Spin-statistics Theorem from the Stuart-Landau Equation

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Abstract

Stuart-Landau (SL) equation describes the universal behavior of nonlinear oscillators near a Hopf bifurcation. Focusing on the ultraviolet sector of field theory, the goal of this brief report is to explore the relationship between the SL equation and the spin-statistics theorem of Quantum Field Theory (QFT).

Key words: Spin-statistics theorem, Stuart-Landau equation, fractal spacetime, complex dynamics, Physics Beyond the Standard Model.

As it is known, QFT treats field excitations as point-like particles identified by their quantum properties. For instance, leptons are fermions characterized by mass (m), electric charge (e), spin ($|s|=1/2$), weak hypercharge (Y_W) and lepton number (L). Renormalization Group asserts that particle masses and charges run with the measurement scale (Λ). A standard example of how electron parameters run in Quantum Electrodynamics can be presented as [1]

$$m = m_0[\alpha, \varepsilon, \ln(\Lambda^2/m^2)]^{-1} \tag{1a}$$

$$\alpha = \alpha_0[\alpha, \varepsilon, \ln(\Lambda^2/m^2)]^{-1} \tag{1b}$$

Here, m_0, α_0 are the bare mass and bare fine-structure constant, respectively, whereas $2\varepsilon = 4 - D$ is the deviation from four spacetime dimensions derived from analytic continuation. Consistency requirements mandate that the other parameters listed above stay independent of Λ. It is currently unclear, however, if spin and lepton numbers retain their original meaning and scale invariance at energies far above the Standard Model (SM) range set by the vacuum expectation value of the Higgs boson ($v = 246$ GeV). Based on the conjecture that spacetime dimensionality flows with Λ above the SM scale [2 - 4], the purpose of this brief analysis is to examine a scenario where the spin-statistics theorem is a consequence of *spin bifurcations* triggered by dimensional instabilities of ultraviolet (UV) field theory.

Let's begin by recalling that the contrasting behavior of fermions and bosons is ultimately tied to their disparity in *canonical dimensions* (CD), which reflect the way fields transform under the Lorentz group. In D spacetime dimensions, the CD of scalars and vector bosons reads,

$$[\Phi]=\frac{D-2}{2} \tag{2a}$$

while the fermionic CD is

$$[\Psi]=\frac{D-1}{2} \tag{2b}$$

We next proceed by introducing several working assumptions:

A1) The deep UV sector of field theory ($\Lambda >> v$) is characterized by *large dimensional fluctuations* $0<|\mu|<1$ of frequency $\omega_0 = O(v)$. Fluctuations are assumed to be unobservable if measurements are taken at time intervals,

$$\Delta t = O(1/\omega_0) \gg O(1/\Lambda) \tag{3}$$

It follows from (3) that dimensional fluctuations are observable only on sufficiently high energy scales, far above v. In this regime, a reasonable expectation is that decoherence turns quantum fields into classical fields [4].

A2) The deep UV sector of field theory $\Lambda >> v$ enables spin to run with Λ.

A3) Spin represents a complex valued parameter whose dynamics is well approximated by the SL equation [see e. g. 5]

$$\frac{dz}{dt} = (\mu + i\omega_0)z - uz|z|^2 \tag{4}$$

$$z(t) = x(t) + i\eta(t)\,; \;\; \eta(t) << x(t) \tag{5}$$

in which $t = \log(\Lambda/\Lambda_0)$ stands for the evolution parameter, Λ_0 for a reference scale and u for a real valued coefficient.

At least in principle and under assumptions A1)-A3), the UV regime of field theory allow bosons and fermions to share a continuous range of spin values.

In particular, (2a) and (2b) turn out to be identical if either one of these conditions is met, namely,

$$D+\mu-2=D-\mu-1 \Rightarrow \mu=\tfrac{1}{2} \tag{6a}$$

or

$$D-\mu-2=D+\mu-1 \Rightarrow \mu=-\tfrac{1}{2} \tag{6b}$$

The bifurcation diagram of the real component of (5) is illustrated in Fig. 1, for the case $u=1$ and $D=4$.

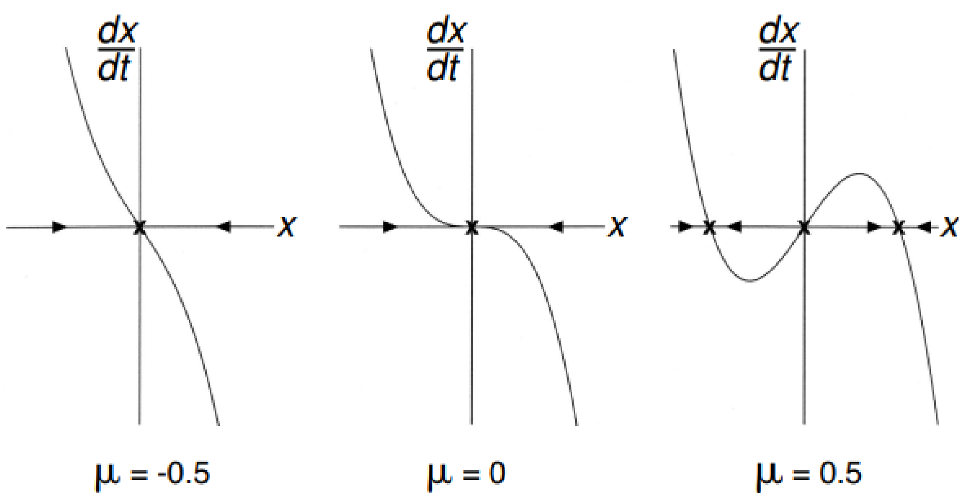

Fig. 1: Bifurcations of the SL model in the $(x, dx/dt)$ plane

It is readily seen that the diagram is partitioned into three spin phases, namely:

1) A stable *scalar phase* develops at $D=4+\mu=4-0.5=3.5$ corresponding to the fixed-point $x=0$. In this phase, bosons and fermions overlap in a spin-zero state.
2) A *continuous spin phase* develops in four dimensional spacetime $D=4$ ($\mu=0$) around the fixed-point $x=0$. <u>This phase automatically segregates boson from fermions, as (2a) and (2b) cannot be simultaneously satisfied if</u> $\mu=0$.
3) A *discrete spin phase* develops at $D=4+0.5=4.5$, which, by (6a), again overlaps bosons and fermions in a state comprising a pair of attractors symmetrically located relative to the repeller $x=0$.

A plausible interpretation of these findings goes as follows:

1) The scalar phase may be associated with the primordial creation of the Higgs field somewhere in the deep UV sector of field theory.

2) The continuous spin phase at $\mu = 0$ motivates the spin-statistics theorem which distinguishes bosons from fermions in $D = 4$ dimensions. By the same token, a continuous spin range may naturally reflect the onset of Dark Matter as topological condensate of continuous dimensions (Cantor Dust) [6 - 7].

3) According to assumption (A1), both scalar and discrete spin phases are unobservable in $D = 4$ dimensions.

A visually suggestive bifurcation diagram of the SL model can be obtained for a system that generalizes (4) to two dimensions (Fig.2). To this end, we set again $u = 1$, $D = 4$ and introduce the additional coordinate y orthogonal to x. This system is described by the differential equations [8 - 9]

$$\frac{dx}{dt} = \mu x - x^3 \tag{7a}$$

$$\frac{dy}{dt} = -y \tag{7b}$$

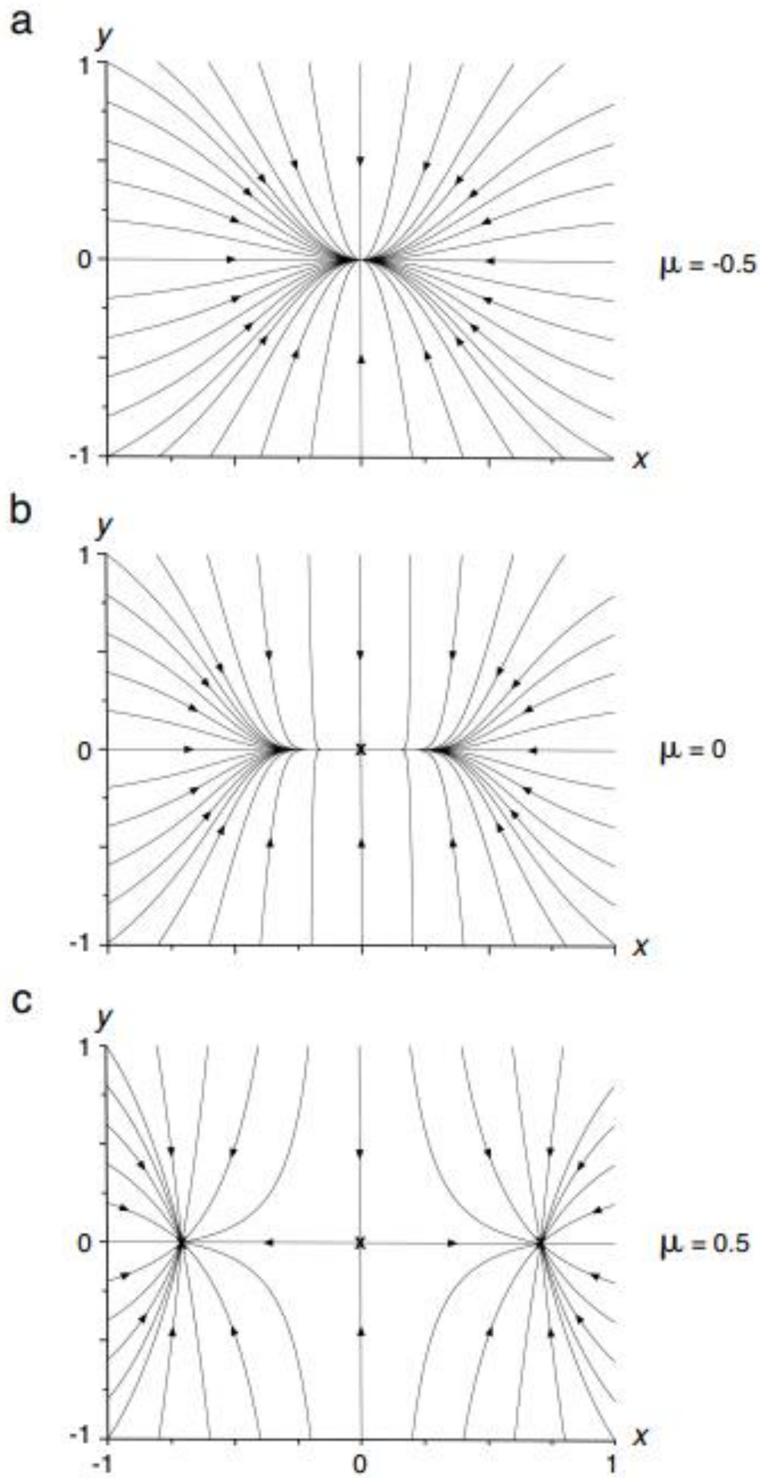

Fig 2: Bifurcations of the SL model in the (x, y) plane

It is again apparent that the spin bifurcation starts to develop in four-dimensional spacetime ($\mu = 0$ and $D = 4$). Iterating the previous arguments, the continuous spin phase shown in Fig. 2b justifies the spin-statistics theorem of QFT and the formation of Dark Matter as topological condensate of continuous dimensions.

References

1. https://arxiv.org/pdf/1901.06573.pdf

2. https://www.researchgate.net/publication/278849474

3. https://www.researchgate.net/publication/344227402

4. https://www.researchgate.net/publication/366580553

5. https://hal.inria.fr/hal-01262570/

6. https://www.researchgate.net/publication/363456790

7. https://www.researchgate.net/publication/358486078

8. Available at http://www.medicine.mcgill.ca/physio/guevaralab/cndsum-bifurcations.pdf

9. Strogatz, S. H., Nonlinear Dynamics and Chaos, Westview Press, 1994.