

THE MASS OF THE OBSERVABLE UNIVERSE ON THE BASIS OF ITS AGE IN BLACK-HOLE COSMOLOGY.

By

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Abstract:

On the assumption of black-hole cosmology, it is straightforward to calculate the mass of the observable Universe on the basis of its age. It will be shown here that this calculation is in agreement with one based on a relationship between fundamental physical constants, providing strong support for black-hole cosmology.

Keywords: cosmology (theory); large-scale structure of the Universe; cosmological parameters; black holes.

Abbreviations used: FLRW = Friedman-Lemaître-Robertson-Walker (metric); Planck 2018 refers to the results from the Planck space observatory released in July 2018.

Conflict of interest: The author asserts that he has no conflict of interest, and has received no funding for his research from any source or sources, public, private or voluntary sector.

Note: All units in S.I. Source of values of physical constants: <https://physics.nist.gov/cuu/Constants/index.html>.

1. Introduction.

Black-hole cosmology is the claim that the Universe we live in is an enormous black hole. It has had few advocates, among them Pathria (1972), Derney and Farnsworth (1983), Zhang (2009), Seshavatharam and Lakshminarayana (2014), Perelman (2020), and Popławski (2020).

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Part of the problem is that these authors, whilst advocating a black-hole cosmology, do not agree with one another regarding its details.

Another part – the bigger one – is that black-hole cosmology, or cosmologies, do not appear to be consistent with the observation that the Universe is not merely expanding, but that its expansion is accelerating (Riess *et al*, 1998; Perlmutter, Turner and White, 1999; Perlmutter, 2000; but see Vishwakarma, 2003).

Even the promoters of the idea of an accelerating expansion of the Universe would be forced to concede that we should not, in fact, use the present tense when discussing its expansion, still less any putative acceleration of it, because what we see of red-shifted distant galaxies, and the Type Ia supernovae in them, is very ancient light indeed – billions of years old (Leibendgut, 2001, p.91)². Any expansion, and any acceleration of it, *if* it took place, took place long ago.

Not all researchers agree that the Universe's expansion is (or *was*) accelerating, not just Vishwakarma (op.cit.): Nielsen, Guffanti and Sarkar (2016), Mohayaee, Rameez and Sarkar (2021), Ni *et al* (2022), *inter alios*, are among those who have questioned the reliability of the observational data on which the assumptions regarding the accelerated expansion have been based.

2. A Black-Hole Cosmology.

A black-hole cosmology is not, in fact, wholly dissimilar to the Newtonian one described by McCrea and Milne (1934, pp.73-75). There would have to be an absolute cosmic time coordinate, for reasons adduced by Gödel (1949, pp.447, 449-450), and is assumed in the 'FLRW' metric (Robertson, 1935, 1936a & b; Walker, 1937), provided that the Universe is isotropic and homogeneous, conforming to the 'cosmological principle' (Einstein, 1917, 1923, 1952; Milne, 1933, pp.1, 3-4; see Barrow, 1989). This, however, is denied by many, if not most, advocates of black-hole cosmology, including Popławski (op.cit.).

Furthermore, the cosmic horizon would *not* be a particle horizon, but an *event* horizon, as they are both defined by Rindler (1956, p.663).

² At a redshift of $z = 1$, the light emitted from galaxies has been travelling towards us for 7.731 billion years (<https://lco.global/spacebook/light/redshift/>).

This point is affirmed by Melia (2007, pp.1919-1920; see Schwarzschild, 1916, 1999). Melia is one of those denying the existence of an absolute cosmic time, but he also claims that the interior of a black hole's space-time should be 'flat'³ and describable using the Minkowski metric (Minkowski, 1909, 2011), citing Birkhoff's theorem (Birkhoff, 1923; see Johansen and Ravndal, 2005, 2006). As Melia points out, the Schwarzschild metric applies to the exterior of the black hole (ibid.). If our space-time is Minkowskian, the appropriate formula for calculating redshifts is that of the *Special*, not General, Theory of Relativity (Melia, 2012, pp.1419-1421), i.e., the formula is *not* for 'gravitational redshifts'⁴. As Melia (2007, p.1919, and 2012, p.1419) points out, it easy to show that the FLRW metric reduces to the Minkowski one when the scale-factor $a(t_0) = H_0(t_0) = 1$, and the Hubble parameter, $H_0 \equiv t_0^{-1} \equiv \tau_0^{-1}$, where τ is the Hubble time. We can also write $R_0 = ct_0 = c\tau_0 = c/H_0$, where R_0 is the Hubble radius. The General Theory applies at the local scale, but not at the cosmic scale.

It is assumed here that the Universe is not merely isotropic and homogeneous, but spherically symmetric and stationary (i.e., not rotating about an axis, as the Gödel universe does [op.cit.]) – that, in other words, it is a Schwarzschild black hole (op.cit.), but its space-time singularity, being in the past, is now at its circumference, not at its centre, which represents the observer's present. We are, as it were, at the centre of a neo- or quasi-Ptolemaic cosmic chronosphere.

If our Universe is, indeed, an enormous black hole, its present age will be given by:

$$t_0 = 2GM/c^3 . \tag{1}$$

As $t_0 = 13.801$ billion years $= 4.355264376 \times 10^{17}$ s (Aghanim *et al*, 2020, Table 1, p.7 pdf.), it is easy to see, by rearrangement of (1), that the mass, M, of the observable Universe will equal

$$c^3 t_0 / 2G = 8.791053 \times 10^{52} \text{ kg}. \tag{2}$$

³ And it is, in fact, flat, as has been empirically determined by Aghanim *et al*, op.cit., Table 2, p.15, although they assume the ' Λ CDM' model.

⁴ The formula, of course, being: $z = [(1 + \beta)/(1 - \beta)]^{1/2} - 1$, where $\beta = v/c$, v being the recessional velocity of distant celestial objects relative to the observer. When $v = c$, $z = \infty$.

As Davies (1982) points out⁵ (pp.77, 82), it is the case that:

$$N_E = (\hbar c/Gm_p^2)(m_p c^2 t_0/\hbar) = c^3 t_0/Gm_p = 1.05117 \times 10^{80} . \quad (3a)$$

Here, N_E is Eddington's number, the number of protons and electrons in the Universe, m_p is the rest-mass of the proton, and the expression $\hbar c/Gm_p^2$ is the inverse of the gravitational fine-structure constant $= \alpha_G^{-1} = M_P^2/m_p^2$, where M_P is the Planck mass $= 2.176434 \times 10^{-8}$ kg.

The expression M_P^2/m_e , where m_e is the rest-mass of the electron, yields a mass, 5.19984×10^{14} kg – that of a modest-sized asteroid. However, multiply this by α_G^{-1} , and one obtains:

$$M = (\hbar c)^2/(Gm_p)^2 m_e = M_P^4/m_p^2 m_e = 8.80435 \times 10^{52} \text{ kg} . \quad (4)$$

Combining equations (2) and (3) gives us

$$M = N_E m_p / 2 = 8.791053 \times 10^{52} \text{ kg} . \quad (5)$$

The figures in (2), (4) and (5), it should be noted, are about a third of that given by Corbeel and Magain⁶ (2023, p.6, pdf. – their figure is 2.7846×10^{53} kg, or 1.4×10^{23} solar masses). How to reconcile equations (2) and (5) with (4)? The simplest answer is to adjust the value of t_0 – and it does not need much adjustment, on the scale we are considering.

Substituting the value for M obtained from (4) in equation (1) gives us a value for t_0 of $4.361851836 \times 10^{17}$ s = 13,821,874,400 years, 20,874,400 years more than the Planck 2018 figure (Aghanim *et al*, *ibid.*). It is possible, of course, that the Planck 2018 figure is correct, and the time in equations (1), (2) and (3) is not the *present* age of the Universe, but the age, t_{MAX} , at which it will reach its maximum size, given by its Schwarzschild radius:

⁵ The present author has substituted the present age of the Universe for the Hubble time, which Davies employs, and drawn out a conclusion implicit in Davies which he does not draw.

⁶ Their figure is clearly excessive, but they *are* advocates of black-hole cosmology.

$$R_S = 2GM/c^2 = 1.307650283 \times 10^{27} \text{ m} = 13,821,874,400 \text{ l.y.} \quad (6)$$

What happens to it after then may depend on the amount of entropy it contains at that time, given the Bekenstein-Hawking formula (Bekenstein, 2008):

$$S_{\text{BH}} = 4GM^2k/\hbar c = 9.037467145 \times 10^{98} \text{ J K}^{-1}. \quad (7)$$

This may well correspond to the ‘heat-death’ of the Universe, or state of thermodynamic equilibrium, envisaged by Lord Kelvin (Kelvin, 1862), provided only that the Universe is closed and finite, rather than infinite⁷, especially if, as Penrose (2010, pp.72, 76-77, 98-99) asserts, the ‘Big Bang’ had very low – even ‘tiny’ – entropy.

3. Conclusion.

We have in this paper seen ample evidence to support the assertion that our observable Universe is an enormous black hole, with maximum horizon surface area = $2.1487856 \times 10^{55} \text{ m}^2$ and maximum volume = $9.3662 \times 10^{81} \text{ m}^3$.

The matter inside that black hole has yet to fill all of the available space, and has just over 20.87 million years to do so, if the suggestion made here is correct. This is just ~0.3865% of the remaining main-sequence life-span of our Sun, which is ~5.4 billion years⁸.

⁷ Kelvin actually thought that it was ‘impossible to conceive a limit to the extent of matter in the universe’, but Sir Isaac Newton, in *De gravitatione* (c.1666-1668), argued the Universe was *spatially* infinite, but only had a finite amount of matter in it. This would entail an overall effective mass-density of zero, with only our part of it containing mass-energy. It should be pointed out, of course, that Kelvin had, through no fault of his own, no knowledge of thermonuclear fusion, and therefore could not know what really generated the Sun’s heat, or how long that heat would go on being produced. His notion of a cooling Sun greatly influenced H.G. Wells, however, when writing *The Time Machine* (1895, 2017).

⁸ See: https://www.atnf.csiro.au/outreach/education/senior/astrophysics/stellarevolution_mainsequence.html for the total lifespan and <http://solar-center.stanford.edu/FAQ/Qage.html> for the Sun’s current age.

Cosmic expansion is here not a question of space expanding, as with the FLRW metric (see above)⁹, but of matter expanding to fill a finite proportion of an infinite space, the bulk of which is empty. Our Universe is thus, on this view, a finite, closed space-time pseudo-Riemannian manifold¹⁰ in an infinite Euclidean¹¹ space-time, but causally isolated from it (Huby, 1971). It is also quasi-Ptolemaic, in that we, as observers, are privileged – being at the centre of it – for our viewpoint is the centre of the sphere that constitutes the black hole, with the singularity at its circumference. At $R_S = ct_{\text{MAX}}$, the redshift $z = \infty$. There is neither ‘dark energy’ nor ‘dark matter’, only the kind with which we are familiar here on Earth. Furthermore, there was no ‘cosmic inflation’: the hypothesis is redundant, because the velocity of cosmic expansion has always been $v_E \leq c$.

The minimum density of matter in the observable Universe will be $\rho_{\text{MIN}} = 9.4 \times 10^{-30} \text{ kg m}^{-3}$, with the energy density $8.4484 \times 10^{-13} \text{ J m}^{-3}$ and the proton-electron particle density, $N_E/V = 0.011239994 \text{ m}^{-3}$, given

$$N_E = c^3 t_{\text{MAX}} / G m_p = 1.05276 \times 10^{80}. \quad (3b)$$

Given the value of $t_{\text{MAX}} \equiv \tau_{\text{MAX}}$, the final value of the Hubble parameter, H_{MAX} , will be $2.2926 \times 10^{-18} \text{ s}^{-1} \simeq 70.743 \text{ km s}^{-1} \text{ Mpc}^{-1}$, compared to its current value, H_0 , calculated from its age (as determined by Aghanim *et al*, op.cit.), of $\sim 70.85 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

The final acceleration parameter of the Universe, q_{MAX} , will be given by:

$$q_{\text{MAX}} = 4\pi G \rho_{\text{MIN}} t_{\text{MAX}}^2 / 3 \simeq 0.0005. \quad (8)$$

⁹ The FLRW space does not expand if it empty of mass-energy, as Cook and Burns (2009) demonstrate.

¹⁰ See: https://en.wikipedia.org/wiki/Pseudo-Riemannian_manifold.

¹¹ The assumption that the General Theory of Relativity – or any other theory or law of physics – is universally applicable is precisely that: an assumption, and not necessarily valid. It may only apply in the observable Universe, and not outside it. If it *does* apply, then an infinite zero-density space-time, having negative curvature, will be hyperbolic, or Bolyai-Lobachevskian.

The final density parameter, $\Omega_{\text{MAX}} = 2q_{\text{MAX}} \simeq 0.001$. This would normally entail a space-time with near-zero curvature, but – as already stated – the General Theory of Relativity, although it applies at local scales, does not apply at the cosmic scale, and thus the curvature of space-time remains zero, or ‘flat’, i.e., Euclidean, regardless of the values of q_{MAX} and Ω_{MAX} . The finite cosmos is thus the Newtonian one of McCrea and Milne (op.cit.), with escape velocity, or – more correctly – speed, as it is independent of direction, given by:

$$v_e = (8\pi G\rho_{\text{MIN}}R_S^2/3)^{1/2} = (2GM/R_S)^{1/2} = c. \quad (9)$$

It is clear, however, that black-hole cosmologies will remain controversial pending further, decisive, empirical evidence in their favour, of the kind supplied by Shamir (2022). It must be accepted, though, that this evidence may well support an anisotropic Universe.

Data Availability Statement: There are no data additional to that contained herein and in the references.

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