E+10	6,1126E+10	5,917E+10	6,0446E+10
58,4	1,2991E+11	1,1372E+11	1,065E+11	1,1791E+11	1,2162E+11	1,1761E+11	1,2028E+11
70,1	2,0481E+11	1,7848E+11	1,6679E+11	1,8527E+11	1,9131E+11	1,8479E+11	1,8922E+11
84,2	2,8768E+11	2,4946E+11	2,3257E+11	2,5928E+11	2,6805E+11	2,5859E+11	2,6515E+11
101,1	3,7898E+11	3,269E+11	3,0403E+11	3,4025E+11	3,5218E+11	3,3931E+11	3,4846E+11
121,4	4,7959E+11	4,1134E+11	3,8156E+11	4,2878E+11	4,444E+11	4,2755E+11	4,3984E+11
145,7	5,9039E+11	5,0332E+11	4,6558E+11	5,2548E+11	5,4539E+11	5,2392E+11	5,4E+11
175	7,1321E+11	6,0407E+11	5,5712E+11	6,3175E+11	6,5666E+11	6,2979E+11	6,5051E+11
210,1	8,4859E+11	7,1372E+11	6,5619E+11	7,4779E+11	7,7853E+11	7,4537E+11	7,7172E+11

252,3	9,9841E+11	8,3347E+11	7,6371E+11	8,7494E+11	9,1248E+11	8,7199E+11	9,0521E+11
302,9	1,164E+12	9,6392E+11	8,801E+11	1,014E+12	1,0594E+12	1,0104E+12	1,052E+12
770	2,331E+12	1,8348E+12	1,6391E+12	1,9551E+12	2,0665E+12	1,9465E+12	2,0708E+12
1000	2,7857E+12	2,1543E+12	1,9104E+12	2,3056E+12	2,4468E+12	2,2947E+12	2,4629E+12

Comparing the Average DM mass placed in the last column with the others columns, it can be checked that ,relatives differences between values of mass at column C at 252 kpc and Average mass are under 1% . See green columns.

So the most suitable parameter C is at 252 kpc C= -3,7771E-24. In my opinion, this is the way to get the best parameter C for M31 galaxy. $C_{M31} = -3,777*10^{-24}$ Because there is no way to know which experimental data is more reliable, so taking the average dark matter mass function it will possible to reduce errors.

P.6 STUDYING CASE WHEN C = 0

Now It will be investigated the conditions to get   C= 0.   Then formula $C = \dfrac{E_0^{-\frac{2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$ leds to

$E_0^{-\frac{2}{3}} = D \cdot R_0 = a^{-\frac{4}{3}} \cdot R_0$ and as $E = a^2 \cdot r^{2b-1}$ then $E_0^{-\frac{2}{3}} = a^{-\frac{4}{3}} \cdot R_0^{\frac{2-4b}{3}} = D * R_0 = a^{-\frac{4}{3}} \cdot R_0$ and by equation of power of R_0 $\dfrac{2-4b}{3} = 1$ it is got b= -1/4

At the beginning of chapter was shown that B= 5/3 leds rightly to b = -1/4. So b= -1/4 is rigorously the power of radius on the rotation curve of galaxy in the halo region, where there is not any baryonic matter. Namely formula is $V = a*r^{-0.25}$. Therefore C= 0 for every point belonging to regression curve whose power is -1 /4.

In the epigraph 9.7 it will be shown that for C= 0 the Bernoulli solution for field becomes direct formula for field, and the same happens with Bernoulli DM density and mass formulas.

9.7 GETTING DIRECT FORMULAS BY BERNOULLI FIELD WHEN PARAMETER C = 0

Thanks demonstration made in previous epigraph it is trustworthy to consider C= 0 in halo región.

FOR FIELD E

When in formula $E(r) = \left(Cr^{\frac{4}{3}} + Dr \right)^{-\frac{3}{2}}$ C= 0 then   it is got $E = a^2 \cdot r^{-\frac{3}{2}}$ being $a^2 = 2.235*10^{21}$ which is precisely direct formula for E.

FOR DM DENSITY

As $D_{DM} = A*E^B$ Using field got by Bernoulli solution it is right to get

$$D_{DM}(r) = A \left(Cr^{\frac{4}{3}} + Dr \right)^{-\frac{5}{2}}$$ Being **A**= 3,488*10^{-6} **D**=5,85*10^{-15} if **C** = 0 then formula becomes

$$D_{DM}(r) = A \cdot D^{-\frac{5}{2}} \cdot r^{-\frac{5}{2}} = L \cdot r^{-\frac{5}{2}}$$ being $L = \dfrac{a^2}{8 \cdot \pi \cdot G} = 1.3326*10^{30}$ which is direct DM density formula.

FOR TOTAL MASS INSIDE A SPHERICAL CORONA

$$M_{DM} = \int_{R1}^{R2} 4\pi r^2 \cdot \rho(r)dr = \int_{R1}^{R2} 4\pi r^2 AE^B dr = 4\pi A\int_{R1}^{R2} r^2 \left[D \cdot r\right]^{\frac{-5}{2}} \cdot dr$$ whose indefinite integral is $M(<r) = \dfrac{a^2 \cdot \sqrt{r}}{G}$

which is direct mass $M_{DIRECT}(<r) = \dfrac{a^2 \cdot \sqrt{r}}{G}$ Being $\dfrac{a^2}{G} = 3.349 \cdot 10^{31}$.

Such formula is only right for radius belonging to halo. Therefore it is only possible to calculate the DM inside a

spherical corona defined by two radius R$_1$ and R$_2$ so $R_1 < M_{DM} < R_2 = \dfrac{a^2}{G} \cdot \left[\sqrt{R_2} - \sqrt{R_1}\right]$

10. MASSES IN M31

In this chapter, It will be calculated some different types of masses related to M31.

10.1 DYNAMICAL MASSES, DIRECT MASS AND DYNAMICAL FITTED MASSES

10.1.1 DYNAMICAL MASSES VERSUS DIRECT MASS

As it is known, dynamical mass represents the mass enclosed by a sphere with a radius r in order to produce a balanced rotation with a specific velocity at such radius.

The formula of dynamical mass is $M_{DYN}(<r) = \dfrac{V^2 \cdot r}{G}$. and $M_{DIRECT}(<r) = \dfrac{a^2 \cdot \sqrt{r}}{G}$ being a^2/ G = 3,35.10^{31}

kpc	m	m/s	Dyn Mass Msun	Direct mass Msun	Rel diff %
40,5	1,250E+21	2,299E+05	4,974E+11	5,95E+11	1,639E+01
49,1	1,515E+21	2,374E+05	6,429E+11	6,55E+11	1,849E+00
58,4	1,802E+21	2,505E+05	8,514E+11	7,14E+11	-1,919E+01
70,1	2,163E+21	2,192E+05	7,825E+11	7,83E+11	1,419E-02
84,2	2,598E+21	2,069E+05	8,373E+11	8,58E+11	2,373E+00
101,1	3,120E+21	2,135E+05	1,071E+12	9,40E+11	-1,392E+01
121,4	3,746E+21	1,978E+05	1,103E+12	1,03E+12	-7,143E+00
145,7	4,496E+21	1,788E+05	1,082E+12	1,13E+12	4,087E+00
175	5,400E+21	1,656E+05	1,115E+12	1,24E+12	9,833E+00
210,1	6,483E+21	1,656E+05	1,339E+12	1,35E+12	1,204E+00
252,3	7,785E+21	1,607E+05	1,514E+12	1,48E+12	-1,951E+00
302,9	9,347E+21	1,508E+05	1,600E+12	1,63E+12	1,629E+00

In the fifth column is tabulated the direct masses in order to be compared with dynamical masses.

Below in the graph are plotted both functions, blue points are dynamical masses and brown point are direct masses.

 The first and third points have the maximum difference regarding fitted curve whereas relative differences decreased as radius increased.

Mdyn vs Mdirect

$y = 3{,}34855E{+}31x^{5{,}00000E{-}01}$
$R^2 = 1{,}00000E{+}00$

$y = 2{,}81100E{+}31x^{5{,}03527E{-}01}$
$R^2 = 9{,}24526E{-}01$
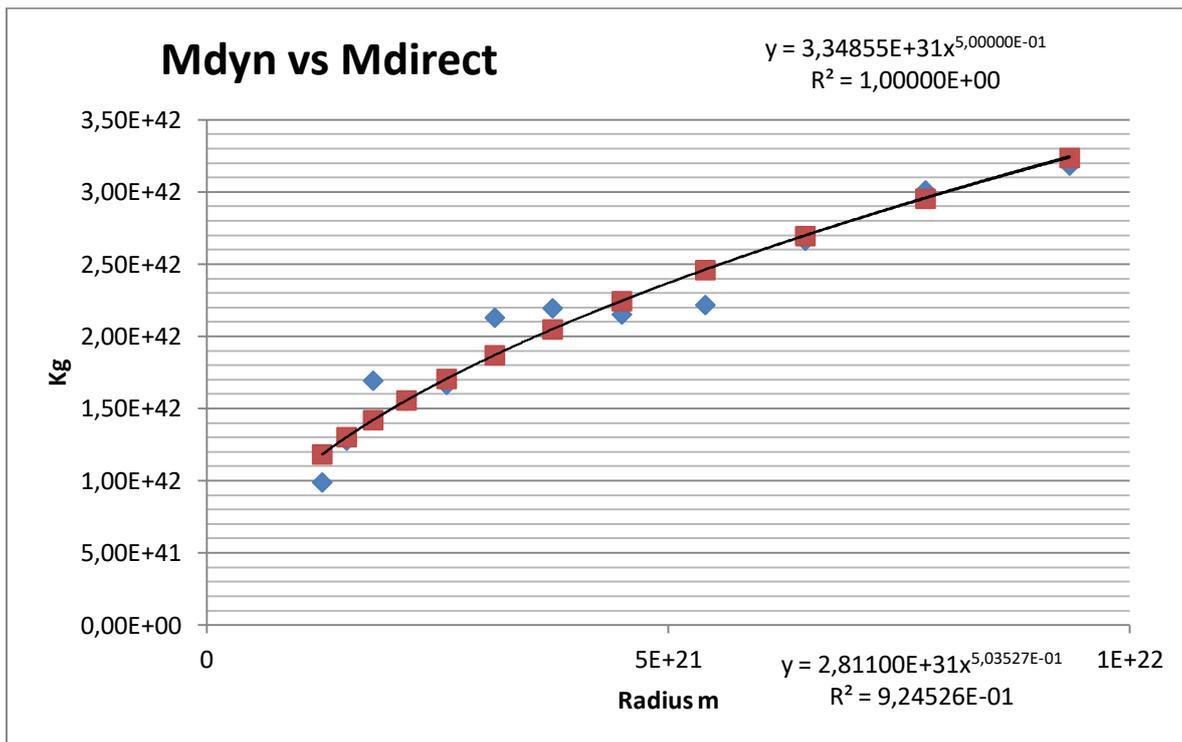
10.1.2 DIRECT MASS VERSUS DYNAMICAL FITTED MASS

Below is plotted dynamical mass. In the table above this paragraph, data are in the grey column. Below in the graph is written the dynamical fitted mass formula as well.

Dyn Mass Vs Dyn Fitted Mass

$y = 2{,}81100E{+}31x^{5{,}03527E{-}01}$
$R^2 = 9{,}24526E{-}01$
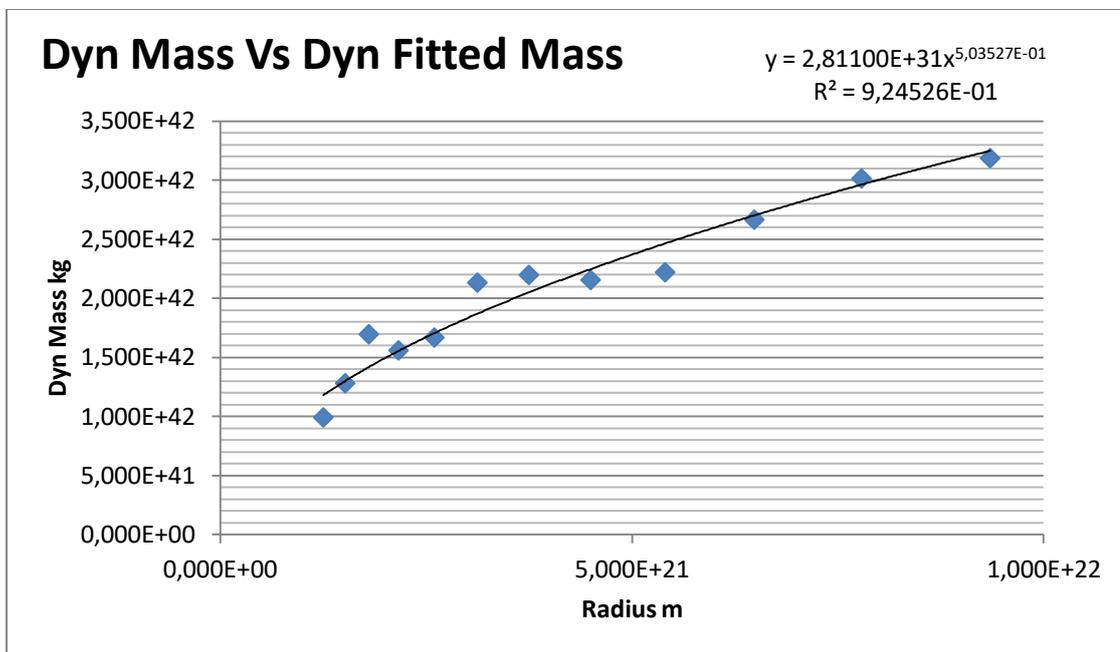
	Direct mass	DynFitted	Relative Diff
kpc	Msun	Mass	%
40,5	5,95E+11	5,928E+11	-3,681E-01
49,1	6,55E+11	6,530E+11	-3,000E-01

58,4	7,14E+11	7,126E+11	-2,387E-01
70,1	7,83E+11	7,812E+11	-1,742E-01
84,2	8,58E+11	8,568E+11	-1,094E-01
101,1	9,40E+11	9,395E+11	-4,481E-02
121,4	1,03E+12	1,030E+12	1,969E-02
145,7	1,13E+12	1,129E+12	8,403E-02
175	1,24E+12	1,238E+12	1,486E-01
210,1	1,35E+12	1,358E+12	2,129E-01
252,3	1,48E+12	1,489E+12	2,773E-01
302,9	1,63E+12	1,632E+12	3,416E-01

$$M_{DYN}^{FIT}(<r) = \frac{v^2 \cdot R}{G} = \frac{\alpha^2 \cdot r^\beta}{G} \quad \text{being} \quad \frac{\alpha^2 \cdot}{G} = 2,811*10^{31}$$

and $\beta = 0,503527$ This function of mass is lightly

different regarding $M_{DIRECT}(<r) = \dfrac{a^2 \cdot \sqrt{r}}{G}$ Being

$\dfrac{a^2}{G} = 3.349 \cdot 10^{31}$. Because parameter **a** was recalculated

using exponent b = - 1/4. See epigraph 9.1.

Relative differences are below 0,4 % throughout dominion, so hereafter it will be used Direct mass instead Fitted dynamical mass function in order to approximate dynamical fitted mass function. For example the dynamical mass at 40,5 kpc is $M_{DYN}(<40,5kpc) = 5,95*10^{11} Msun$ which may be considerate as the total mass of M31 at radius 40,5 kpc, despite the fact that $M_{MEASURED}(<40,5kpc) = 4,97*10^{11} Msun$ and its relative difference is 16,4%. As it is well known, the first and third point of rotation curve have the biggest relative differences regarding the fitted function as it is seen at graphic.

Obviously, dominion of dynamic fitted mass is 40,5 kpc up to 303 kpc, as well as for direct mass, because parameter **a** was got with a statistical procedure at such dominion. However in following chapter will be shown that direct mass is a very good approximation for Bernoulli mass at halo region, even for cluster scale (3 Mpc).

10.2 BERNOULLI MASS VERSUS DIRECT MASS

Below are both function formulas.

$$M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$$ being a²/ G = 3,35.10³¹ . As was pointed in previous paragraph, will be used to calculate

dynamical mass $M_{DYN}(<40,5kpc) = 5,95*10^{11} Msun$ may be considerate the total mass of M31 at 40,5 kpc because direct mass is equivalent to dynamical fitted mass.

Hereafter $M_{DYN}(<40,5kpc) = 5,95*10^{11} Msun$ will be renamed $M_{TOTAL}(<40,5kpc)$

According the theory, carefully developed in previous chapters, Bernoulli mass formula is the primitive to calculate the dark matter in the halo region by the Barrow`s rule.

$$M(<r) = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$$

M31 PARAMETERS C & D
$C_{M31} = -3,777*10^{-24}$
$D_{M31} = a^{\frac{-4}{3}} = 5,85*10^{-15}$

Despite the fact that Bernoulli mass function works only at halo region, at 40,5 kpc differs only 1,04 % regarding direct mass at 40,5 kpc.

Below are tabulated both function and its relative difference. It is remarkable that even at 3 Mpc its difference is only 4,35 %, despite the fact that its dominion has been extended 30 times¡

kpc	m	Direct mass Msun	Bernoulli mass Msun	Rel diff %
40,5	1,250E+21	5,949E+11	6,011E+11	1,04E+00
60	1,851E+21	7,240E+11	7,327E+11	1,19E+00
80	2,469E+21	8,361E+11	8,471E+11	1,31E+00
100	3,086E+21	9,347E+11	9,481E+11	1,41E+00
200	6,171E+21	1,322E+12	1,346E+12	1,77E+00
385	1,188E+22	1,834E+12	1,875E+12	2,20E+00
500	1,543E+22	2,090E+12	2,142E+12	2,40E+00
770	2,376E+22	2,594E+12	2,668E+12	2,77E+00
1000	3,086E+22	2,956E+12	3,048E+12	3,02E+00
1500	4,629E+22	3,620E+12	3,750E+12	3,46E+00
2000	6,171E+22	4,180E+12	4,346E+12	3,80E+00
3000	9,257E+22	5,120E+12	5,353E+12	4,35E+00

10.3 DM MASS OF SPHERICAL CORONA IN HALO BY BERNOULLI FORMULA AND TOTAL MASS

According theory developed, using Barrow`s rule to Bernoulli formula it will be calculated DM inside a spherical corona with Radius R1 and R2 belonging at halo region i.e. DM (R1,R2) = Bernoulli (R2)- Bernoulli(R1)

Being $M_{BER}(<r) = \dfrac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ the Bernoulli formula for M31 whose parameters are:

M31 PARAMETERS C & D
$C_{M31} = -3,777*10^{-24}$
$D_{M31} = a^{-\frac{4}{3}} = 5,85*10^{-15}$

Therefore, if it is considerate R1 = 40,5 kpc then DM-Spherical Corona (R1,R) = $M_{BER}(<R)$ (Cyan data)- 6,011E+11 (grey data) and total mass at R radius will be got adding M_{TOTAL} (<40,5kpc) = 5,95E+11 (pink data) to the DM inside spherical corona defined by 40,5 kpc and R (green data). See table below. Obviously the differences between total mass (yellow column) and Bernoulli mass (cyan column) are negligible because the difference between grey and pink values is 1% only.

Radius kpc	Radius m	Bernoulli Mass Msun	DM-SphCorona M(R)-M<40,5	Total Mass M(<R) Msun	Relative diff % Bernoulli Mass Versus Total Mass
40,5	1,250E+21	6,011E+11	0,000E+00	5,95E+11	1 %

60	1,851E+21	7,327E+11	1,316E+11	7,266E+11	8,4E-1
80	2,469E+21	8,471E+11	2,460E+11	8,410E+11	7,2E-1
100	3,086E+21	9,481E+11	3,470E+11	9,420E+11	6,4E-1
200	6,171E+21	1,346E+12	7,446E+11	1,340E+12	4,5E-1
385	1,188E+22	1,875E+12	1,274E+12	1,869E+12	3,3E-1
500	1,543E+22	2,142E+12	1,540E+12	2,135E+12	2,9E-1
770	2,376E+22	2,668E+12	2,067E+12	2,662E+12	2,3E-1
1000	3,086E+22	3,048E+12	2,447E+12	3,042E+12	2E-1
1500	4,629E+22	3,750E+12	3,149E+12	3,744E+12	1,6E-1
2000	6,171E+22	4,346E+12	3,744E+12	4,339E+12	1,4E-1
3000	9,257E+22	5,353E+12	4,751E+12	5,346E+12	1,1E-1

In conclusion, Bernoulli mass formula may be considerate as a very good estimation for total mass at a specific radius R belonging to halo region i.e. for R > 40,5 kpc as according the theory the halo region is unlimited.

Hereafter Bernoulli mass function will be considerate as synonymous of total mass at a specific radius.

11. DARK GRAVITATION THEORY IN MILKY WAY

In [17] Abarca M.2019. was introduced the extended halo of M31 whose radius was defined as the distance between M31 and Milky Way. Similarly the extended halo for Milky Way is the same distance. The reason to consider this extension is backed by the concept of DM nature according with theory of gravitational field as generator of DM.

This concept will be discussed widely at the end of the chapter.

11. 1 ROTATION CURVE OF MILKY WAY BY SOFUE 2020 DATA
This table of rotation curve of Milky Way comes from [2] Sofue.

kpc	km/s	radius m	vel m/s
30,448	229,6	9,40E+20	229600
33,493	222,5	1,03E+21	222500
36,842	215	1,14E+21	215000
40,527	207,1	1,25E+21	207100
44,579	200,3	1,38E+21	200300
49,037	194,7	1,51E+21	194700
53,941	189,8	1,66E+21	189800
59,335	186,2	1,83E+21	186200
65,268	184,7	2,01E+21	184700
71,795	183,9	2,22E+21	183900
78,975	181,4	2,44E+21	181400
86,872	175,5	2,68E+21	175500
95,56	167,7	2,95E+21	167700

This new set of Sofue data is very important for the theory of Dark gravitation theory because gives a rotation curve at halo region with a power for radius very close to -1/4 which is the same for M31. This fact back strongly the hypothesis of this theory.

**11.2 FITTED FUNCTION VELOCITY VERSUS RADIUS AND PARAMETER a AT HALO REGION**

As it is known, Bernoulli solution is only right into the halo region, where the baryonic mass is negligible.
So it is needed to consider a radius where the ratio baryonic matter versus DM density is negligible. To calculate baryonic volume density has been used model provided by Sofue [6] for baryonic disc.

 See pg 12. Table 3 below. Parameter for baryonic matter at disc in Milky Way.

Table 3. Best-fit parameters of the direct SMD by NFW halo and exponential disk.

Component	Parameter	Fitted Value	χ^2
Expo. disk	a_d	4.38 ± 0.35 kpc	
	Σ_0	$(1.28 \pm 0.09) \times 10^3 M_\odot pc^{-2}$	

With this parameters is possible to calculate baryonic volume density at 30,5 kpc $D_{BARYONIC}$(30,5Kpc) = 1,5E-24 kg/m^3. With the Direct DM density D_{DM}(30,5 Kpc) = 3,3E-23 kg/m^3. So ratio Baryonic versus DM at 30,5 kpc is 0,045. Therefore at 30,5 kpc is possible to considerate negligible Baryonic matter.

Velocity vs Radius $y = 3{,}689182E{+}10 x^{-2{,}487176E{-}01}$
$R^2 = 9{,}632100E{-}01$
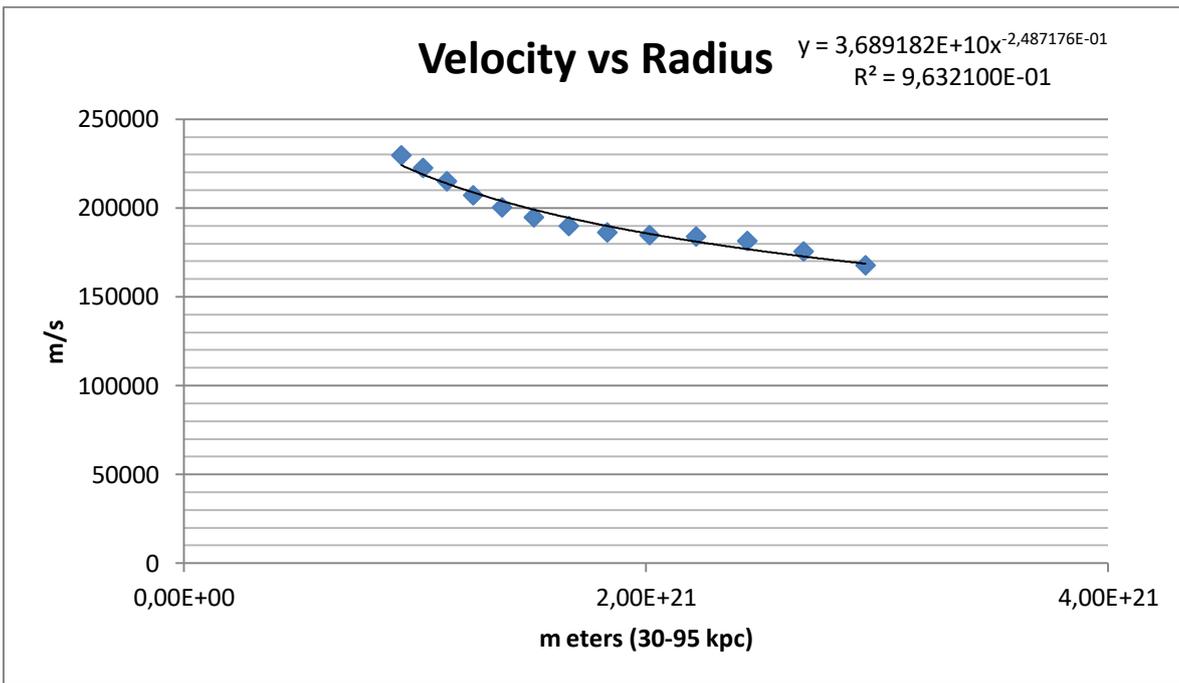
According the statistical procedure   v=a*R^b Being a = 3.68918E+10 and   b = - 0.248717

Using Buckingham theorem it has been stated b= - 1/4 donc it is needed recalculated parameter **a** through the formula as it was made with M31 rotation curve.

$$a = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}}$$

In table below the fourth column is calculated the numerator values. In the fifth column are denominator values. At the bottom is found parameter **a** optimal **a**=3,90787373E+10

Which is lightly bigger compared with which one associated to   b= - 0.248717

kpc	m	m/s	Parameter a optimal when b=-1/4	
30,448	9,40E+20	229600	1,31E+00	3,26245E-11
33,493	1,03E+21	222500	1,24E+00	3,11061E-11
36,842	1,14E+21	215000	1,17E+00	2,96587E-11
40,527	1,25E+21	207100	1,10E+00	2,82781E-11
44,579	1,38E+21	200300	1,04E+00	2,69624E-11
49,037	1,51E+21	194700	9,87E-01	2,57076E-11
53,941	1,66E+21	189800	9,40E-01	2,45111E-11
59,335	1,83E+21	186200	9,00E-01	2,33705E-11
65,268	2,01E+21	184700	8,72E-01	2,2283E-11
71,795	2,22E+21	183900	8,48E-01	2,12459E-11
78,975	2,44E+21	181400	8,16E-01	2,02571E-11
86,872	2,68E+21	175500	7,71E-01	1,93145E-11
95,56	2,95E+21	167700	7,20E-01	1,84156E-11
		a optimal	3,90787373E+10	with b= -1/4
	D= a^(-4/3)	D parameter	7,54072E-15	

According the theory of dark matter by gravitation, each galaxy has two parameters C and D.

Parameter D is similar for similar galaxies, for example D= 5.85E-15 for M31.

Dark matter by gravitation theory stated that B has to be the same for all galaxies. However parameter **a** depend on each galaxy because it depend on baryonic matter enclosed by the galaxy.

New parameters a&b - A&B for Milky Way	
B	5/3
$b = \dfrac{B-2}{2B-2}$	b = -1/4
a optimal	$3,90787373 * 10^{10}$
$A = \dfrac{a^{\frac{-4}{3}}}{8\pi G}$	New parameter A $4,496262 * 10^{-6}$
$D = 8\pi G A = a^{\frac{-4}{3}}$	$7,54 * 10^{-15}$

11.3 PARAMETER C FOR MILKY WAY

C is the initial condition associated to a differential equation for the gravitational field in the galaxy whose formula depend on D and Eo.

$$C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$$

D is a global parameter because it has been calculated statistically, but Eo is the field at a

specific radius Ro, so Eo is very dependent of error measure associated at such radius. In table below, it will be calculated parameter C for each data point because each one may be considerate as a initial condition.

This is the reason why in the following epigraph will be calculated Bernoulli mass for every parameter C and afterwards will be calculated the average Bernoulli mass function in order to calculate its parameter C associated.

Point	kpc	m	m/s	Eo v^2/R	Parameter C
1	30,448	9,40E+20	229600	5,61E-11	-2,8439E-23
2	33,493	1,03E+21	222500	4,79E-11	-2,0251E-23
3	36,842	1,14E+21	215000	4,07E-11	-9,7430E-24
4	40,527	1,25E+21	207100	3,43E-11	3,2002E-24
5	44,579	1,38E+21	200300	2,92E-11	1,1840E-23
6	49,037	1,51E+21	194700	2,51E-11	1,5517E-23
7	53,941	1,66E+21	189800	2,16E-11	1,6475E-23
8	59,335	1,83E+21	186200	1,89E-11	1,2028E-23
9	65,268	2,01E+21	184700	1,69E-11	-9,8889E-25
10	71,795	2,22E+21	183900	1,53E-11	-1,5770E-23
11	78,975	2,44E+21	181400	1,35E-11	-2,2598E-23
12	86,872	2,68E+21	175500	1,15E-11	-1,5434E-23
13	95,56	2,95E+21	167700	9,54E-12	-2,1145E-36

11.4 FITTING PARAMETER C BY BERNOULLI MASS FORMULA

Below are tabulated the different Bernoulli masses for each parameter C calculated in previous epigraph. The radius dominion extend up to 3 Mpc because according Dark matter by gravitation theory, this phenomenon is associated to gravitational field, so its dominion is unlimited.

$$M_{BER}(r) = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$$

There are13 parameters C. The column in cyan colour is the average mass function. The column in yellow shows the Bernoulli mass with a fitted parameter C in order to minimize the relative difference between the average mass function and Bernoulli mass function with a parameter C fitted. C= -5E-24 . The last column shows that relative differences are below 0,1%

Radius kpc	C at 30,4 kpc	C at 33,5 kpc	C at 36,8 kpc	C at 40,5kpc	C at 44,5 kpc	C at 49 kpc	C at 54 kpc	C at 59kpc
30,448	3,729E+11	3,668E+11	3,59E+11	3,502E+11	3,445E+11	3,420E+11	3,414E+11	3,443E+11
33,493	3,918E+11	3,852E+11	3,77E+11	3,673E+11	3,610E+11	3,584E+11	3,577E+11	3,609E+11
36,842	4,117E+11	4,046E+11	3,96E+11	3,851E+11	3,783E+11	3,755E+11	3,748E+11	3,782E+11
40,527	4,327E+11	4,249E+11	4,15E+11	4,038E+11	3,965E+11	3,934E+11	3,927E+11	3,963E+11
44,579	4,548E+11	4,463E+11	4,36E+11	4,234E+11	4,155E+11	4,122E+11	4,114E+11	4,153E+11
49,037	4,780E+11	4,688E+11	4,57E+11	4,440E+11	4,354E+11	4,319E+11	4,309E+11	4,353E+11
53,941	5,024E+11	4,924E+11	4,80E+11	4,656E+11	4,563E+11	4,524E+11	4,514E+11	4,561E+11
59,335	5,281E+11	5,173E+11	5,04E+11	4,882E+11	4,781E+11	4,740E+11	4,729E+11	4,779E+11

65,268	5,552E+11	5,434E+11	5,29E+11	5,119E+11	5,010E+11	4,965E+11	4,954E+11	5,008E+11
71,795	5,837E+11	5,709E+11	5,55E+11	5,367E+11	5,250E+11	5,201E+11	5,189E+11	5,247E+11
78,975	6,137E+11	5,998E+11	5,83E+11	5,628E+11	5,501E+11	5,448E+11	5,435E+11	5,498E+11
86,872	6,453E+11	6,302E+11	6,12E+11	5,901E+11	5,763E+11	5,707E+11	5,692E+11	5,761E+11
95,56	6,787E+11	6,622E+11	6,42E+11	6,187E+11	6,039E+11	5,977E+11	5,961E+11	6,035E+11
770	2,105E+12	1,999E+12	1,88E+12	1,740E+12	1,659E+12	1,626E+12	1,618E+12	1,657E+12
1000	2,440E+12	2,305E+12	2,15E+12	1,980E+12	1,879E+12	1,839E+12	1,829E+12	1,877E+12
2000	3,639E+12	3,380E+12	3,09E+12	2,786E+12	2,610E+12	2,540E+12	2,523E+12	2,606E+12
3000	4,631E+12	4,249E+12	3,83E+12	3,400E+12	3,156E+12	3,061E+12	3,037E+12	3,151E+12

C at 65kpc	C at 72kpc	C at 79 kpc	C at 87 kpc	C at 95,5 kpc	Average mass	Fitted C C= -5E-24	Relative Diff %
3,531E+11	3,635E+11	3,685E+11	3,633E+11	3,524E+11	3,556E+11	3,55896E+11	0,0920024
3,704E+11	3,817E+11	3,871E+11	3,814E+11	3,696E+11	3,730E+11	3,73386E+11	0,09259705
3,885E+11	4,007E+11	4,066E+11	4,004E+11	3,877E+11	3,914E+11	3,91737E+11	0,09305282
4,075E+11	4,207E+11	4,271E+11	4,204E+11	4,066E+11	4,106E+11	4,11E+11	0,09335478
4,274E+11	4,418E+11	4,487E+11	4,414E+11	4,264E+11	4,308E+11	4,31207E+11	0,09348646
4,483E+11	4,639E+11	4,714E+11	4,635E+11	4,473E+11	4,520E+11	4,52417E+11	0,0934304
4,702E+11	4,871E+11	4,952E+11	4,867E+11	4,691E+11	4,742E+11	4,74677E+11	0,09316754
4,932E+11	5,115E+11	5,203E+11	5,111E+11	4,920E+11	4,976E+11	4,98036E+11	0,09267747
5,173E+11	5,371E+11	5,467E+11	5,367E+11	5,160E+11	5,221E+11	5,22549E+11	0,09193812
5,426E+11	5,641E+11	5,745E+11	5,636E+11	5,412E+11	5,478E+11	5,48279E+11	0,09092555
5,691E+11	5,924E+11	6,038E+11	5,919E+11	5,676E+11	5,748E+11	5,75285E+11	0,08961411
5,969E+11	6,222E+11	6,345E+11	6,216E+11	5,953E+11	6,031E+11	6,03625E+11	0,08797638
6,261E+11	6,535E+11	6,669E+11	6,529E+11	6,244E+11	6,328E+11	6,33375E+11	0,08598213
1,782E+12	1,945E+12	2,029E+12	1,941E+12	1,772E+12	1,827E+12	1,82424E+12	-0,1464653
2,032E+12	2,236E+12	2,342E+12	2,231E+12	2,020E+12	2,089E+12	2,08444E+12	-0,22487616
2,879E+12	3,251E+12	3,451E+12	3,242E+12	2,856E+12	2,989E+12	2,97243E+12	-0,54427538
3,530E+12	4,061E+12	4,353E+12	4,047E+12	3,498E+12	3,693E+12	3,66183E+12	-0,83889186

In conclusion, parameter C = -5E-24 is the value that fit better the average mass of the other 13 Bernoulli mass functions.

So the parameters for Milky Way are $D = 7,54*10^{-15}$ and $C= -5*10^{-24}$

| Parameter for Milky Way |
|---|
| $C= -5*10^{-24}$ |
| $D = 7,54*10^{-15}$ |

11.5 BERNOULLI MASS FORMULA VERSUS DIRECT MASS

Below are tabulated and compared both types of formulas for masses. As it was expected Bernoulli mass is a bit bigger than the other one, but even at 3 Mpc the relative difference is about 4%.

Radius kpc	Radius m	Bern Mass	Direct Mas	Rel diff %

3,045E+01	9,40E+20	3,55896E+11	3,52E+11	0,9725524
3,349E+01	1,03E+21	3,73386E+11	3,70E+11	1,00389568
3,684E+01	1,14E+21	3,91737E+11	3,88E+11	1,03624245
4,053E+01	1,25E+21	4,11E+11	4,07E+11	1,06963952
4,458E+01	1,38E+21	4,31207E+11	4,26E+11	1,10409792
4,904E+01	1,51E+21	4,52417E+11	4,47E+11	1,13967098
5,394E+01	1,66E+21	4,74677E+11	4,69E+11	1,17638927
5,934E+01	1,83E+21	4,98036E+11	4,92E+11	1,2142852
6,527E+01	2,01E+21	5,22549E+11	5,16E+11	1,25339682
7,180E+01	2,22E+21	5,48279E+11	5,41E+11	1,29376995
7,898E+01	2,44E+21	5,75285E+11	5,68E+11	1,33544217
8,687E+01	2,68E+21	6,03625E+11	5,95E+11	1,37844797
9,556E+01	2,95E+21	6,33375E+11	6,24E+11	1,42284204
7,700E+02	2,38E+22	1,82424E+12	1,77E+12	2,84565461
1,000E+03	3,09E+22	2,08444E+12	2,02E+12	3,10333024
2,000E+03	6,17E+22	2,97243E+12	2,86E+12	3,90459723
3,000E+03	9,26E+22	3,66183E+12	3,50E+12	4,46534223

Blue points belong to Bernoulli mass formula.
Brown points to Direct mass formula.

Bernoulli Mass Vs Direct Mass

$y = 6{,}27475E+10x^{5{,}07436E-01}$
$R^2 = 9{,}99995E-01$

$y = 6{,}38703E+10x^{5{,}00000E-01}$
$R^2 = 1{,}00000E+00$

Msun — Radius Kpc
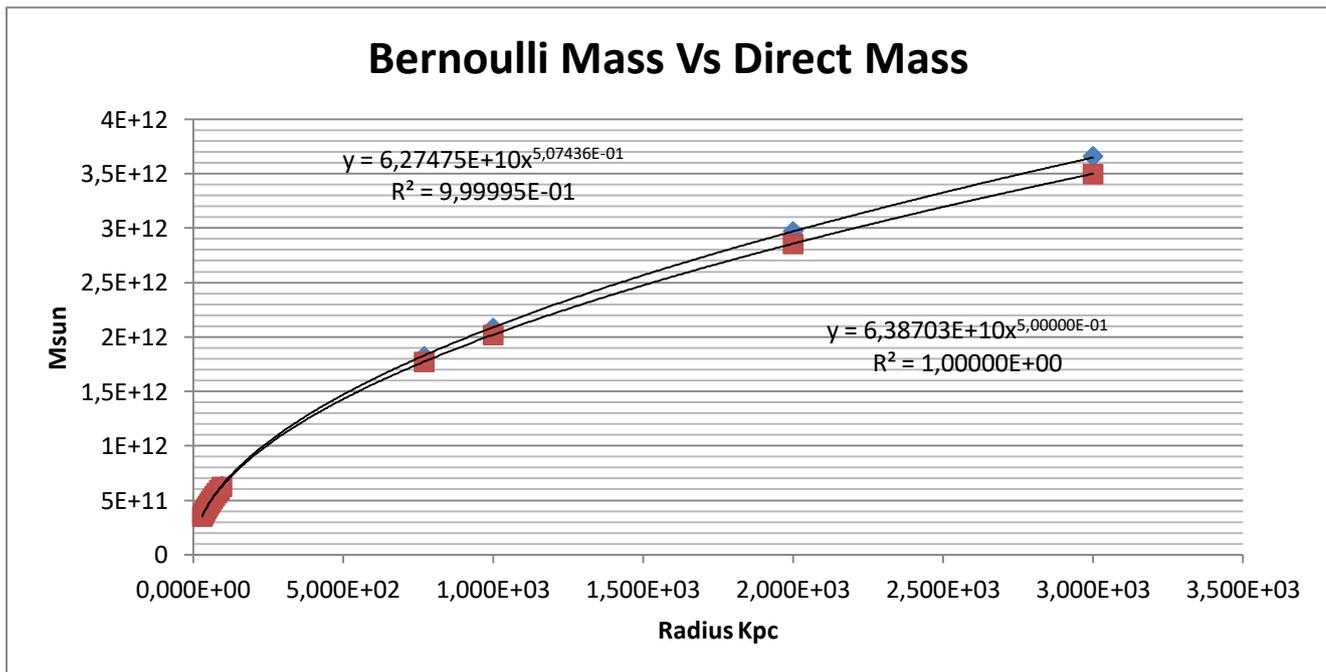
As direct mass is by far more simple than Bernoulli mass hereafter it will be used to estimate dynamical mass for M33 in the following chapter.

12. DIRECT MASS FORMULA FOR M33 GALAXY

Table and graphic has been taken from [4] Corbelli,E.2014 data.

As it is clear from the graphic, the rotation curve from 10 kpc to 23 kpc is quite horizontal.

radius kpc	Vel km/s	Mdyn Msun
8	116,3	2,514E+10
9	118,7	2,947E+10
10	118,7	3,274E+10
11	118,7	3,601E+10
12	119,3	3,969E+10
13	118,7	4,256E+10
14	119,3	4,630E+10
15	119,88	5,009E+10
16	119,88	5,343E+10
17	119,88	5,677E+10
18	119,88	6,011E+10
19	119,88	6,345E+10
20	119,88	6,679E+10
21	119,88	7,013E+10
22	119,3	7,276E+10
23	119	7,568E+10
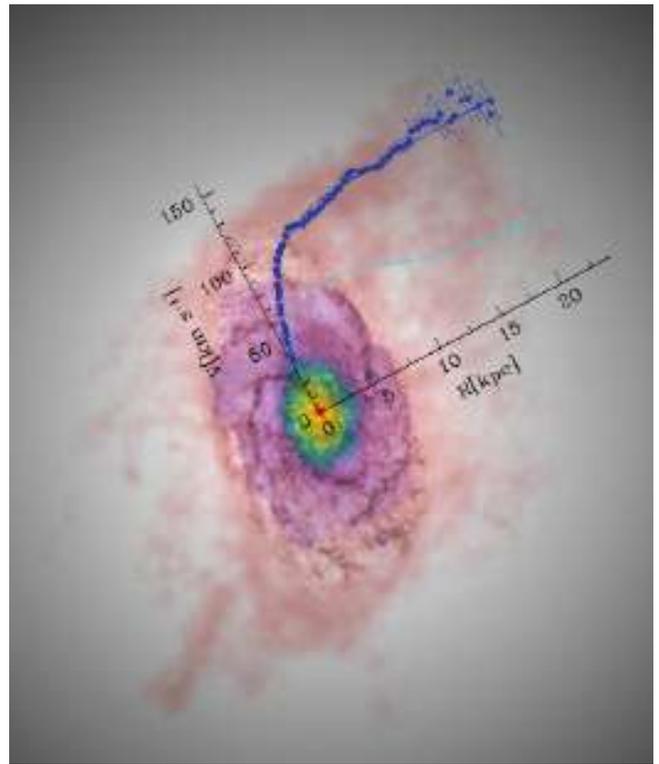
In the above graph, the inner stellar surface density ranges from red to magenta colours, the pink cloud represents the HI gas distribution.

According [7] Carignan.2017 Mass dynamic < 23 kpc is 8E+10 Msun, which is a bit large than value of table above.

As $M_{DIRECT}(<r) = \dfrac{a^2 \cdot \sqrt{r}}{G}$ depend on parameter **a** it is right to get such parameter using only a value for the mass

at a specific radius. So taking the Carignan data it is got $a^2 = 3{,}987E20$ which is rounded to $a^2 = 4E20$ (I.S. units).

Radius kpc	radius m	Mass Msun
23,00	7,10E+20	8,025E+10
50,00	1,54E+21	1,183E+11
200,00	6,17E+21	2,366E+11
500,00	1,54E+22	3,742E+11
1000,00	3,09E+22	5,291E+11
1500,00	4,63E+22	6,480E+11
2000,00	6,17E+22	7,483E+11

According the Dark gravitation theory parameter **a** have to be calculated inside the region where the baryonic matter is negligible. Obviously, with these data it is not possible to accept fully such hypothesis, but it is clear that 23 kpc is a distance quite large for a galaxy whose baryonic matter is below a tenth of M31, so to calculate parameter **a** with data from Carignan is an approximation acceptable. Below are calculated some masses at different radii, using Direct mass.

$D_{DM}(r) = L \cdot r^{\frac{-5}{2}}$ being $L = \dfrac{a^2}{8 \cdot \pi \cdot G} = 2{,}385E+29$. So it is right to

calculate $D_{DM}(23\ kpc) = 1{,}78E\text{-}23$ kg/m³. [4] Corbelli,E.2014 does not give information for density of HI cloud, but it is plausible to consider it negligible such baryonic matter so far away, 23 kpc, for a little galaxy as M33 it is.

13 THE MASS CALCULUS FOR THE LOCAL GROUP OF GALAXIES

According [5] Sofue the relative velocity between M31 and Milky Way is 170 km/s. Assuming that both galaxies are linked gravitationally, it is possible to calculate the total mass of the Local group by a simple formula because of the Virial theorem.

$$M = \frac{v^2 \cdot r}{G}$$ As r = 770 kpc and v= 170 km/s then $M_{LOCAL\ GROUP} = 5.17 *10^{12}$ Msun

According [5] Sofue, the total mass of M31 and Milky Way is approximately $3*10^{12}$ Msun, so there is a mass lack of $2*10^{12}$ Msun which is a considerable amount of matter. Namely read epigraph 4.6 of [5] Sofue paper.

Up to now, in order to do calculus with data of rotation curve, the border of M31 is right to be placed at a half the distance to Milky Way because it is supposed that up to such distance its gravitational field dominates whereas for bigger distances is Milky Way field which dominates.

This hypothesis is right when it is considered rotation curves of different systems bounded to each galaxy i.e. stars or dwarf galaxies. However when it is considered the gravitational interaction between both giant galaxies it is needed to extend their haloes up to 770 kpc, because according Dark gravitation theory the phenomenon of Dark matter is linked to gravitational field, which is unlimited.

Therefore the M31 halo is extend up to 770 kpc and reciprocally the Milky Way halo is extend up to 770 kpc, when it is calculated the gravitational interaction between both galaxies.

13.1 AN ESTIMATION OF TOTAL MASS OF THE LOCAL GROUP AT DIFFERENT RADII

In the previous version of this paper was considered M31 and MW only, now it will be considered M33 too. The rest of galaxies have a mass negligible in order to estimate the total mass of Local Group. In fact M33 add a 5% approximately only. In this version it will be used Direct mass instead Bernoulli mass because it is more simple and both give results quite similar.

Radius	Direct mass M31	Direct mass MW	Direct Mass M33	Direct Mass Total
kpc	msun	Msun	Msun	Msun
40,00	5,913E+11	4,040E+11	1,058E+11	1,1011E+12
100,00	9,349E+11	6,388E+11	1,673E+11	1,7410E+12
300,00	1,619E+12	1,106E+12	2,898E+11	3,0155E+12
770,00	2,594E+12	1,773E+12	4,643E+11	4,8310E+12
1000,00	2,956E+12	2,020E+12	5,291E+11	5,5055E+12
1500,00	3,621E+12	2,474E+12	6,480E+11	6,7428E+12
2000,00	4,181E+12	2,857E+12	7,483E+11	7,7859E+12
2500,00	4,674E+12	3,194E+12	8,366E+11	8,7049E+12
3000,00	5,121E+12	3,499E+12	9,165E+11	9,5358E+12

The values of parameters a^2 have been got in previous chapters.

Galaxies	Parameter a^2
M31	2,235E+21
MW	1,527E21
M33	4E20
Parameter a^2 Local Group	4,162E21

By the formula of Direct mass it is right to get the table of masses using parameters a^2 associated to galaxies.

The fourth column is got by the parameter equivalent or adding the three column masses.

So a^2 for Local Group is 4,16E21 which is got adding the a^2 parameter associated to the galaxies.

In the previous version of this paper mass MW+M31 at 770 kpc was 4,5E12 Msun, now adding direct mass M33 the mass reach 4,8E12 Msun, which is almost the dynamical mass commonly associated to Local Group 5E12 Msun.

Obviously this is a magnificent success of *Dark Matter by Gravitation* theory.

If it is added the masses, in the table above, at 770 kpc associated to MW and M31, it is got 4,37E12 which is 3% lower the mass got with Bernoulli formula, used in version 1 of this paper. So it worth use Direct mass because of its simplicity and rightness of calculus.

When it is considered the universal expansion with a Hubble constant $H_o = 70$ km/s/Mpc, the local expansion between M31 and MW should be 54 Km/s, therefore to justify an approach velocity equal to 170 km/s it is needed even more mass than 5E+12 Msun. However dark gravitation theory stated that the more distance between galaxies the more mass have both galaxies. For example the total mass if both galaxies would have at 1,5 Mpc distance, would be 6,7E+12Msun.

At 100 Km/s the time to traverse 730 kpc, from 1,5 Mpc to current distance 770 kpc, would be 7,1 Gy which is a half of the Universe age according current cosmology.

This calculus are only an estimation to shows that it is needed much more mass that 5E+12 Msun to justify the current approach velocity of M31 and MW. However Dark gravitation theory may support such extra amount of mass.

To study evolution of cluster dynamics it will be needed a model to consider the universal expansion form the ancient Universe to current universe taking in consideration the total DM associated to the gravitational field.

In chapter 11 of paper [2] Abarca,M.2019. it was made an estimation about the DM in the ancient Universe 9300 million years ago, when the Universe was decreased by 0,5 factor. As in the theory of DM by gravitation, the total DM is related to the distances, it was estimated that DM associated to galaxies was reduced a 30% regarding the current value.It is sure that such studies will be a magnificent test to check the *Theory of dark matter by gravitation.*

14. DARK MATTER IS COUNTER BALANCED BY DARK ENERGY
The basic concepts about DE in the current cosmology can be studied in [9] Chernin,A.D.

In the current model ΛCDM dark energy has an effect equivalent to antigravity i.e. the mass of dark energy is negative and the dark energy have a constant density for all the Universe equal to

$$\varphi_{DE} = \varphi_C \bullet \Omega_{DE} = -6.26 \cdot 10^{-27} \, kg/m^3 \quad \text{being } \Omega_{DE} = 0.68 \text{ and } \rho_C = \frac{3H^2}{8\pi G} \text{ the critic density of the Universe },$$

where H is the Hubble constant H=70km/s/Mpc.

As DE density is constant, the total DE is proportional to Radius with power 3. Therefore if it is considered a volume of space, whose radius is measured in Mega parsecs then dark energy is not negligible. According [9] Chernin,A.D.

$$M_{DE} = -\varphi_{DE} \frac{8\pi R^3}{3} \text{ a good approximation for total mass of DE is given by the formula}$$

$$M_{DE} = -5,24 \bullet 10^{-26} \bullet R^3 kg \quad \text{and as Direct mass } M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G} \text{ being } a_{L-G}^2 = 4,162 \bullet 10^{21} \, I.S \, units \text{then}$$

by equation it is possible to get the radius when the total mass DM+DE=0,which is the volume where the Local Group has a total mass zero, in other words such radius is a good estimation for Local Group size.

It is a simple calculus which gives a radius approximate to 2,2 Mpc, so for radius under 2,2 Mpc dark matter dominates and for bigger radius dark energy dominates and it is not possible to link by gravitation a galaxy more than 2,2 Mpc far away to centre of mass of Local Group, because there is more dark energy than dark matter.

In order to compare with one of the biggest cluster of galaxies, according [13] Joo Heon Yoon, the Coma cluster has a Virial radius R=2,9 Mpc with a Virial Mass 1,4E15 Msun. Using the formula of direct mass

$$M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$$ it is possible to get an approximate value for a^2 = 6,215E23 in I.S. units.

With such value for a^2 it is possible to get an equation between dark energy mass and dark matter by direct mass formula. The radius where dark energy is equal to dark matter is 16,2 Mpc in other words 16,2 Mpc is the radius of region where the DM of Coma Cluster dominates by gravitation.

In fact, it is known that the majority of clusters in the Universe have a size that ranges between 5 Mpc and 10 Mpc.

With these examples it has been illustrated how DM by the formula of direct mass $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ which

grows with the square root of radius, it is counter balanced by dark energy at mega parsecs scale. The exact radius depend on each galaxy or cluster, namely by its parameter a^2.

15. CONCLUSION

As it has been outlined at the introduction, this work is the consequence of the new set of data for rotation curve in Milky Way halo. With these new data, the fitted function of rotation curve at halo region of Milky Way has exactly the same exponent that fitted function associated to M31 galaxy. This fact back strongly the main hypothesis of Dark gravitation theory i.e. Dark matter is generated according an unknown quantum gravitational mechanism, which depend on the gravitational field, so is a universal law.

Through the firsts ten chapters it is developed the theory using M31 rotation curve. This chapters are identical to the previous paper [2] Abarca,M.2019, excepting the getting of parameter C , the initial condition of solution got for Bernoulli differential equation associated to field E. See chapter 9. This parameter associated to a specific point of rotation curve is quite problematic because of experimental errors. In this work the value got has been C = - 3,77E-24 whereas in the previous work was C = 6,36E-24. Both are little numbers, but with the value negative the field is lightly bigger and consequently DM density is a bit bigger. In fact Bernoulli mass formula becomes Direct mass formula when parameter C is equal to zero. Remember that direct mass is possible to be calculated with a point at region halo belonging to rotation curve. $P(R_0, V_0)$, so it is by far more simple than Bernoulli mass, and its values are very close.

The new results published in this work are in chapter 11 because there are calculated parameters C & D associated to Milky Way halo. With them is calculated total mass function in Milky Way halo, Bernoulli mass and Direct mass formulas.

In the chapter 12 is calculated the total mass of M33 at different radii which is approximately a 10 % of masses asscociated to M31 plus MW. In this calculus has been used Direct mass formula, which has only one parameter but gives values very closed to Bernoulli mass formula.

In the 13 chapter is calculated the mass of Local Group by Dark matter by gravitation being $4,8*10^{12}$ Msun at 770 kpc, which is equivalent to dynamical mass $5*10^{12}$ Msun if it is considered the range of errors of experimental measures.

The current theories of dark matter give a total mass for Local group about $3*10^{12}$ Msun which is very different to dynamical mass of Local Group.

In addition it is shown how the theory gives a total mass 5.5E+12Msun at 1Mpc of radius , 6.7E+12Msun at 1.5 Mpc, and 7.8E+12 Msun at 2 Mpc. In brief, that theory works quite well at cluster scale, as it was widely developed in previous paper [3] Abarca,M.2019 where it was studied dark matter in Coma cluster.

In the chapter 14, it is shown how at 2.1 Mpc the total dark energy with an antigravity effect is able to counter balance the dark matter. In fact DE matter grows with power 3 of radius whereas DM grows with 0,5 power of radius.

In my opinion, this theory cannot be developed any more from Newtonian framework. It will be needed to considered General Relativity to introduce into the tensor energy T, the energy associated to dark matter.

A natural way to do it will be to consider the density of dark matter multiplied by c^2 i.e. $D_{DM}^{ENERGY} = \dfrac{a^2 \cdot c^2}{8 \cdot \pi \cdot G} \cdot r^{-5/2}$

16.   BIBLIOGRAPHYC REFERENCES

[1] Abarca,M.2014,viXra:1410.0200.   *Dark matter model by quantum vacuum*

[2] Abarca,M.2019. viXra:1909.0594
   A dark matter formula for Milky Way and M31 halo got by Buckingham theorem.V4

[3] Abarca,M.2019. viXra:1912.0233
   Dark Gravitation Theory for M31 and Coma Cluster

[4] Corbelli,E. et all. 2014.Arxiv
   Dynamical signatures of a Lambda-CDM halo and the distribution of the baryons in M33

[5] Sofue, Y.2015. arXiv:1504.05368v1   *Dark halos of M31 and the Milky Way.*

[6] Sofue,Y.2020.MDPI/Galaxies
   Rotation curve of the Milky Way and the dark matter density.

[7] Carignan et all. .2017.Arxiv
   HI kinematics and mass distribution of Messier 33

[8] Abarca,M.2016. viXra:1606.0279v1
    Dark matter density on big galaxies depend on gravitational field as Universal law.

[9] Chernin,A.D. et all.Arxiv 2013.
     Dark energy and the structure of the Coma cluster of galaxies

[10] Abarca,M.2016.vixra:1601.0242
     A New DM Density Profile for M31 Galaxy to Demonstrate that DM is Generated by the Gravitational Field.

[11] Nieuwenhuizen,T.M. 2010 .arXiv:1011.2530v1
Do micro brown dwarf detections explain the galactic dark matter?

[12] Nieuwenhuizen,T.M. 2012. arXiv:1210.0489v2
Do the Herschel cold clouds in the Galactic halo embody its dark matter?

[13] Joo Heon Yoon & M.E. Putman. arXiv:1704.07839v1
Lyα Absorbers and the Coma cluster