

# A simple claim about teeth, toothpaste, and toothbrushes

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ABSTRACT: I think toothpastes ought to contain abrasives with Mohs hardness strictly between 4 and 5 so that brushing your teeth will get rid of tartar and plaque without damaging your tooth enamel. But as of 2023 toothpaste-makers do not do that.

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Tooth enamel is the hardest substance in the human body. Minerologically it is 96% hydroxyapatite  $\text{Ca}_5(\text{PO}_4)_3(\text{OH})$ , which has Mohs hardness 5 and density 3.16 gram/cc. It can be improved by exposure to fluoride ions in water or toothpaste, causing admixture of some fluorapatite  $\text{Ca}_5(\text{PO}_4)_3\text{F}$  which also is Mohs 5 and density 3.15, but less soluble, more stable, and more resistant to acids.

Undesirable bacterial "plaque" formed on your teeth is made of polysaccharides, glycoproteins, and water (I am guessing Mohs 3-4 and density 1.2 for it), while "tartar," aka "calculus," is made of both inorganic minerals and organic (former plaque) components. The mineral proportion is 40-60%, and mainly consists of these 3 kinds of calcium phosphate minerals:

- whitlockite,  $\text{Ca}_9(\text{Mg,Fe})(\text{PO}_4)_6(\text{PO}_3\text{OH})$ , Mohs 5, density 3.13
- hydroxyapatite,  $\text{Ca}_5(\text{PO}_4)_3\text{OH}$ , Mohs 5, density 3.16
- octacalcium phosphate,  $\text{Ca}_8\text{H}_2(\text{PO}_4)_6 \cdot 5\text{H}_2\text{O} = \text{Ca}_8\text{H}_{12}\text{P}_6\text{O}_{29}$ , [sometimes called "tetracalcium hydrogentriphosphate"; IUPAC name "octacalcium bis(hydrogenphosphate) tetrakis(phosphate) pentahydrate"], Mohs unknown (but presumably below 5, I'd guess 3), density 2.69.

The following 3 minerals have also been detected, but are rarer:

- brushite,  $\text{CaHPO}_4 \cdot 2\text{H}_2\text{O}$ , Mohs 2.5, density 2.33
- calcite,  $\text{CaCO}_3$ , Mohs 3, density 2.71
- monetite,  $\text{CaHPO}_4$ , Mohs 3.5, density 2.92

The "Mohs hardness" scale is based on the empirical claim that substances with greater Mohs value will scratch substances with lesser Mohs, but not the reverse. For a polycrystalline substance made of *multiple* kinds and orientations of crystals (e.g. granite or dental tartar), its *susceptibility* to scratch-damage is largely governed by its *least*-Mohs component, while its ability to *inflict* such damage is largely governed by its *max*-Mohs component.

The **obvious conclusion** suggested by those facts is that your toothbrush bristles, and/or (more likely, and probably more importantly) abrasive particles inside toothpaste, optimally should have Mohs hardness *intermediate* between that of plaque/tartar and tooth enamel, i.e. strictly between Mohs 4 and 5. Then brushing your teeth would grind away plaque and tartar, but not your tooth

enamel.

However, as of year 2023, the makers of commercial toothbrushes and toothpastes appear unacquainted with that – because they simply do not do it! One could speculate whether that is because of (i) stupidity, (ii) an intentional conspiracy to keep dentists and dental hygienists (and themselves?) in business, (iii) some unknown but actually-valid technical reason.

I now list some nontoxic substances, some pretty common, claimed to have Mohs hardnesses in the right range:

**Mohs 4.5-5:** Apophyllite  $(K,Na)Ca_4Si_8O_{20}(F,OH)\cdot 8H_2O$ , Pectolite  $NaCa_2Si_3O_8(OH)$ , Phillipsite  $(Ca,Na_2,K_2)_3Al_6Si_{10}O_{32}\cdot 12H_2O$ , Yugawaralite  $CaAl_2Si_6O_{16}\cdot 4H_2O$ , and Wollastonite  $CaSiO_3$  (\$1.19/lb).

**Mohs 4.5:** Gmelinite  $Na_4(Si_8Al_4)O_{24}\cdot 11H_2O$

**Mohs 4-5:** Iron Fe and soft steels (\$4/lb), Nickel Ni, Purpurite aka manganese phosphate  $MnPO_4$ , Zincite  $ZnO$  (\$6/lb), Mordenite  $(Ca,Na_2,K_2)Al_2Si_{10}O_{24}\cdot 7H_2O$ , Friedelite  $Mn^{2+}_8Si_6O_{15}(OH,Cl)_{10}$ , Chabazite  $(Ca,K_2,Na_2,Mg)_2(Al_2Si_4O_{12})_2\cdot 12H_2O$ , and Augelite  $Al_3(PO_4)(OH)_3$ .

**Mohs 4:** Fluorite  $CaF_2$  (\$6/lb?).

At least several of those are commercially available in powder form in large quantity, indicated by price quotes I found on ads targeted to hobbyists on the internet for 0.1-10 lb quantities.