Inconsistency Defended

By Jim Rock

Abstract: Here two contradictory arguments are defended. They can be developed in any formal system containing sets, arithmetic, and relations between the rational numbers.

Introduction. For all rational numbers $\boldsymbol{a}$ in the closed interval $[0, 1]$
let the collection of all R_a sets be { y is a rational number | $0 \le y \le a$ }

Consider the entire collection of R_a sets. They form a hierarchy of sets.

Each set contains all the elements in sets below it in the set hierarchy.

Each set contains a single element that is not in any set below it in the set hierarchy.

We take the largest element out of each set in the entire collection.

Argument #1: Each R_a contains a largest element.

1) The set containing zero becomes the null set.
2) Their largest element is now missing. But, all other sets remain in the same relative position in the set hierarchy as { y is a rational number | $0 \le y < \boldsymbol{a}$ }
3) For each set below an R_a in the set hierarchy R_a contains the former largest element from the specified set.
4) From 2) 3) Each R_a contains elements not in the sets below it in the set hierarchy.
5) Let $\boldsymbol{c}$ and $\boldsymbol{d}$ be two elements of a single R_a set with $\boldsymbol{c} > \boldsymbol{d}$.
6) $\boldsymbol{d}$ is an element of R_c, which is a proper subset of R_a.
7) For any two elements in R_a the smaller element is contained in a proper subset of R_a.
8) From 4) 7) Each R_a set contains a largest element not in any set below it in the set hierarchy.

Argument #2: No R_a contains a largest element.

1) Suppose there is a largest element a' in some individual R_a.
2) $a' < (a + a')/2 < a$.
3) Let $b = (a + a')/2$.
4) Then b is in R_a and $a' < b$.

The difference between the two arguments is no attempt is made to specify a largest element in Argument #1. It is an existence argument only. When a largest element is assumed in Argument #2 it leads to a contradiction so there is no largest element. A valid proof by contradiction. Using actual rational numbers shows that Argument #2 is true and Argument #1 is false. But, the question is whether there are any steps in Argument #1 that do not follow logically from prior true statements. In statement 4) in Argument #1 no such elements can be specified. But, does 4) follow logically from 2) and 3).

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