Non-Equilibrium Dynamics and High Energy Physics

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Abstract

The time dependent Ginzburg Landau equation (TGLE) is a prototype model of non-equilibrium statistical physics and critical phenomena. This brief report points out that, applying TGLE to the chaotic dynamics of interacting fields hints to unexpected solutions to the challenges confronting high-energy theory.

Key words: Time dependent Ginzburg-Landau equation, dynamic critical phenomena, bifurcations, chaotic strings, Standard Model, High energy physics.

1. Brief overview of TGLE

Consider a classical scalar field in 1+1 dimensions evolving in nonequilibrium conditions, which are likely to develop far beyond the

Standard Model (SM) scale. The field dynamics near its critical point is described by the TGLE as in [1]

$$\frac{\partial\varphi(x,t)}{\partial t} = -\frac{\delta H}{\delta\varphi} + h(x,t) + \eta(x,t) \tag{1}$$

in which

$$H[\varphi] = \int d^2x \, [\frac{1}{2}(\frac{\partial\varphi}{\partial x})^2 + V(\varphi)] \tag{2}$$

$V(\varphi)$ is the potential function, $h(x,t)$ the external field and $\eta(x,t)$ a Gaussian white noise source with correlations,

$$\langle \eta(x,t)\eta(x',t') \rangle \propto \delta(x-x')\delta(t-t') \tag{3}$$

In particular, the potential function of self-interacting scalar field theory is given by,

$$V_S(\varphi) = \frac{1}{2} r_0 \varphi^2 + u_0 \varphi^4 \tag{4}$$

The model (1) can be alternatively formulated in terms of a probability distribution $P[\varphi,t]$ via the Fokker-Planck equation,

$$\frac{\partial P}{\partial t}=\int d^2x\frac{\delta}{\delta\varphi}[\frac{\delta P}{\delta\varphi}+P\frac{\delta H}{\delta\varphi}] \tag{5}$$

For time-independent fields, the equilibrium solution of (5) amounts to

$$P_{eq}[\varphi]=\frac{1}{Z}\exp(-H[\varphi]) \tag{6a}$$

in which the functional integral Z takes the form associated with equilibrium field theory, namely,

$$Z=\int D\varphi\exp(-H[\varphi]) \tag{6b}$$

A reasonable assumption is that the equilibrium settings (6a) and (6b) - matching the low-energy regime of field theory - are recovered in the long-time limit, i.e.

$$\lim_{t\to\infty}P[\varphi]=P_{eq}[\varphi] \tag{6c}$$

2. Field bifurcations as analog scenario of the Higgs mechanism

Consider now the case where (1) works with the potential function (4) and where the noise η, external field h and the spatial gradient $\partial\varphi/\partial x$ all vanish away. One obtains the cubic field equation,

$$\frac{\partial\varphi(x,t)}{\partial t}=-\frac{\partial V_S(\varphi)}{\partial\varphi}=-r_0^2\varphi-4\mu_0\varphi^3 \tag{7}$$

It can be shown that (7) represents a dynamic analog of electroweak symmetry breaking using the term-by-term identification [4-5]

$$r_0^2=-2\lambda_H v^2 \tag{8a}$$

$$u_0=\lambda_H \tag{8b}$$

Here, v stands for the vacuum expectation value of the Higgs boson, λ_H is the Higgs self-interaction coupling and time is interpreted as analog of the Renormalization Group scale as in

$$t=\log(\mu/\mu_0) \tag{9}$$

Equations (7) and (8) provide an alternative scenario to the standard Higgs paradigm of the SM, approximately matching its content and predictions. The mass-generating mechanism proposed in [4-5] is driven by a cascade of bifurcations initiated by the running of $\lambda_H = \lambda_H(t)$. The progressive splitting and morphing of the scalar field are displayed below.

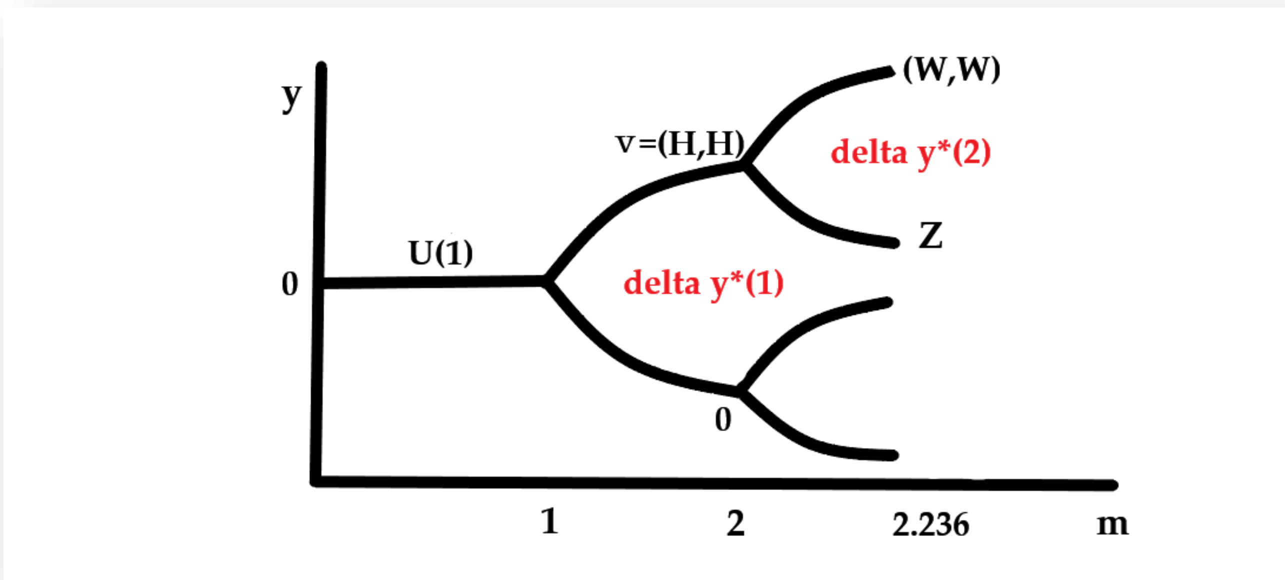

Fig. 1 Bifurcation diagram of electroweak symmetry breaking.

There is an alternative way of interpreting the sequential generation of SM via bifurcations, namely, the quartet of electroweak bosons splits into the gluon octet and the lepton multiplet into the quark multiplet according to the following process:

$$\boxed{\left(\gamma \quad W^- \quad W^+ \quad Z^0\right) \Rightarrow (gluons_{1-8})}$$

$$\boxed{\left(\nu_e \quad \nu_\mu \quad \nu_\tau \quad e \quad \mu \quad \tau\right) + antiparticles \Rightarrow \left(u \quad d \quad c \quad s \quad b \quad t\right)_{r,g} + antiparticles}$$

The flowchart points out that the transition $U(1) \times SU(2) \Rightarrow SU(3)$ is a transformation of a *stable cycle of period 4* in the electroweak sector to a *stable cycle of period 8* in the strong sector.

3. Chaotic Strings and SM parameters

A familiar embodiment of the TGLE equation (1) is the *random walk model*, applicable to a variety of contexts, from classical diffusion and Brownian motion to the Path Integral formulation of quantum theory. The 1+1 diffusion equation reads,

$$\frac{\partial \varphi(x,t)}{\partial t} = \frac{1}{2} \frac{\partial^2 \varphi(x,t)}{\partial x^2} \tag{10}$$

Adding random fluctuations and a deterministic “force” to the diffusive term (10) yields

$$\frac{\partial\varphi(x,t)}{\partial t}=\frac{1}{2}\frac{\partial^2\varphi(x,t)}{\partial x^2}+\frac{\partial V_0(\varphi)}{\partial\varphi}+N(x,t) \tag{11}$$

in which $V_0(\varphi)$ is the deterministic potential and $N(x,t)$ the noise contribution related to (3). Comparative evaluation of (1) and (11) leads to

$$h(x,t)=\frac{\partial^2\varphi(x,t)}{\partial x^2} \tag{12a}$$

$$V(\varphi)=-V_0(\varphi) \tag{12b}$$

Numerical analysis of differential equation (11) requires converting it into a discrete map. To this end, we introduce the parameterization,

$$t=n\tau\,,\ \ x=i\,\delta\,,\ \ n,i=1,2,... \tag{13}$$

which gives

$$\frac{\varphi_{n+1}^i-\varphi_n^i}{\tau}=\frac{\varphi_n^{i+1}-2\varphi_n^i+\varphi_n^{i-1}}{2\delta^2}+\frac{\partial V(\varphi_n^i)}{\partial\varphi}+noise \tag{14}$$

Labeling the coupling constant,

$$\alpha = \frac{\tau}{\delta^2} \tag{15}$$

renders (14) in the form,

$$\varphi_{n+1}^{i} = (1-\alpha)[\varphi_n^i + \frac{\tau}{1-\alpha}\frac{\partial V_0(\varphi_n^i)}{\partial \varphi}] + \frac{\alpha}{2}(\varphi_n^{i+1} + \varphi_n^{i-1}) + \tau \cdot noise \tag{16}$$

It can be shown [2 - 3] that (16) is equivalent to a *coupled map lattice* [6], which is a spatially extended dynamical system governed by the equation,

$$\Phi_{n+1}^{i} = (1-\alpha)T(\Phi_n^i) + \frac{\alpha}{2}(\Phi_n^{i+1} + \Phi_n^{i-1}) + \tau \cdot noise \tag{17}$$

Here, the field Φ is a normalized version of φ at some energy scale Δ given by

$$\varphi_n^i = \Phi_n^i \Delta \tag{18}$$

and the local map T assumes the form,

$$T(\Phi) = \Phi + \frac{\tau}{\Delta^2(1-\alpha)}\frac{\partial V_0(\Delta\Phi)}{\partial\Phi} \tag{19}$$

Drawing from the framework of *stochastic quantization* and *coupled map lattices*, [2] shows that SM parameters can be fixed using a model of self-interacting scalar field theory in 1+1 dimensions. This model is built on the analogy with (17)-(19) and is referred to as describing *chaotic strings* (CS). Specifically, the SM parameters are fixed by a Renormalization flow equation for the coupling constants α having the form,

$$\frac{d\alpha}{dt} = bW(\alpha) + noise \tag{20}$$

$$\frac{d\alpha}{dt} = -b\frac{\partial V_0(\alpha)}{\partial\alpha} + noise \tag{21}$$

where $W(\alpha)$ is the interaction potential of chaotic strings, $V_0(\alpha)$ their expectation potential and $b > 0$ a positive constant.

Tab. 1 lists the values for some of the actual versus predicted SM parameters derived from the CS model.

parameter	chaotic string prediction	measured
$\alpha_{el}(0)$	0.0072979(17)	0.00729735253(3)=1/137.036
$\bar{s}_l^2$	0.23177(7)	0.2318(2)
$\alpha_s(m_Z)$	0.117804(12)	0.1185(20)
m_e	0.5117(8) MeV	0.51099890(2) MeV
m_μ	105.6(3) MeV	105.658357(5) MeV
m_τ	1.782(7) GeV	1.7770(3) GeV
m_{ν_1}	$1.452(3) \cdot 10^{-5}$ eV	?
m_{ν_2}	$2.574(3) \cdot 10^{-3}$ eV	?
m_{ν_3}	$4.92(1) \cdot 10^{-2}$ eV	$\sim 5 \cdot 10^{-2}$ eV ?
m_u	5.07(1) MeV	$\sim$ 5 MeV
m_d	9.35(1) MeV	$\sim$ 9 MeV
m_s	164.4(2) MeV	$\sim$ 170 MeV
m_c	1.259(4) GeV	1.26(3) GeV
m_b	4.22(2) GeV	4.22(4) GeV
m_t	164.5(2) GeV	164(5) GeV
m_W	80.36(2) GeV	80.37(5) GeV
m_H	154.4(5) GeV	?

Tab. 1: Actual vs. predicted SM parameters using the CS model [2].

4. Comparing the Bifurcation Model (BM) to the CS Model

The object of this section is to compare the two approaches discussed in sections 2 and 3. It is instructive to see that, although both theories can be traced to the onset of TGLE at high energies, they differ in many respects.

1) CS incorrectly estimates the mass of the Higgs boson whereas BM asserts that the Higgs mass naturally follows from the bifurcation dynamics of the cubic map.
2) BM makes no reference to quantized vacuum fluctuations, but to the evolution of classical fields.
3) The dynamics of BM is controlled by the flow of the scalar self-coupling (λ_H), whereas in CS by the flow of the coupling between chaotic strings (α).
4) In contrast with BM, CS is based upon stochastic quantization which introduces an unphysical time coordinate.
5) It's unclear how the SM gauge structure, three generations, and chirality of weak interactions develop in CS.
6) BM makes no reference to string theory or to Feynman webs.
7) BM applies to the *start* of the transition to chaos whereas CS to the *end* of this transition corresponding to fully developed chaos.

8) At least in principle, BM can accommodate particle decay channels as resulting from a *random walk* on the bifurcation tree.

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