# Systems of State-Set Permutated Replicators Walking Ergodically and Non-Ergodically in Permutation Space

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**Abstract**

In this paper I describe collections of state-set permutated Byl replicators replicating under common state-transition functions as systems of replicators, and show that systems can be walked through a permutation space by replacement or deletion and addition of system members. By doing this, a process of homochiral replication systems exploring an "adjacent possible" is modelled. Ergodicity corresponds to the maximum possible number of replacements in a system at some or all steps, but assuming gradualism of system change by limiting the number of changes in each step to the minimum possible, walks within the comprehensive permutation space become restricted in range.

*Keywords*: adjacent possible, artificial life, cellular automata, ergodicity, biochirality, origin of life, permutation, replicator

## Introduction

### The concept of the Adjacent Possible

The *adjacent possible* is defined by Stuart A. Kauffman as the range of possibilities immediately available to a dynamic system [6] with relevance to all hierarchical levels of biology – from systems of molecules to the entire biosphere. As possibilities are integrated, further possibilities become available, so the adjacent possible continually expands as systems incorporate possibilities into reality. In addition to the ever-increasing adjacent possible, the rate of exploring the adjacent possible is constrained by the continuation of viability of an evolving system, where viability is threatened by too-rapid change. It follows that the historical pathways of biological systems are nonergodic, *i.e.*, the space of all possibilities can never be fully explored. As an illustration, we can observe that there are far more possible proteins than there are in the set of existing proteins.

The observation that similar solutions appear in distinct evolutionary lineages is interpreted as *convergent evolution*. One long-recognized example is the functional similarity of cetaceans (marine mammals) to sharks (cartilaginous fish) within a shared aquatic environment. The many observed instances of convergent evolution may indicate that evolutionary exploration of adjacent possibility space is nonergodic, meaning much of the adjacent possible is not visited in the continuing historical record of biology.

In this paper I describe collections of state-set permutated Byl replicators [2][11] replicating under common state-transition functions as *systems* of replicators, and show that systems can be walked through a permutation space by sequential processes of replacement, or deletion and addition of system members. By doing this, a process of simple homochiral replication systems exploring an *adjacent possible* is modelled.

### Multiple permutated systems of the Byl replicator

Table 1 below lists five systems of state-set permutated instances of the Byl replicator [2] each including the original form of the replicator, designated as R-12345 [11]. The members of each three- or four-member system replicate under one consistent system-specific cell state transition function consisting of Moore neighbourhood rules. For convenience of further discussion, the 120 permutations of the active state set {1, 2, 3, 4, 5} are indexed by sequential assignment of integer indices 1 to 120 as shown in the Appendix Table following **References**.

**Table 1.** Five systems (in columns) of common-chirality (R-) state set-permutated Byl replicators, all including R-12345. The members of each system replicate under one system-specific state-transition function. This Table is adapted from the version in [11] by inclusion of bracketed numbers indicating the permutation indices corresponding to each system member. The comprehensive mapping of permutation transforms/permutated replicators to corresponding indices is shown in the Appendix Table.

| | | | | |
|---|---|---|---|---|
| **R-12345 (1)** | **R-12345 (1)** | **R-12345 (1)** | **R-12345 (1)** | **R-12345 (1)** |
| **R-25413 (47)** | **R-12354 (2)** | **R-12354 (2)** | **R-12354 (2)** | **R-12354 (2)** |
| **R-25431 (48)** | **R-41532 (78)** | **R-51432 (102)** | **R-14523 (17)** | **R-15423 (23)** |
| | | | **R-14532 (18)** | **R-15432 (24)** |

As an alternative to tabulation, systems can be represented graphically as in Figure 1 below.

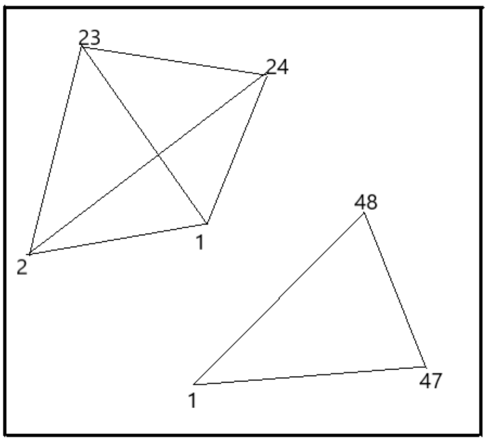


**Figure 1**. Nodes labelled by permutation indices represent permutation instances of replicators, and each edge connecting two nodes represents replication of its two nodes under a common state-transition function. A three-member system is represented by three-nodes connected with three-edges, *e.g.*, the triangle 1,47,48 corresponds to column 1 of Table 1. The quadrilateral 1,2,23,24 corresponds to the four-member system of column 5 in Table 1, with the six edges connecting all four nodes to each other indicating that all four members belong to a system replicating under one system-specific state-transition function.

Extrapolating Figure 1, the five systems of Table 1 can be visualized together as a graph (Figure 2, below).

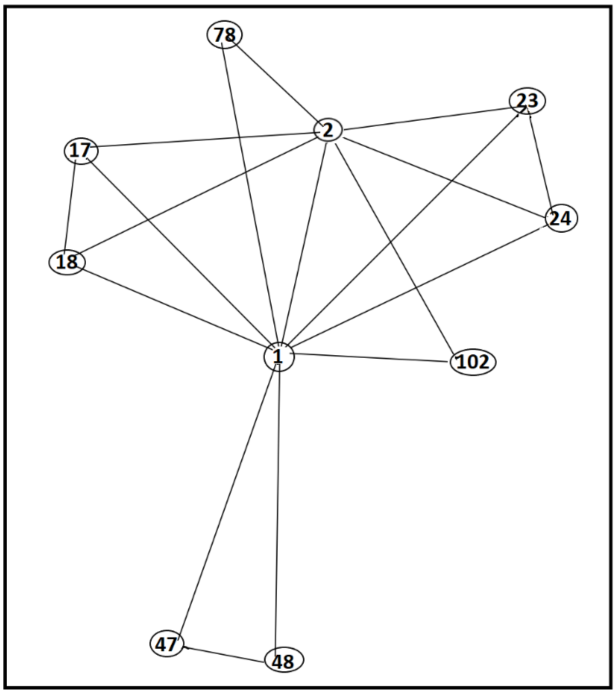


**Figure 2**. A graphical representation of all five systems shown in Table 1. The two systems shown in isolation in Figure 1 are easily seen here in relation to each other and to the other three systems in Table 1.

There are 120 permutations of the active state set {1,2,3,4,5}, so there are 120 state-assignment permutations of the Byl replicator. Each system of replicators has state-set permutation equivalents derived from the uniform application of a permutation transform to all members of the system.

There are 280,840 combinations of three permutations from 120 permutations, but only 120 of the combinations correspond to three-member systems replicating under a system-specific common state-transition function. Three of these are listed in Table 1. There are 8,214,570 combinations of four permutations from 120 permutations, but only 60 of these combinations correspond to four-member systems, with two of them listed in Table 1. Why are there only 60 different four-member system permutations, not 120? The answer is that the 120 permutation transforms correspond to 60 pairs, each of which applied to one four-member system produce the same permutated result. As an example, applying permutation transformation 12345 → 12354 (index 2) to system 1, 2, 17, 18 (by indices) delivers the permutated system 1, 2, 23, 24. Applying the different permutation transformation 12345 → 15423 (index 23) to system 1, 2, 17, 18 also delivers system 1, 2, 23, 24. As a second example, applying permutation transformation 12345 → 31425 (index 51) to system 1, 2, 17, 18 delivers the permutated system 51, 52, 59, 60. Applying the different permutation transformation 12345 → 32514 (index 59) to system 1, 2, 17, 18 delivers the same result.

There are 120 permutated-equivalent versions of Table 1. Table 2 below shows the example of the permutation transformation 12345 → 53124 (index 109) applied uniformly to Table 1.

**Table 2.** An example of a permutated equivalent of Table 1 derived by application of the permutation 12345 → 53124 (index 109) to all replicator instances in Table 1. There are 120 permutation-equivalents of Table 1.

| | | | | |
|---|---|---|---|---|
| **R-53124 (109)** | **R-53124 (109)** | **R-53124 (109)** | **R-53124 (109)** | **R-53124 (109)** |
| **R-34251 (64)** | **R-53142 (110)** | **R-53142 (110)** | **R-53142 (110)** | **R-53142 (110)** |
| **R-34215 (63)** | **R-25413 (47)** | **R-45213 (93)** | **R-52431 (108)** | **R-54231 (118)** |
| | | | **R-52413 (107)** | **R-54213 (117)** |

The 120 permutated versions of Table 1 are all equivalent if we consider that the reassignments of states corresponding to permutations of the state set are merely reassignment of cell state labels, so in this sense Table 1 is a complete and comprehensive compilation of systems of coexisting right-handed (R-) permutated replicators.

## Systems can be walked through permutation space

Considering an environment of systems of coexisting state-permutated replicators, a walk within the environment can be defined by swapping out one or more state permutated replicators in systems of three or four coexisting permutated instances. As an example:

Table 1, column 3 (the system of permutated replicator instances 1,2,102 by indices) can be walked within a space of system permutations to system 1,2,78 (Table 1, column 2) by the one replacement of permutated replicator R-51432 (102) with R-41532 (78). This step is equivalent to applying the state set permutation 12345 → 12354 to the system 1,2,102. Relative to the system 1,2,102, the replicator R-41532 (78) external to it can be considered different from replicators 1,2 or 102 only by reassignment of state labels, but by replacing 102 and becoming a component of the system, replicator 78 becomes functionally coexistent with permutated replicators 1 and 2, and in that context (*i.e.*, replicating with 1 and 2 under a common state-transition function), the permutated assignment of cell state labels now corresponds to a distribution of state functions different from each of the permutated replicators 1 and 2.

The systems shown as columns of Table 1, and equivalently as triangles and quadrilaterals in Figure 2, are interconvertible by application of appropriate permutation transforms listed in Table 3 below. Applying the permutation transformations walk the three- or four-member systems between each other within the system permutation space.

**Table 3.** Permutation transforms between columns in Table 1

| **Table 1 column transformations** | **Columns content (systems) by indices** | **Applied permutation transform 12345 -->** | **Transform inverse 12345 -->** |
|---|---|---|---|
| Column 1 to 2 | 1, 47, 48 to 78, 1, 2 | 41532 | 25413 |
| 1 to 3 | 1, 47, 48 to 102, 2, 1 | 51432 | 25431 |
| 2 to 3 | 1, 2, 78 to 2, 1, 102 | 12354 | 12354 |
| 4 to 5 | 1, 2, 17, 18 to 23, 24, 2, 1 | 15423 | 12354 |

Considering the conversion of the Table 1, column 1 system to the column 2 system (Table 3, line 1), application of the permutation 12345 → 41532 to the system 1,47,48 transforms it to the system R-

41532, R-12345 and R-12354 (indices: 78, 1 ,2). The inverse operation of recovering the system 1, 47, 48 is achieved by application of the permutation transform 12345 → 25413 (index 47) to system 78, 1, 2. Table 3 shows that all of the three systems of three coexisting replicators each (columns 1, 2 and 3, Table 1) are equivalent under permutations of the state labels. The bottom line of Table 3 shows also that the two four-member systems are equivalent under specific permutations of the state labels. Within columns, the permutation transforms are not merely state-labelling variations – their coexistence of replication under a common state transition function corresponds to exchanges of cell state functions. At this point we can recognize that just one three-member system and one four-member system is sufficient as a compact compilation of systems of coexisting right-handed (R-) permutated replicators, as these can be transformed to a complete set of permutated systems by application of the 120 permutation transforms.

By substituting or adding and subtracting permutated replicators in three- or four-member systems, the systems can be walked through a permutation space of systems of coexisting replicators. The comprehensive permutation space is illustrated in Figure 3 below.

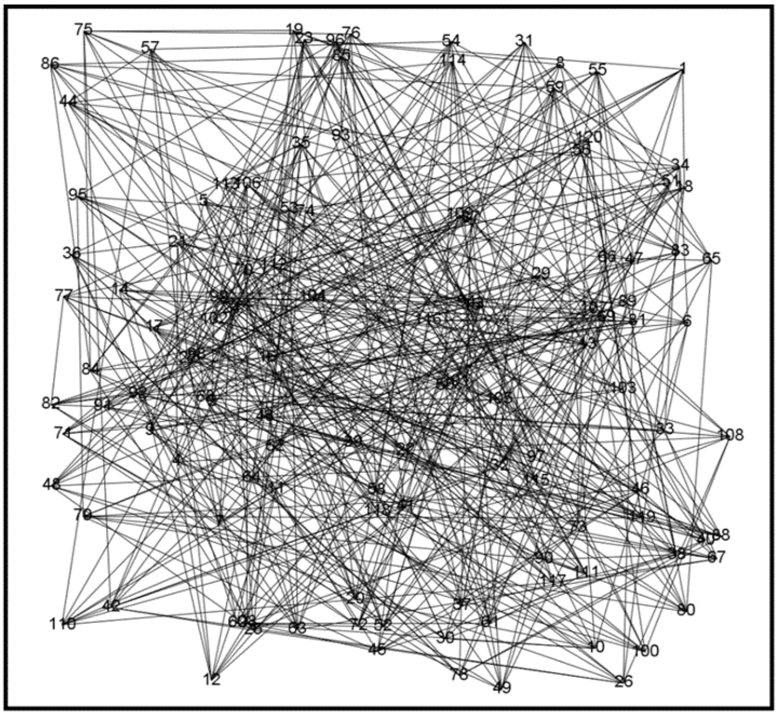

**Figure 3**. 120 nodes representing the 120 state set permutations of R-12345 with 540 edges corresponding to all groupings of replication under single system-specific state-transition functions, *i.e.*, all permutations of three- and four-member R-systems are represented. Tracing out all or any systems from this Figure as shown is not practically possible by sight, but the Figure indicates the detail of the permutation space systems can walk within by substitution or addition and subtraction of replicator instances, while preserving coexistence of replication under system-specific state transition functions.

Diversification in the course of biological evolution has historically been considered to be gradual, but dissenters have argued for existence of long periods of stasis punctuated by short periods of rapid speciation (*punctuated equilibrium*, [5]). The principle of gradualism is assumed for the walks in permutation space numbered 2 to 5 in the descriptions below. Observing a gradualism principle of minimizing the number of changes to a system at each step of a walk greatly limits the range of the adjacent possible for the next step.

## Results

A walk traversing all possible permutations of a three-member or four-member system is achievable if there is no limit applied to the number of permutation substitutions occurring at each step, *i.e.*, replacements of up to all members of a system in one step are all acceptable steps. In dynamics, ergodic walks are of indefinite length with no limitation on how often states can be visited, but for this study, a conveniently short walk in which every possible permutated system is visited just once is recognized as a sufficient proxy for identifying ergodicity. At every step of a system's ergodic walk, the adjacent possible is maximum, each step being a choice from the set of all possible system permutations.

### Walk 1, an example walk of a three-member system.

The permutation space in which a three-member system can be walked is defined by all 120 nodes and 300 of the 540 edges included in Figure 3. There are 120! (120 factorial) walks in permutation space corresponding to all possible ordered sequences of the 120 possible three-member system permutations. A walk organized to maximise the number of substitutions at each step (all three replicators changed each time) to visit each system permutation once is shown in Table 4 below.

**Table 4**. A walk visiting all 120 state-set permutations of the right-handed (R-) three-member system starting at system permutation 78, 1, 2 by indices.

| Step | coexisting replicators by indices | Step | coexisting replicators by indices | Step | coexisting replicators by indices |
|---|---|---|---|---|---|
| 0 | 78, 1, 2 | 40 | 102, 2, 1 | 80 | 1, 47, 48 |
| 1 | 55, 48, 47 | 41 | 47, 109, 110 | 81 | 93, 110, 109 |
| 2 | 109, 64, 63 | 42 | 11, 63, 64 | 82 | 63, 92, 91 |
| 3 | 39, 91, 92 | 43 | 91, 112, 111 | 83 | 23, 111, 112 |
| 4 | 111, 62, 61 | 44 | 35, 61, 62 | 84 | 62, 84, 83 |
| 5 | 16, 83, 84 | 45 | 83, 31, 32 | 85 | 107, 32, 31 |
| 6 | 31, 72, 71 | 46 | 7, 71, 72 | 86 | 71, 103, 104 |
| 7 | 95, 104, 103 | 47 | 104, 34, 33 | 87 | 6, 33, 34 |
| 8 | 33, 67, 68 | 48 | 87, 68, 67 | 88 | 67, 118, 117 |
| 9 | 21, 117, 118 | 49 | 117, 86, 85 | 89 | 41, 85, 86 |
| 10 | 85, 70, 69 | 50 | 9, 69, 70 | 90 | 69, 116, 115 |
| 11 | 45, 115, 116 | 51 | 116, 82, 81 | 91 | 18, 81, 82 |
| 12 | 81, 44, 43 | 52 | 57, 43, 44 | 92 | 43, 120, 119 |
| 13 | 19, 119, 120 | 53 | 119, 80, 79 | 93 | 65, 79, 80 |
| 14 | 79, 46, 45 | 54 | 3, 45, 46 | 94 | 46, 101, 102 |
| 15 | 70, 102, 101 | 55 | 101, 8, 7 | 95 | 77, 7, 8 |
| 16 | 8, 65, 66 | 56 | 32, 66, 65 | 96 | 66, 73, 74 |
| 17 | 120, 74, 73 | 57 | 73, 22, 21 | 97 | 27, 21, 22 |
| 18 | 22, 107, 108 | 58 | 68, 108, 107 | 98 | 108, 26, 25 |
| 19 | 84, 25, 26 | 59 | 25, 23, 24 | 99 | 49, 24, 23 |
| 20 | 24, 105, 106 | 60 | 92, 106, 105 | 100 | 105, 38, 37 |
| 21 | 59, 37, 38 | 61 | 37, 96, 95 | 101 | 13, 95, 96 |
| 22 | 96, 98, 97 | 62 | 72, 97, 98 | 102 | 97, 16, 15 |
| 23 | 29, 15, 16 | 63 | 15, 93, 94 | 103 | 61, 94, 93 |
| 24 | 94, 100, 99 | 64 | 48, 99, 100 | 104 | 99, 14, 13 |
| 25 | 53, 13, 14 | 65 | 14, 89, 90 | 105 | 38, 90, 89 |
| 26 | 89, 55, 56 | 66 | 113, 56, 55 | 106 | 56, 42, 41 |
| 27 | 2, 41, 42 | 67 | 42, 75, 76 | 107 | 118, 76, 75 |
| 28 | 75, 20, 19 | 68 | 51, 19, 20 | 108 | 20, 113, 114 |
| 29 | 44, 114, 113 | 69 | 114, 50, 49 | 109 | 90, 49, 50 |
| 30 | 50, 18, 17 | 70 | 26, 17, 18 | 110 | 17, 87, 88 |
| 31 | 115, 88, 87 | 71 | 88, 54, 53 | 111 | 34, 53, 54 |
| 32 | 54, 3, 4 | 72 | 100, 4, 3 | 112 | 4, 35, 36 |
| 33 | 80, 36, 35 | 73 | 36, 51, 52 | 113 | 112, 52, 51 |
| 34 | 52, 5, 6 | 74 | 76, 6, 5 | 114 | 5, 39, 40 |
| 35 | 103, 40, 39 | 75 | 40, 77, 78 | 115 | 64, 78, 77 |
| 36 | 10, 59, 60 | 76 | 86, 60, 59 | 116 | 60, 27, 28 |
| 37 | 106, 28, 27 | 77 | 28, 11, 12 | 117 | 74, 12, 11 |
| 38 | 12, 57, 58 | 78 | 110, 58, 57 | 118 | 58, 29, 30 |
| 39 | 82, 30, 29 | 79 | 30, 9, 10 | 119 | 98, 10, 9 |
| | | | | | |

The walk shown in Table 4 above is achieved by applying a substitution of all three replicators at each and every step, *i.e.*, ignoring the gradualism requirement of disallowing replacement of all three system members at any step.

This walk can be compared with Walk 2 described below. Walk 2 illustrates that avoiding the replacement of all three system members at any step excludes ergodicity. The walk cannot be maintained through all of the 120 system permutation possibilities without allowing at least one step of all-member replacements.

**Walk 2, an example walk of a three-member system indicating non-ergodicity.**

This walk (Table 5 below) is directed by replacement of one of the three members at odd-numbered steps, and replacement of two members at even-numbered steps, with one of the two replaced members being one of the two members common to the previous two steps. Avoidance of replacing all three system members at any one step is intended to minimise the number of replacements to make change over time as gradual as possible. Replacement choices are made which keep the walk away from previously-visited systems for as long as possible.

**Table 5**. A walk determined by minimizing replacements of permutated replicators at each step (limited to one replacement at odd-numbered steps and two replacements at even-numbered steps) and avoiding transition choices which cause a revisit to a previous permutated system. Colour coding indicates unavoidable returns to previously-visited permutated systems. Breaking of the direct path through all of the permutation possibilities is unavoidable at Step 108.

| Step | coexisting replicators by index | Step | coexisting replicators by index | Step | coexisting replicators by index | …or… | |
|---|---|---|---|---|---|---|---|
| 0 | 78, 1, 2 | 40 | 119, 80, 79 | 80 | 56, 42, 41 | | |
| 1 | 102, 2, 1 | 41 | 65, 79, 80 | 81 | 2, 41, 42 | | |
| 2 | 1, 47, 48 | 42 | 79, 46, 45 | 82 | 42, 75, 76 | | |
| 3 | 55, 48, 47 | 43 | 3, 45, 46 | 83 | 118, 76, 75 | | |
| 4 | 47, 109, 110 | 44 | 46, 101, 102 | 84 | 75, 20, 19 | | |
| 5 | 93, 110, 109 | 45 | 70, 102, 101 | 85 | 51, 19, 20 | | |
| 6 | 109, 64, 63 | 46 | 101, 8, 7 | 86 | 20, 113, 114 | | |
| 7 | 11, 63, 64 | 47 | 77, 7, 8 | 87 | 44, 114, 113 | | |
| 8 | 63, 92, 91 | 48 | 8, 65, 66 | 88 | 114, 50, 49 | | |
| 9 | 39, 91, 92 | 49 | 32, 66, 65 | 89 | 90, 49, 50 | | |
| 10 | 91, 112, 111 | 50 | 66, 73, 74 | 90 | 50, 18, 17 | | |
| 11 | 23, 111, 112 | 51 | 120, 74, 73 | 91 | 26, 17, 18 | | |
| 12 | 111, 62, 61 | 52 | 73, 22, 21 | 92 | 17, 87, 88 | | |
| 13 | 35, 61, 62 | 53 | 27, 21, 22 | 93 | 115, 88, 87 | | |
| 14 | 62, 84, 83 | 54 | 22, 107, 108 | 94 | 88, 54, 53 | | |
| 15 | 16, 83, 84 | 55 | 68, 108, 107 | 95 | 34, 53, 54 | | |
| 16 | 83, 31, 32 | 56 | 108, 26, 25 | 96 | 54, 3, 4 | | |
| 17 | 107, 32, 31 | 57 | 84, 25, 26 | 97 | 100, 4, 3 | | |
| 18 | 31, 72, 71 | 58 | 25, 23, 24 | 98 | 4, 35, 36 | | |
| 19 | 7, 71, 72 | 59 | 49, 24, 23 | 99 | 80, 36, 35 | | |
| 20 | 71, 103, 104 | 60 | 24, 105, 106 | 100 | 36, 51, 52 | | |
| 21 | 95, 104, 103 | 61 | 92, 106, 105 | 101 | 112, 52, 51 | | |
| 22 | 104, 34, 33 | 62 | 105, 38, 37 | 102 | 52, 5, 6 | | |
| 23 | 6, 33, 34 | 63 | 59, 37, 38 | 103 | 76, 6, 5 | | |
| 24 | 33, 67, 68 | 64 | 37, 96, 95 | 104 | 5, 39, 40 | | |
| 25 | 87, 68, 67 | 65 | 13, 95, 96 | 105 | 103, 40, 39 | | |
| 26 | 67, 118, 117 | 66 | 96, 98, 97 | 106 | 40, 77, 78 | | |
| 27 | 21, 117, 118 | 67 | 72, 97, 98 | 107 | 64, 78, 77 | | |
| 28 | 117, 86, 85 | 68 | 97, 16, 15 | 108 | 10, 59, 60 | 78, 1, 2 (Step 0) | 77, 7, 8 (Step 47) |
| 29 | 41, 85, 86 | 69 | 29, 15, 16 | 109 | 86, 60, 59 | | |
| 30 | 85, 70, 69 | 70 | 15, 93, 94 | 110 | 60, 27, 28 | | |
| 31 | 9, 69, 70 | 71 | 61, 94, 93 | 111 | 106, 28, 27 | | |
| 32 | 69, 116, 115 | 72 | 94, 100, 99 | 112 | 28, 11, 12 | | |
| 33 | 45, 115, 116 | 73 | 48, 99, 100 | 113 | 74, 12, 11 | | |
| 34 | 116, 82, 81 | 74 | 99, 14, 13 | 114 | 12, 57, 58 | | |
| 35 | 18, 81, 82 | 75 | 53, 13, 14 | 115 | 110, 58, 57 | | |
| 36 | 81, 44, 43 | 76 | 14, 89, 90 | 116 | 58, 29, 30 | | |
| 37 | 57, 43, 44 | 77 | 38, 90, 89 | 117 | 82, 30, 29 | | |
| 38 | 43, 120, 119 | 78 | 89, 55, 56 | 118 | 30, 9, 10 | | |
| 39 | 19, 119, 120 | 79 | 113, 56, 55 | 119 | 98, 10, 9 | | |
| | | | | 120 | 9, 69, 70 (Step 31) | 10, 59, 60 (Step 108) | |

The system 1,2,78 at Step 0 is walked to system 1,2,102 (Table 1, column 3) by application of permutation transformation 12345 → 12354 (Appendix permutation index 2), which in this case restricts replacement to just one of the system members (78 → 102), preserving replicators 1 and 2 in the first step. In the next step, replacement is restricted to replacement of the system members 2 and 102 to give system 1,47,48. This step corresponds to application of permutation 12345 → 25431 (index 48) to the system 1,2,102. At Step 107, the gradualist step-protocol requires replacement of 64, 78, 77 with either 78, 1, 2 (a return to Step 0), or with 77, 7, 8 (a return to Step 47). The walk from Time 107 can only be continued to unvisited permutations by a single-step substitution of all three system members which is achieved here by replacement of 64, 78, 77 by the system 10, 59, 60 at Step 108. From here, the gradualist protocol can be applied again at each remaining step up to Step 119 at which all possible system permutations have been visited once. To summarize, the minimum-replacement protocol does not support the ergodicity which is possible only by allowing substitution of all three system members per step at one or more steps.

This instance of a gradual walk is just one possibility of many. Up to and including the unavoidable all-member replacement to 10, 59, 60 at Step 108, there are 22 alternative choices allowed by the walk protocol which cut short an otherwise-conceivable direct path through all system permutations. After Step 108, there are five alternative choices allowed by the gradualist walk protocol which divert the walk from completion of a direct path visiting all permutation possibilities. The possible permutation path loops range in length from nine steps to 97 steps.

There are two nine-step loops identifiable in the walk shown in Table 5. The system at Step 49 is 32, 66, 65 by permutation indices. This transitions to 66, 73, 74 at Step 50. The adjacent possible at Step 49 includes the one alternative possibility of retaining permutated replicator 65 rather than 66, which gives at Step 50 the system 65, 79, 80, but this system was already visited at Step 41, closing a nine-step loop. Similarly, the transition from Step 79 to Step 88 arrives at system 114, 50, 49 at Step 88. At Step 88, the alternative adjacent possible Step is to 113, 56, 55, but this was already visited at Step 79, closing another nine-step loop.

To contrast with these nine-step loops, a long loop of 97 steps can be seen in the walk shown in Table 5. The transition from Step 105 to the system 40, 77, 78 at Step 106 can alternatively be to the system 39, 91, 92 by preservation of permutated replicator 39 instead of 40. However, system 39, 91, 92 was visited at Step 9, closing a loop of length 97 steps.

**Walk 3, an example walk of a four-member system.**

Table 6 below shows a walk constrained to systems of four co-replicators. As for the walks of three-member systems, a walk of a four-member system visiting all permutations is possible if replacement of all system members in a single step is permitted. The only possible change of less than all members at each step which transforms a four-member system to another valid four-member system is a replacement of two system members at each step. Changing one or three members does not produce a valid system of permutated replicators replicating under one consistent state transition function.

**Table 6.** Walk 3, constrained to systems of four coexisting replicators.

| step | System with corresponding (index) |
|---|---|
| 1 (start) | R-12345 (1) |
| | R-12354 (2) |
| | R-14523 (17) |
| | R-14532 (18) |
| | |
| 2 | R-12354 (2) |
| | R-12345 (1) |
| | R-15423 (23) |
| | R-15432 (24) |
| | |
| 3 | R-13254 (8) |
| | R-13245 (7) |
| | R-15432 (24) |
| | R-15423 (23) |
| | |
| 4 | R-13245 (7) |
| | R-13254 (8) |
| | R-14532 (18) |
| | R-14523 (17) |
| | |
| 5 | R-12345 (1) |
| (same as step 1: | R-12354 (2) |
| walk loop closed) | R-14523 (17) |
| | R-14532 (18) |
| | |

The transition from step 1 to step 2 occurs by replacement of replicators 17 and 18 with replicators 23 and 24. The corresponding permutation transform is 12345 → 15423 applied to 1,2,17,18 (as shown in Table 3). For a walk of a system of four permutated replicators, the minimum number of replacements per step which conserves replication of all system members under one state transition function is two. There is an alternative choice for the replacement of replicators at each step, *e.g.*, step 1 to step 2 can alternatively be the transition 1,2,17,18 → 7,8,18,17 by replacement of replicators (1) and (2) with (7) and (8) at step 2.

The walk shown in Table 6 illustrates that a walk visiting all permutations by means of minimum replicator replacements (two) at each step while maintaining a valid system is not possible.

An interesting property of the four-member systems is that they each have a "racemic" form, *e.g.*, if we look at the system 1,2,17,18 at step 1 in Table 6, the L- equivalents of permutated replicators 17 and 18 (L-14523 and L-14532 respectively) also form a valid system with R-12345 (1) and R-12354 (2), all replicating under one common state transition function (see [11]). A "step 0" preceding step 1 in Table 6 from racemic system R-12345 (1), R-12354 (2), L-14523, L-14532 to step 1 system R-12345 (1), R-12354 (2), R-14523 (17), R-14532 (18) may represent an abstraction of a transition from a racemic protobiology to the beginning of the homochiral biology we observe today.

### Walk 4: Alternating three- and four-member systems

By addition or subtraction of system members, a system can be toggled between a three-member and a four-member system, so a walk of alternating three- and four-member systems can be defined. The minimum changes per step which support an alternating walk are an alternation of: delete one member, add two, and delete two members and add one. All 120 three-member and 60 four-member permutated systems occur within the complete permutation space of 120 nodes and 540 edges visualised in Figure 3, but like the previous walks of minimal-replacements per step, the alternating walk exemplified in Table 7 excludes visiting all permutation possibilities.

**Table 7.** A walk of alternating three- and four-member systems

| Step | System with corresponding (index) | additions/deletions |
|---|---|---|
| 1 (start) | R-12345 (1) | |
| | R-25413 (47) | |
| | R-25431 (48) | |
| | | delete 1 (1), add 2 (26,25) |
| 2 | R-21354 (26) | |
| | R-21345 (25) | |
| | R-25413 (47) | |
| | R-25431 (48) | |
| | | delete 2 (47,48), add 1 (84) |
| 3 | R-42531 (84) | |
| | R-21345 (25) | |
| | R-21354 (26) | |
| | | delete 1 (84), add 2 (41,42) |
| 4 | R-21345 (25) | |
| | R-21354 (26) | |
| | R-24513 (41) | |
| | R-24531 (42) | |
| | | delete 2 (25,26), add 1 (2) |
| 5 | R-12354 (2) | |
| | R-24513 (41) | |
| | R-24531 (42) | |
| | | delete 1 (2), add 2 (31,32) |
| 6 | R-23145 (31) | |
| | R-23154 (32) | |
| | R-24531 (42) | |
| | R-24513 (41) | |
| | | delete 2 (41,42), add 1 (83) |
| 7 | R-42513 (83) | |
| | R-23145 (31) | |
| | R-23154 (32) | |
| | | delete 1 (83), add 2 (48,47) |
| 8 | R-23154 (32) | |
| | R-23145 (31) | |
| | R-25431 (48) | |
| | R-25413 (47) | |
| | | delete 2 (32,31), add 1 (55) |
| 9 | R-32145 (55) | |
| | R-25431 (48) | |
| | R-25413 (47) | |
| | | delete 1 (55), add 2 (26,25) |
| 10 | R-21354 (26) | |
| (same as Step 2: | R-21345 (25) | |
| walk loop closed) | R-25413 (47) | |
| | R-25431 (48) | |
| | | |

**Discussion**

The motivation for conducting this work is that the simplicity of these minimal systems exploring a small-sized adjacent possible may correspond to a level of simplicity of systems existing during an abiogenesis process. The problem remains that deterministic loop replicators such as the Byl replicator are brittle and do not grow in size or complexity, but as this study and previous work shows, *e.g.* [11], they are variable by permutation of cell states.

The Byl replicator [2] was derived as a simplification of the larger Langton replicator [7], so in the context of an evolutionary timeline in a cellular automata (CA) world supporting these families of 2D replicators we might consider that the family of permutated Byl replicators as temporally ancestral to a later family of Langton replicators. If we entertain this scenario further, we could provisionally consider further development through the Codd replicator [4] to the huge von Neumann self-reproducing machine [8].

By reversing the historical programme of progressive simplification of CA replicators, a prospective abstraction of evolution immediately following abiogenesis might be constructed. Such an abstraction might be useful in identifying some fundamental universal logic inherent in abiogenesis and subsequent early evolution. As a first step, we can imagine a Langton replicator appearing in a CA world previously occupied only by simpler Byl replicators, but the problem which immediately presents is to derive an expanded state-transition function which effects the transformation of a Byl replicator to a Langton replicator and simultaneously supports Langton replication with continuing support for Byl replication.

Precedents relevant to this prospective programme already exist, *e.g.*, [3][10]. Chou and Reggia [3] developed an impressive diversifying ecosystem of loop replicators, but the large sizes of the state-transition function and the state-set necessary to support the system defy simple state transition function analysis. H. Sayama's later evoloop system [10] successfully incorporates evolution of loop replicators within a much-simpler CA environment. In that study the pathway from an ancestral loop replicator leads to dominance of the environment by a smaller loop species, but the potential of the evoloop system has inspired further work, *e.g.* [9].

Expansion into the adjacent possible opens more possibilities, indicating the conjecture that the adjacent possible can expand faster than the rate of systems exploring it, *i.e.*, exclusion of ergodicity. In biological systems, there appears to be no identifiable ceiling to the size of an adjacent possible – biology appears to be endlessly surprising, even given the observation of many examples of convergent evolution. By comparison, the permutation space shown in Figure 3 represents a low definite ceiling to the absolute number of possibilities a system of state-permutated Byl replicators can explore. A higher-than-minimum rate of member substitutions within a system per walk-step (up to all members replaced per step) corresponds to dynamic ergodicity within the fixed finite permutation space. In these cases, any system defined within the available permutation space is available for the next step, so the adjacent possible is maximized. By contrast, the minimum-possible number of substitutions per step applied to enforce gradualism can correspond to as few as only two possibilities available at each step, but even with choices consciously made to prolong a gradualist walk toward the ideal of visiting all permutations of systems, short loops excluding many permutation possibilities are unavoidable.

In comparing biology today and its history deducible from preserved evidence with these simple abstractions, the question which arises is: what determines and defines a point within abiogenesis and subsequent early evolution at which the adjacent possible inflates to indefinite size?

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**Appendix:** 120 (5!) state-set permutations 12345 → ***** table entry, with corresponding indices 1 to 120. The entries can equivalently describe a replicator in which the state labels 1,2,3,4,5 have been permutated, *e.g.*, with respect to replicator R-12345 indexed as replicator (1), permutated replicator R-12354 (state labels 4 and 5 exchanged) is indexed as (2).

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 12345 | 1 | 15324 | 21 | 24513 | 41 | 34125 | 61 | 42315 | 81 | 51423 | 101 |
| 12354 | 2 | 15342 | 22 | 24531 | 42 | 34152 | 62 | 42351 | 82 | 51432 | 102 |
| 12435 | 3 | 15423 | 23 | 25134 | 43 | 34215 | 63 | 42513 | 83 | 52134 | 103 |
| 12453 | 4 | 15432 | 24 | 25143 | 44 | 34251 | 64 | 42531 | 84 | 52143 | 104 |
| 12534 | 5 | 21345 | 25 | 25314 | 45 | 34512 | 65 | 43125 | 85 | 52314 | 105 |
| 12543 | 6 | 21354 | 26 | 25341 | 46 | 34521 | 66 | 43152 | 86 | 52341 | 106 |
| 13245 | 7 | 21435 | 27 | 25413 | 47 | 35124 | 67 | 43215 | 87 | 52413 | 107 |
| 13254 | 8 | 21453 | 28 | 25431 | 48 | 35142 | 68 | 43251 | 88 | 52431 | 108 |
| 13425 | 9 | 21534 | 29 | 31245 | 49 | 35214 | 69 | 43512 | 89 | 53124 | 109 |
| 13452 | 10 | 21543 | 30 | 31254 | 50 | 35241 | 70 | 43521 | 90 | 53142 | 110 |
| 13524 | 11 | 23145 | 31 | 31425 | 51 | 35412 | 71 | 45123 | 91 | 53214 | 111 |
| 13542 | 12 | 23154 | 32 | 31452 | 52 | 35421 | 72 | 45132 | 92 | 53241 | 112 |
| 14235 | 13 | 23415 | 33 | 31524 | 53 | 41235 | 73 | 45213 | 93 | 53412 | 113 |
| 14253 | 14 | 23451 | 34 | 31542 | 54 | 41253 | 74 | 45231 | 94 | 53421 | 114 |
| 14325 | 15 | 23514 | 35 | 32145 | 55 | 41325 | 75 | 45312 | 95 | 54123 | 115 |
| 14352 | 16 | 23541 | 36 | 32154 | 56 | 41352 | 76 | 45321 | 96 | 54132 | 116 |
| 14523 | 17 | 24135 | 37 | 32415 | 57 | 41523 | 77 | 51234 | 97 | 54213 | 117 |
| 14532 | 18 | 24153 | 38 | 32451 | 58 | 41532 | 78 | 51243 | 98 | 54231 | 118 |
| 15234 | 19 | 24315 | 39 | 32514 | 59 | 42135 | 79 | 51324 | 99 | 54312 | 119 |
| 15243 | 20 | 24351 | 40 | 32541 | 60 | 42153 | 80 | 51342 | 100 | 54321 | 120 |