Eigenwertproblem of advanced Lorentz-Einstein-Factor

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Abstract:

The differential equation (DE) of second order for advanced Lorentz-Einstein-factor in fourth order can be written in a Sturm-Liouville form (STL). Therefore it can be formulated as an eigenwertproblem (eigenvalueproblem) for local flat spacetime-states.These eigenvalues are calculated.

Key-words: eigenwerte; Sturm-Liouville equation; differental equation of second order; Einstein-Lorentz-factor; model of damped resonance; eigenfunctions; ftl-velocity; eigenresonance of space-time.

1. Introduction:

In the following paper there will be shown, that a second-oder differential-equation can represent equations of eigenvalues („eigenwerte“) [5.] for advanced Lorentz-term of fourth order whose eigenvalues only exist under special conditions.

For the advanced SRT with ftl there can be written a Lorentz-Einstein -factor of fourth-order in root [1.].

$$\Gamma = \sqrt[4]{\left(1-\frac{v^2}{c^2}\right)^2+\frac{n\cdot v^2\cdot a^2}{c^4}} \qquad (1.)$$

For this term there exists a differential equation of second order which means, this DE is the generating equation for that factor [2.].

$$A\cdot\ddot{\psi}+AB\dot{\psi}+(A\cdot C-D)\cdot\psi=0 \qquad (2.)$$

The limiting cases to standard-SRT are fulfilled, because for $B\equiv 0$ follows $a\equiv 0$ and in this case there is generated by this DE the ordinary, common Lorentz-factor of [3.] and his counterpart of Feinberg for classical tachyons, see [4.] and Appendix:

$$\gamma=\sqrt{1-\frac{v^2}{c^2}} \quad (3.)$$

This DE (2.) can be written in a Sturm-Liouville-form. This means, there is a description of an eigenwert-problem of this equation.This description follows now.

2.1. General Calculation:

The Sturm-Liouville-form of a DE can be formulated in its eigenform as:

$$-(p\cdot\dot{\psi})^{\circ}+q\cdot\psi=\lambda_n\cdot\omega\cdot\psi \quad (4a.)$$

This means:

$$-p\cdot\ddot{\psi}-\dot{p}\cdot\dot{\psi}+q\cdot\psi=\lambda_n\cdot\omega\cdot\psi \quad (4b.)$$

Then can this equation be written as an eigenwertproblem:

$$L\cdot\psi=\lambda\cdot\psi \quad (4c.)$$

with the operator:

$$L=\frac{1}{\omega}\cdot((-\ddot{p})+q) \quad (4d.)$$

For the DE of advanced Einstein-Lorentz-factor (2.) this model can be written as:

$$-A\cdot\ddot{\psi}-A\cdot B\,\dot{\psi}+D\,\psi=A\cdot C\cdot\psi \quad (5a.)$$

with

$$\psi(r)=A\cdot e^{i\cdot(\frac{r}{r_{PL}}-\theta)} \quad (5.b)$$

(All dots above an variable or bracket are derivations after time in Newtonian notation).

Comparison of coefficients leads to:

$$\begin{aligned} A&=p\\ A\cdot B&=\dot{p}\\ D&=q\\ A\cdot C&=\lambda_n\cdot\omega \end{aligned} \quad (5c.)$$

Boundary condition after Sturm (BC) for this STL-system is the equation:

$$\psi(r)\cdot\sin(\theta)+\dot{\psi}(r)\cdot\cos(\theta)=0 \quad (6.)$$

This leads especially to the BCs:

$$\psi(r_{Pl})\cdot\sin(0)+\dot{\psi}(r_{Pl})\cdot\cos(0)=0 \quad (6a.)$$

$$\psi(r_{Pl})\cdot\sin(\pi)+\dot{\psi}(r_{Pl})\cdot\cos(\pi)=0 \quad (6b.)$$

with

$$r\in[r_{Pl};n\cdot r_{Pl}];n\in\mathbb{N};\theta\in[0;\pi] \quad (6c.)$$

Then the eigenwert-problem of a STL-equation can be written as:

$$\lambda_n=\pi^2\cdot n^2\cdot\left(\int_a^b\sqrt{\frac{\omega}{p}}dx\right)^{-2} \quad (7a.)$$

This leads in this case here to a selfreferential condition of:

$$\lambda_n=\pi^2\cdot n^2\cdot\left(\int_a^b\sqrt{\frac{C}{\lambda_n}}dr\right)^{-2} \quad (7b.)$$

After some boring calculations this formulation leads to an eigenwert-solution of:

$$\lambda_n=e^{2\cdot\pi\cdot n\cdot\int\frac{1}{\sqrt{C}}d\omega+k} \quad (8a.)$$

Since C= $\omega^2{}_{PL}$, the integration has to be after the frequency ω of the oscillating system because of condition of absence of units of measurement in exponent of the value. Especially this integration leads to $\omega=\Omega$, the frequency of the modelled damping system, which is the only quantity which makes sense to use in these circumstances. Finally this equation leads to the expression of:

$$\lambda_n=e^{\frac{2\cdot\pi\cdot n\cdot\Omega}{\omega_{PL}}+k} , \quad (8b.)$$

where k is an unknown, constant integration variable, which could be set to zero (or not).

2.2. Special calculation:

With

$$\Omega=\frac{a}{R}=\frac{1}{T};R=m\cdot r_{PL};m=const.;m\in\mathbb{N};\omega_{PL}=\frac{1}{t_{PL}}=\frac{c}{r_{PL}} \quad (9.)$$

follows as a solution for the equation of eigenvalues:

$$\lambda_n = e^{2\cdot\pi\cdot\frac{n}{m}\cdot\frac{a}{c}+k} \quad ; \qquad (10.)$$

The factor a is damping velocity of outer model- system, which reduces to $a \equiv 0$ for states of common, classical SRT.

$m = n\, ;\, m, n \in \mathbb{N}$ possible.

So the equation for eigenfunction is finally:

$$\widehat{\psi_n}(t) = \lambda_n \cdot \psi(t) = \lambda_n \cdot A \cdot e^{i\cdot(\frac{v}{c}\cdot\frac{t}{t_{PL}} - \theta)} = \frac{e^{2\cdot\pi\cdot\frac{n}{m}\cdot\frac{a}{c}+k} \cdot e^{i\left(\frac{v}{c}\cdot\frac{t}{t_{PL}} - \theta\right)}}{\sqrt{(1 - \frac{v^2}{c^2})^2 + \frac{n\cdot a^2\cdot v^2}{c^4}}} \qquad (11.)$$

$k = \pm i\cdot\theta$ or $k = \pm\theta$ possible.

3. Conclusion:

There are calculated the eigenwerte for the DE of second order which represents in its solution the advanced Lorentz-Einstein-factor as an amplitude.They exist and are real e-functions.

Critical remark:

For $a \equiv 0$ this equation (8b.) leads to

$$\lambda_{(n)} = e^k \quad . \qquad (11.)$$

This formula doesn‘t depend from an n (see low brackets in lambda), so it could be reinterpreted as the classical form of SRT-Lorentz-factor because of $a \equiv 0$ – but on the other hand this form of a DE of second order can only be written as a STL-problem for constant value of p .Because p includes the velocity v of the inertial system (common classical Lorentz-factor), this velocity can only be supposed as a constant. So in this case acceleration in resp. of inertial systems can‘t be described.

Possibly k could be interpreted as constant supposed phaseangle $\pm i\cdot\theta$, but in case of classical SRT this phase-angle is identical equal to zero (or multiplies of Pi for classical tachyons), so the eigenwert will be reduced to a $\lambda = |1|$ identity (see Appendix).

4. Summary:

The differential equation of second order for the advanced Lorentz-Einstein-factor in fourth order can be written as a Sturm-Liouville-problem. In this case the eigenvalue(s) can be calculated.They are real e-functions and depend from the velocity a of the outer model of a damping system.For $a \equiv 0$ the differential equation will reduce to the form for classical SRT-factor and possibly the eigenvalue be only the trivial case of $\lambda = |1|$ or at least $\lambda = e^k ; k = const.$.

5. Appendix: Deduction of classical Lorentz-Einstein-factor from a DE

The aim is to determine the amplitude factor of A from the following DE of second order:

$$A\cdot\ddot{\psi}(t)+C\cdot(A-D)\cdot\psi(t)=0 \qquad \text{(A1.)}$$

with:

$$C=\omega_{PL}^2 ; D=e^{i\theta} \qquad \text{(A2.)}$$

and the plane-wave function

$$\psi(r)=A\cdot e^{i\cdot(\frac{r}{r_{PL}}-\theta)} \qquad \text{(A3.)}$$

With the following common relations and the transition from r to t :

$$(r=v\cdot t ; r_{PL}=c\cdot t_{PL})\Rightarrow \psi(r)\rightarrow\psi(t) \qquad \text{(A4.)}$$

there is the function of a planewave depending only of time t :

$$\psi(t)=A\cdot e^{i\cdot(\frac{v}{c}\cdot\frac{t}{t_{PL}}-\theta)} \qquad \text{(A5.)}$$

with its derivations:

$$\dot{\psi}(t)=i\cdot A\cdot\left(\frac{\dot{v}}{c}\cdot\frac{t}{t_{PL}}+\frac{v}{c}\cdot\frac{1}{t_{PL}}\right)\cdot e^{i\left(\frac{v}{c}\cdot\frac{t}{t_{PL}}-\theta\right)} \qquad \text{(A6.)}$$

and

$$\ddot{\psi}(t)=A\cdot\left[i\left(\frac{\ddot{v}}{c}\cdot\frac{t}{t_{PL}}+2\cdot\frac{\dot{v}}{c}\cdot\frac{1}{t_{PL}}\right)-\left(\frac{\dot{v}}{c}\cdot\frac{t}{t_{PL}}+\frac{v}{c}\cdot\frac{1}{t_{PL}}\right)^2\right]\cdot e^{i(\frac{v}{c}\cdot\frac{t}{t_{PL}}-\theta)} \qquad \text{(A7.)}$$

For $v=const.$ between inertial-systems of classical SRT, this relation of second derivation reduces to:

$$\ddot{\psi}(t)=-A\cdot\frac{v^2}{c^2}\cdot\frac{1}{t_{PL}^2}\cdot e^{i(\frac{v}{c}\cdot\frac{t}{t_{PL}}-\theta)} \qquad \text{(A8.)}$$

Setting $\psi(t) \wedge \ddot{\psi}(t)$ in the DE, this leads to calculation of amplitude A of the wave-function $\psi(t)$:

$$A = \frac{e^{i\cdot\theta}}{1 - \frac{v^2}{c^2}} \qquad \text{(A9.)}$$

Remark:

Since $e^{(i\cdot\theta)} = \cos(\theta) + i\cdot\sin(\theta)$ and comparison of coefficients in

$e^{(i\cdot\theta)} = A\cdot(1 - \frac{v^2}{c^2})$ leads to the conclusion for $i\cdot\sin(\theta) = 0$

and the conditions for calculation of θ :

$i\cdot\sin(\theta) = 0 \, for \, \theta = k\cdot\pi \,; k \in \mathbb{Z}$ but

$$\begin{aligned} I \cos(\theta) &= 1 \, for \, k = 2\cdot n \,; n \in \mathbb{Z} \,; \\ II \cos(\theta) &= -1 \, for \, k = 2\cdot n + 1 \,; n \in \mathbb{Z} \end{aligned} \qquad \text{(A10.)}$$

I leads to square of Einstein-Lorentz-factor, II to square of Feinberg-factor of classical, common SRT.

6. References:

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7. Verification: this paper is written without using a chatbot like ChatGPT- 4 or other chatbots. It is fully human work .