Elementary proof of the Syracuse conjecture

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Abstract :

In application of the Collatz algorithm (3* + 1) we have :

x > 0 , x + V > 0 (x + V successor of x, V variable of adjustment)

and **V < x** (evaluation of V is in the main text, in first step V = 2 and x > 2).

And as by hypothesis x gives a sequence of Syracuse **S(x) = [x, … ,1], x --> 1**.

We deduce the rules :

(x > 0) ^ (**V < x**)^ (x + V > 0) ==> (**0 < x + V < 2x**)

(**0 < x + V < 2x**) ==> [(x --> 1) ==> (x + V --> 1)]

The two sequences S(x) and S(x+V) converge to the only trivial cycle : [4, 2, 1].

So by recurrence, every positive integer gives a sequence of Syracuse.

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Syracuse conjecture (Collatz conjecture)

Algorithm of Collatz :

Let x a positive integer number.

1 - if x is even then x := x/2

2 - if x is odd then x := x * 3 + 1

We repeat 1 - 2 until obtain a cycle (is only cycle?) or x tends to infinity.

The symbol := means : assign value on right to variable on left.

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Representation of numbers :

Let V a variable which, added to variable x, gives the successor x + V.

The variable V is a variable of adjustment.

Variables x and V are written in the form :

x := a*(y) with a := 2^{α} and α is integer >= 0, y is an odd positive variable.

V := b*(z) with b :=2^{β} and β is integer >= 0, z is an odd positive variable.

x + V := a*(y) + b*(z).

Application of the algorithm of Collatz :

The coefficient a being power of 2, the algorithm is applied to the odd part y of x := a*(y) giving a sequence of Syracuse S(x) = [x, … ,1] and the odd part z of V := b*(z) is multiplied by 3 plus an adjustment.

In operation 3* + 1 , x := a*(3*y + 1) = a'*(y'), x is increased by (a - 1) to subtract from V and we have for V in x + V : V := b*(3*z) - (a-1) = b'*(z') .

So we have the equality
a*(3*y+1) + b*(3*z) - (a-1) = a*(3*y) +1 + b*(3*z) = 3 * (a*(y) + b*(z)) + 1, giving 3*(x + V) + 1, with x and V of before the operation 3* + 1, according to the rule 2 of the algorithm.

a' et b' are power of 2 which can be equal to 1, y' and z' are odd numbers.
In the line a'*(y') + b'*(z'), a' and b' are divided by gcd(a',b') according to the rule 1 of the algorithm.
If gcd(a',b') = 1, the division by 2 is deferred.

Evaluation of the variable of adjustment V :

When x is multiplied by 3 then + 1, V is multiplied by 3.
When x is divided by 2, V is divided by 2.
When x = a(3*y+1), x is increased by (a - 1), V is decreased by (a - 1).
We deduce that V is always less than x.

Conclusion :

In application of the Collatz' algorithm we have :
x > 0 , x + V > 0 and **V < x** (in first step V = 2 and x > 2).
And as by hypothesis x gives a sequence of Syracuse **S(x) = [x, … ,1], x --> 1**.
We deduce the rules :
(x > 0) ^ (**V < x**)^ (x + V > 0) ==> (**0 < x + V < 2x**)
(**0 < x + V < 2x**) ==> [(x --> 1) ==> (x + V --> 1)]
The two sequences S(x) and S(x+V) converge to the only trivial cycle : [4, 2, 1].

So by recurrence, every positive integer gives a sequence of Syracuse.