Elementary proof of the Syracuse‘ conjecture

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Syracuse’ conjecture (Collatz’ conjecture)

Algorithm of Collatz (C):

Let x a positive integer number.

1 - if x is even then x := x/2

2 - if x is odd then x := x * 3 + 1

We repeat 1 - 2 until obtain a cycle (is only cycle?) or x tends to infinity.

The symbol := means : assign value on right to variable on left.

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Representation of numbers :

Let V a variable which, added to variable x, gives the successor x + V.

The variable V is a variable of adjustment.

Variables x and V are written in the form :

x := a(y) with a := 2^{α} and α is integer >= 0, y is an odd variable.

V := b(z) où b =2^{β} and β is integer >= 0, z is an odd variable.

x + V := a(y) + b(z) ; a, b, y, z are positive integer variables.

Application of the algorithm of Collatz :

The algorithm is applied to the odd part y of x := a(y) giving a sequence of Syracuse C(x) = 1 and the odd part z of V := b(z) is multiplied by 3 plus an adjustment.

The aim is to prove if C(x) = 1 then C(x + V) = 1.

In operation 3* + 1 , x := a(3*y+1) = a’(y’), x is increased by (a - 1) to subtract from V and we have for V in x + V : V := b(3*z) - (a-1) = b'(z') .

We have the equality

a(3*y+1) + b(3*z) - (a-1) = a(3*y) +1 + b(3*z) = 3 * (a(y) + b(z)) + 1

according to the rule 2 of the algorithm.

a’ et b' are power of 2 which can be equal to unity, y’ and z' are odd numbers.

In the line a'(y') + b'(z'), a’ and b’ are divided by gcd(a',b') according to the rule1 of the algorithm.

If gcd(a',b') = 1, the division by 2 is deferred.

Let's give an example of calculation :
As initial data of the algorithm :
x = 13, V = 2 et x + 2 = 15 is the successor number of x at the first step.

x :=	V :=		x + V :=		
1(13)	+ 2(1)	=	15	=	1(15)
1(40)	+ 2(3)	=	46	=	2(23)
8(5)	+ 2(3)	=	46	=	2(23)
4(5)	+ 1(3)	=	23	=	1(23)
4(16)	+ 1(9) - 3	=	70	=	2(35)
64(1)	+ 2(3)	=	70	=	2(35)
32(1)	+ 1(3)	=	35	=	1(35)
32(4)	+ 1(9) - 31	=	106	=	2(53)
128(1)	+ 2(-11)	=	106	=	2(53)
64(1)	+ 1(-11)	=	53	=	1(53)
64(4)	+ 1(-33) - 63	=	160	=	32(5)
256(1)	+ 32(-3)	=	160	=	32(5)
8(1)	+ 1(-3)	=	5	=	1(5)
8(4)	+ 1(-9) - 7	=	16	=	16(1)
32(1)	+ 16(-1)	=	16	=	16(1)
2(1)	+ 1(-1)	=	1	=	1(1)

When x is multiplied by 3 then + 1, V is multiplied by 3.
When x is divided by 2, V is divided by 2.
When x = a(3*y+1), x is increased by (a - 1), V is decreased by (a - 1).
We deduce that V is always less than x for x > 1.
In the application of the algorithm of Collatz :
x > 0 , x + V > 0 et V < x.
As x gives a sequence of Syracuse, when C(x) Є [4, 2, 1] (« trivial cycle»), it implies V < 4 and, as x + V > 0, V > -4 and C(x + V) Є [7, 6, 5, 4, 2, 1], and ultimately C(x + V) Є [4, 2, 1].
So by recurrence, every positive integer gives a sequence of Syracuse.