THE REINTERPRETATION OF THE ELECTROMAGNETIC WAVE EQUATION

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ABSTRACT

This publication contains a mathematical approach for a reinterpretation of the electromagnetic wave equation given a magnetic and electric field density. The basis for this is the essay "The Reinterpretation of the 'Maxwell Equations'" (Martin, 2021). In this paper it is shown that there is a magnetic field density due to the fact that $\operatorname{div}\vec{B}$ is equal to $(\mathrm{Sp})\operatorname{grad}\vec{B}$. The same approach applies to the electric field density. The consequence of this is that both the magnetic field density and the electric field density not only play an important role in the "Maxwell equations", but also in the calculation of the electromagnetic wave equation.

In this publication, the electromagnetic wave equation is calculated with the help of vector calculus. First, the individual components of the magnetic wave and the individual components of the electric wave are derived.

Furthermore, it is shown that the individual components of the two types of waves result in three different directions of movement, which the respective field can theoretically achieve in the direction of propagation. In addition, the Poynting vector shows a longitudinal energy wave in the direction of propagation of the electromagnetic wave, which is suitable for energy transport.

As already mentioned, the calculations made in this elaboration are based on the principles of vector calculation and show a transverse wave component, a longitudinal and a combined wave component of the electromagnetic wave.

1. INTRODUCTION

The German physicist Heinrich Rudolf Hertz (1857 - 1894) succeeded in proving the existence of electromagnetic waves in 1887. The term for electromagnetic waves at the time was radio waves. Hertz experiments suggested that the electromagnetic wave is a transverse wave. Previously, the English mathematician James Clerk Maxwell assumed that electromagnetism must propagate through space in the form of waves.
The Croatian experimenter Nikola Tesla also dealt with the phenomenon of electromagnetic waves. According to Tesla, however, the electromagnetic wave propagates in the longitudinal direction, i.e. as a longitudinal wave in space.
In this paper, the electromagnetic wave equation is analyzed with the help of vector calculation and reinterpreted under the assumption of a magnetic and electric field density. The assumption of these two field densities are based on the paper "The Reinterpretation of the 'Maxwell Equations'" (Martin, 2021).

2. IDEAS AND METHODS

2.1 IDEA FOR REINTERPRETING THE ELECTROMAGNETIC WAVE EQUATION

The basic idea for the reinterpretation of the electromagnetic wave equation is based on the elaboration "The Reinterpretation of the 'Maxwell Equations'" (Martin, 2021). There it is shown that the previous law of induction $\operatorname{rot}\vec{E} = \frac{\delta\vec{B}}{\delta t}$ only works if there is a magnetic field density $\operatorname{div}\vec{B} = \rho_m$. This connection is established via the mathematical principle $(\mathrm{Sp})\operatorname{grad}\vec{B} = \operatorname{div}\vec{B}$, since the terms of $(\mathrm{Sp})\operatorname{grad}\vec{B}$ are the same terms required for $\frac{\delta\vec{B}}{\delta t}$. On the one hand, this means that the law of induction in the undeformed space medium and in the undistorted magnetic field must be expanded to the following form $\operatorname{rot}\vec{E} = \frac{\delta\vec{B}}{\delta t} + \rho_m$ and, on the other hand, that a magnetic and an electric field density must also be taken into account in the electromagnetic wave equation.
The notations of the physical symbols used in this elaboration are shown below. Also here are the basic sets of equations that are needed to reinterpret the wave equation. These come from the elaboration "The Reinterpretation of the 'Maxwell Equations'" (Martin, 2021).

$\vec{E}$ = electric field strength
$\vec{v}$ = velocity
$\vec{B}$ = magnetic flux density
$\vec{H}$ = magnetic field strength
$\vec{D}$ = electric flux density
$\times$ = cross product
$\vec{s}$ = route
$\vec{f}$ = deflection
t = time
c = speed of light
ρ_{el} = electrical space charge density
ρ_m = magnetic space charge density
δ = delta
rot = rotation
div = divergence
grad = gradient

Unipolar induction according to Farady:

$$\vec{E} = \vec{v} \times \vec{B} \tag{2.1.1}$$

Rotation of the electric field:

$$\operatorname{rot} \vec{E} = \operatorname{rot}(\vec{v} \times \vec{B}) \tag{2.1.2}$$

$$\operatorname{rot} \vec{E} = (\operatorname{grad} \vec{v})\ \vec{B} - (\operatorname{grad} \vec{B})\vec{v} + \vec{v}\ \operatorname{div} \vec{B} - \vec{B}\ \operatorname{div}\ \vec{v} \tag{2.1.3}$$

Basic rule of vector calculation (magnetic field):

$$(\mathrm{Sp})(\operatorname{grad} \vec{B}) = \operatorname{div}(\vec{B}) \tag{2.1.4}$$

Unipolar induction for magnetic fields:

$$\vec{H} = -(\vec{v} \times \vec{D}) \tag{2.1.5}$$

Rotation of the magnetic field:

$$\operatorname{rot} \vec{H} = -\operatorname{rot}(\vec{v} \times \vec{D}) \tag{2.1.6}$$

$$\text{rot}\,\vec{H} = -(\text{grad}\,\vec{v})\;\vec{D} + (\text{grad}\,\vec{D})\;\vec{v} - \vec{v}\;\text{div}\;\vec{D} + \vec{D}\;\text{div}\;\vec{v} \tag{2.1.7}$$

Basic rule of vector calculation (electric field):

$$(\text{Sp})(\text{grad}\,\vec{D}) = \text{div}(\vec{D}) \tag{2.1.8}$$

Wave equation from classical mechanics:

$$\frac{\delta^2}{\delta t^2}\cdot\vec{f} = c^2\cdot\frac{\delta^2}{\delta s^2}\cdot\vec{f} \tag{2.1.9}$$

2.2 VECTOR CALCULA BASICS

In order to be able to derive the electromagnetic wave equation from vector calculation, the basics used for this are described in this chapter.

First, three meta-vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are introduced at this point. The three meta-vectors will be used in the following mathematical basic descriptions. In Equation 2.2.1, these three meta-vectors are used to map the cross product.

$$\vec{c} = \vec{a} \times \vec{b} \tag{2.2.1}$$

The rot - operator is applied to Equation 2.2.1 on both sides of the equation. This creates Equation 2.2.2.

$$\text{rot}\,\vec{c} = \text{rot}(\vec{a} \times \vec{b}) \tag{2.2.2}$$

Now the right-hand side of Equation 2.2.2 is rewritten according to the rules of vector calculation. Equation 2.2.3 results from this.

$$\text{rot}\,\vec{c} = \text{rot}(\vec{a}\times\vec{b}) = (\text{grad}\,\vec{a})\;\vec{b} - (\text{grad}\,\vec{b})\;\vec{a} + \vec{a}\;\text{div}\,\vec{b} - \vec{b}\;\text{div}\,\vec{a} \tag{2.2.3}$$

On the right-hand side of Equation 2.2.3, two vector gradients (grad) are created, each of which forms a matrix and two vector divergences (div).

If a minus sign is now applied to all sides of Equation 2.2.3, Equation 2.2.3 changes to Equation 2.2.4.

$$-\text{rot}\,\vec{c} \;=\; -\,\text{rot}\left(\vec{a}\times\vec{b}\right) \;=\; -\left(\text{grad}\,\vec{a}\right)\,\vec{b} \;+\; \left(\text{grad}\,\vec{b}\right)\,\vec{a} \;-\; \vec{a}\;\text{div}\,\vec{b} \;+\; \vec{b}\;\text{div}\,\vec{a} \qquad (2.2.4)$$

In the following, the two Equations 2.2.3 and 2.2.4 are calculated a second time with the rotation operator (rot). The two Equations 2.2.5 and 2.2.6 arise.

$$\text{rot rot}\;\vec{c} \;=\; \text{rot rot}\left(\vec{a}\times\vec{b}\right) \;=\; \text{grad div}\;\vec{c} \;-\; \text{div grad}\;\vec{c} \;=\; \text{grad div}\;\left(\vec{a}\times\vec{b}\right) \;-\; \text{div grad}\;\left(\vec{a}\times\vec{b}\right) \qquad (2.2.5)$$

$$-\text{rot rot}\;\vec{c} \;=\; -\text{rot rot}\left(\vec{a}\times\vec{b}\right) \;=\; -\text{grad div}\;\vec{c} \;+\; \text{div grad}\;\vec{c} \;=\; -\text{grad div}\;\left(\vec{a}\times\vec{b}\right) \;+\; \text{div grad}\;\left(\vec{a}\times\vec{b}\right) \qquad (2.2.6)$$

If the last term of each of the two Equations 2.2.5 and 2.2.6 is rewritten with the help of the La-Place-operator, Equations 2.2.7 and 2.2.8 result.

$$\text{rot rot}\;\vec{c} \;=\; \text{grad div}\;\vec{c} \;-\; \text{div grad}\;\vec{c} \;=\; \text{grad div}\;\vec{c} \;-\; \Delta\,\vec{c} \qquad (2.2.7)$$

$$-\text{rot rot}\;\vec{c} \;=\; -\text{grad div}\;\vec{c} \;+\; \text{div grad}\;\vec{c} \;=\; -\text{grad div}\;\vec{c} \;+\; \Delta\,\vec{c} \qquad (2.2.8)$$

If Equations 2.2.7 and 2.2.8 are now rearranged, Equation 2.2.9 results.

$$\Delta\vec{c} \;=\; \text{grad div}\;\vec{c} \;-\; \text{rot rot}\;\vec{c} \qquad (2.2.9)$$

2.3 DERIVATION OF THE ELECTROMAGNETIC WAVE EQUATION

The rot - operator is applied to Equation 2.1.2 and Equation 2.1.6 according to the calculation rules from Equation 2.2.5 and 2.2.6. Taking Equations 2.2.7 and 2.2.8 into account, the expressions from Equations 2.3.3, 2.3.4, 2.3.5 and 2.3.6 arise.

$$\text{rot}\,\vec{E} \;=\; \text{rot}\left(\vec{v}\times\vec{B}\right) \qquad (2.1.2)$$

$$\text{rot rot}\;\vec{E} \;=\; \text{grad div}\;\vec{E} \;-\; \text{div grad}\;\vec{E} \;=\; \text{grad div}\;\left(\vec{v}\times\vec{B}\right) \;-\; \text{div grad}\;\left(\vec{v}\times\vec{B}\right) \qquad (2.3.3)$$

$$\text{rot rot}\;\vec{E} \;=\; \text{grad div}\;\vec{E} \;-\; \text{div grad}\;\vec{E} \qquad (2.3.4)$$

$$\text{rot}\,\vec{H} \;=\; -\text{rot}\left(\vec{v}\times\vec{D}\right) \qquad (2.1.6)$$

$$\text{rot rot}\;\vec{H} \;=\; -\text{grad div}\;\vec{H} \;+\; \text{div grad}\;\vec{H} \;=\; -\text{grad div}\;\left(\vec{v}\times\vec{B}\right) \;+\; \text{div grad}\;\left(\vec{v}\times\vec{B}\right) \qquad (2.3.5)$$

$$\text{rot rot } \vec{H} = \text{-grad div } \vec{H} + \text{div grad } \vec{H} \qquad (2.3.6)$$

Die Gleichung 2.3.4 bildet die elektrische Wellengleichung ab. Demnach zeigt die Gleichung 2.3.6 die magnetische Wellengleichung.
In einem nächsten Schritt werden nun die einzelnen Terme aus den Gleichungen 2.3.4 und 2.3.6, im Detail berechnet und analysiert.

2.4 THE ELECTRICAL WAVE EQUATION

In order to be able to understand the calculations for the electric and later also for the magnetic wave equation, the following descriptions first deal with the basics of electromagnetic waves. Then the mathematical derivation of the electric wave is discussed and finally the individual types of electric waves are derived.

2.4.1 FUNDAMENTALS OF THE ELECTROMAGNETIC WAVE EQUATION

First, the electromagnetic wave equation is mapped, this is referred to below as Equation 2.4.3 and 2.4.4 and calculated taking into account Equations 2.4.1 and 2.4.2.

Gaussian law:

$$\text{div} \vec{D} = \rho_{el} \qquad (2.4.1)$$

Dirac's law:

$$\text{div} \vec{B} = \rho_m \qquad (2.4.2)$$

Simplified electric wave equation:

$$\Delta \vec{E} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{E}}{\delta t^2} \qquad (2.4.3)$$

Simplified magnetic wave equation:

$$\Delta \vec{H} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{H}}{\delta t^2} \qquad (2.4.4)$$

2.4.2 MATHEMATICAL DERIVATION OF THE ELECTRICAL WAVE EQUATION

In this chapter, Equation 2.4.3 is derived mathematically from Equation 2.3.4. The derivation is based on the physical assumption that there is an electric field density. Equations 2.4.1 is the mathematical-physical expression for this. The result of this is that both the gradients occurring in the equations and divergences have an influence on the overall result.
First, at this point, the first term from Equation 2.3.4 is examined. This is shown in Equation 2.4.2.1. In Equation 2.4.2.1, the vector $\vec{E}$ is rewritten into its component notation.

$$\text{rot rot } \vec{E} = \text{rot rot} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \qquad (2.4.2.1)$$

Next, the first rot - arithmetic-operation is also written in its component notation in Equation 2.4.2.2. This shows that the individual components of vector $\vec{E}$, namely E_x , E_y and E_z , are offset against the individual components of vector $\vec{\nabla}$, namely $\frac{\delta}{\delta x}$, $\frac{\delta}{\delta y}$ and $\frac{\delta}{\delta z}$ in the cross product.

$$\text{rot rot } \vec{E} = \text{rot } \left(\begin{pmatrix} \frac{\delta}{\delta x} \\ \frac{\delta}{\delta y} \\ \frac{\delta}{\delta z} \end{pmatrix} \times \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \right) \qquad (2.4.2.2)$$

The cross product from Equation 2.4.2.2 has been rewritten into summation form in Equation 2.4.2.3.

$$\text{rot rot } \vec{E} = \text{rot } \left(\begin{pmatrix} \frac{\delta E_z}{\delta y} - \frac{\delta E_y}{\delta z} \\ \frac{\delta E_x}{\delta z} - \frac{\delta E_z}{\delta x} \\ \frac{\delta E_y}{\delta x} - \frac{\delta E_x}{\delta y} \end{pmatrix} \right) \qquad (2.4.2.3)$$

In the next step, the above rot - operator is rewritten in component notation so that the individual components of the second $\vec{\nabla}$ - operator, namely $\frac{\delta}{\delta x}$, $\frac{\delta}{\delta y}$ and $\frac{\delta}{\delta z}$ in the cross product, can be calculated with the rest of the right-hand side of Equation 2.4.2.3. This is how Equation 2.4.2.4 is created.

$$\text{rot rot } \vec{E} = \left(\begin{pmatrix} \frac{\delta}{\delta x} \\ \frac{\delta}{\delta y} \\ \frac{\delta}{\delta z} \end{pmatrix} \times \begin{pmatrix} \frac{\delta E_z}{\delta y} - \frac{\delta E_y}{\delta z} \\ \frac{\delta E_x}{\delta z} - \frac{\delta E_z}{\delta x} \\ \frac{\delta E_y}{\delta x} - \frac{\delta E_x}{\delta y} \end{pmatrix} \right) \qquad (2.4.2.4)$$

After the individual components of the $\vec{\nabla}$ - operator, namely $\frac{\delta}{\delta x}$, $\frac{\delta}{\delta y}$ and $\frac{\delta}{\delta z}$ in the cross product, have been calculated with the remainder of the right-hand side of Equation 2.4.2.4, Equation 2.4.2.5 follows.

$$\text{rot rot } \vec{E} = \begin{pmatrix} \delta\frac{(\frac{\delta E_y}{\delta x} - \frac{\delta E_x}{\delta y})}{\delta y} - \delta\frac{(\frac{\delta E_x}{\delta z} - \frac{\delta E_z}{\delta x})}{\delta z} \\ \delta\frac{(\frac{\delta E_z}{\delta y} - \frac{\delta E_y}{\delta z})}{\delta z} - \delta\frac{(\frac{\delta E_y}{\delta x} - \frac{\delta E_x}{\delta y})}{\delta x} \\ \delta\frac{(\frac{\delta E_x}{\delta z} - \frac{\delta E_z}{\delta x})}{\delta x} - \delta\frac{(\frac{\delta E_z}{\delta y} - \frac{\delta E_y}{\delta z})}{\delta y} \end{pmatrix} \qquad (2.4.2.5)$$

Equation 2.4.2.5 is now simplified to Equation 2.4.2.6. In Equation 2.4.2.6, a notation was chosen for the double directional derivative that is clear and therefore easy to understand. This is useful because in the case of field sizes that do not change over time, but change in space, it doesn't matter which direction is derived first.

$$\text{rot rot } \vec{E} = \begin{pmatrix} \frac{\delta^2 E_y}{\delta x \delta y} - \frac{\delta^2 E_x}{\delta y \delta y} - \frac{\delta^2 E_x}{\delta z \delta z} + \frac{\delta^2 E_z}{\delta x \delta z} \\ \frac{\delta^2 E_z}{\delta y \delta z} - \frac{\delta^2 E_y}{\delta z \delta z} - \frac{\delta^2 E_y}{\delta x \delta x} + \frac{\delta^2 E_x}{\delta y \delta x} \\ \frac{\delta^2 E_x}{\delta z \delta x} - \frac{\delta^2 E_z}{\delta x \delta x} - \frac{\delta^2 E_z}{\delta y \delta y} + \frac{\delta^2 E_y}{\delta z \delta y} \end{pmatrix} \qquad (2.4.2.6)$$

Equation 2.4.2.6 shows that each term of the matrix represents a double directional derivative. If Equation 2.4.2.6 is now simplified in the form that two different directional derivations are shown separately and two directional derivations in the same direction are combined, Equation 2.4.2.7 results.

$$\text{rot rot } \vec{E} = \begin{pmatrix} \frac{\delta^2 E_y}{\delta x \delta y} - \frac{\delta^2 E_x}{\delta y^2} - \frac{\delta^2 E_x}{\delta z^2} + \frac{\delta^2 E_z}{\delta x \delta z} \\ \frac{\delta^2 E_z}{\delta y \delta z} - \frac{\delta^2 E_y}{\delta z^2} - \frac{\delta^2 E_y}{\delta x^2} + \frac{\delta^2 E_x}{\delta y \delta x} \\ \frac{\delta^2 E_x}{\delta z \delta x} - \frac{\delta^2 E_z}{\delta x^2} - \frac{\delta^2 E_z}{\delta y^2} + \frac{\delta^2 E_y}{\delta z \delta y} \end{pmatrix} \qquad (2.4.2.7)$$

Equation 2.4.2.7 was used to mathematically derive the first term from Equation 2.3.4. In the next step, the second term from Equation 2.3.4 is examined. This is shown in Equation 2.4.2.8. On the right side of Equation 2.4.2.8 the vector $\vec{E}$ is shown in component notation.

$$\text{grad div } \vec{E} = \text{grad div} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \qquad (2.4.2.8)$$

First of all, it can be seen that the divergence of the electric field vector $\text{div}(\vec{E})$ must be determined in Equation 2.4.2.8. In the paper "The Reinterpretation of the 'Maxwell Equations'" (Martin, 2021) it was already shown why these must have a value from a mathematical point of view. This is in a direct connection via the mathematical expression $(\text{Sp})(\text{grad}\,\vec{E}) = \text{div}(\vec{E})$. The trace from the matrix of the electric field gradient $(\text{Sp})(\text{grad}\,\vec{E})$ is composed of the values $\frac{\delta E_x}{\delta x}$, $\frac{\delta E_y}{\delta y}$ and $\frac{\delta E_z}{\delta z}$. These become $\frac{\delta \vec{E}}{\delta t}$ when offset against the velocity vector $\frac{\delta \vec{s}}{\delta t}$. This means that if $\text{div}(\vec{E})$ has no value, $\frac{\delta \vec{E}}{\delta t}$ would have no value either.

The electric wave is therefore a wave which, according to this interpretation, is also based on density states. The different density states result in potential differences in the electric field, from which the direction and length of the electric field pointer follow.

Next, from Equation 2.4.2.8, the div arithmetic operation is applied to the individual components of the vector $\vec{E}$, namely E_x, E_y and E_z. To do this, the individual components of the $\vec{\nabla}$ - operator, namely $\frac{\delta}{\delta x}$, $\frac{\delta}{\delta y}$ and $\frac{\delta}{\delta z}$, are calculated in the form shown in Equation 2.4.2.9.

$$\text{grad div } \vec{E} = \text{grad} \left(\frac{\delta E_x}{\delta x} + \frac{\delta E_y}{\delta y} + \frac{\delta E_z}{\delta z} \right) \tag{2.4.2.9}$$

In the next step, the grad arithmetic operation is performed on the right-hand side of Equation 2.4.2.9. To do this, the individual components of the $\vec{\nabla}$ - operator, namely $\frac{\delta}{\delta x}$, $\frac{\delta}{\delta y}$ and $\frac{\delta}{\delta z}$, are calculated with the expression $\left(\frac{\delta E_x}{\delta x} + \frac{\delta E_y}{\delta y} + \frac{\delta E_z}{\delta z} \right)$ as shown in Equation 2.4.2.10.

$$\text{grad div } \vec{E} = \begin{pmatrix} \delta \dfrac{\left(\dfrac{\delta E_x}{\delta x} + \dfrac{\delta E_y}{\delta y} + \dfrac{\delta E_z}{\delta z}\right)}{\delta x} \\ \delta \dfrac{\left(\dfrac{\delta E_x}{\delta x} + \dfrac{\delta E_y}{\delta y} + \dfrac{\delta E_z}{\delta z}\right)}{\delta y} \\ \delta \dfrac{\left(\dfrac{\delta E_x}{\delta x} + \dfrac{\delta E_y}{\delta y} + \dfrac{\delta E_z}{\delta z}\right)}{\delta z} \end{pmatrix} \tag{2.4.2.10}$$

Now the right-hand side of Equation 2.4.2.10 is simplified for the first time to the form shown in Equation 2.4.2.11. For the purpose of standardization, the same notation was chosen for this as was used to derive the first term from Equation 2.3.4.

$$\text{grad div } \vec{E} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta x \delta x} + \frac{\delta^2 E_y}{\delta y \delta x} + \frac{\delta^2 E_z}{\delta z \delta x} \\ \frac{\delta^2 E_x}{\delta x \delta y} + \frac{\delta^2 E_y}{\delta y \delta y} + \frac{\delta^2 E_z}{\delta z \delta y} \\ \frac{\delta^2 E_x}{\delta x \delta z} + \frac{\delta^2 E_y}{\delta y \delta z} + \frac{\delta^2 E_z}{\delta z \delta z} \end{pmatrix} \quad (2.4.2.11)$$

Finally, Equation 2.4.2.11 is simplified once more. Equation 2.4.2.12 results from this.

$$\text{grad div } \vec{E} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + \frac{\delta^2 E_y}{\delta y \delta x} + \frac{\delta^2 E_z}{\delta z \delta x} \\ \frac{\delta^2 E_x}{\delta x \delta y} + \frac{\delta^2 E_y}{\delta y^2} + \frac{\delta^2 E_z}{\delta z \delta y} \\ \frac{\delta^2 E_x}{\delta x \delta z} + \frac{\delta^2 E_y}{\delta y \delta z} + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} \quad (2.4.2.12)$$

Equation 2.4.2.12 was used to derive the second term from Equation 2.3.4. At this point, finally, the third term from Equation 2.3.4 is examined. This is shown in Equation 2.4.2.13. Here, too, the component notation for the vector $\vec{E}$ was chosen on the right-hand side of the equation.

$$\text{div grad } \vec{E} = \text{div grad} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \quad (2.4.2.13)$$

First the grad - operation is applied to the vector $\vec{E}$. To do this, the individual elements of the $\vec{\nabla}$ - operator are calculated in the form with the individual elements of the vector $\vec{E}$, which is shown in Equation 2.4.2.14.

$$\text{div grad } \vec{E} = \text{div} \begin{pmatrix} \frac{\delta E_x}{\delta x} & \frac{\delta E_x}{\delta y} & \frac{\delta E_x}{\delta z} \\ \frac{\delta E_y}{\delta x} & \frac{\delta E_y}{\delta y} & \frac{\delta E_y}{\delta z} \\ \frac{\delta E_z}{\delta x} & \frac{\delta E_z}{\delta y} & \frac{\delta E_z}{\delta z} \end{pmatrix} \quad (2.4.2.14)$$

Now the div - operation is calculated in the right part of Equation 2.4.2.14. To do this, the individual components of the $\vec{\nabla}$ - operator are calculated using the matrix in the right-hand part of Equation 2.4.2.14. This results in the Equation 2.4.2.15.

$$\text{div grad } \vec{E} = \begin{pmatrix} \frac{\delta(\frac{\delta E_x}{\delta x})}{\delta x} + \frac{\delta(\frac{\delta E_x}{\delta y})}{\delta y} + \frac{\delta(\frac{\delta E_x}{\delta z})}{\delta z} \\ \frac{\delta(\frac{\delta E_y}{\delta x})}{\delta x} + \frac{\delta(\frac{\delta E_y}{\delta y})}{\delta y} + \frac{\delta(\frac{\delta E_y}{\delta z})}{\delta z} \\ \frac{\delta(\frac{\delta E_z}{\delta x})}{\delta x} + \frac{\delta(\frac{\delta E_z}{\delta y})}{\delta y} + \frac{\delta(\frac{\delta E_z}{\delta z})}{\delta z} \end{pmatrix} \qquad (2.4.2.15)$$

Equation 2.4.2.15 can be simplified to Equation 2.4.2.16. Both equations consist of terms that represent a double directional derivative in the same direction.

$$\text{div grad } \vec{E} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + \frac{\delta^2 E_x}{\delta y^2} + \frac{\delta^2 E_x}{\delta z^2} \\ \frac{\delta^2 E_y}{\delta x^2} + \frac{\delta^2 E_y}{\delta y^2} + \frac{\delta^2 E_y}{\delta z^2} \\ \frac{\delta^2 E_z}{\delta x^2} + \frac{\delta^2 E_z}{\delta y^2} + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} \qquad (2.4.2.16)$$

Equation 2.4.2.16 was used to derive the third term from Equation 2.3.4.
In a final step, the results from Equations 2.4.2.7, 2.4.2.12 and 2.4.2.16 are inserted into Equation 2.3.4. Equation 2.4.2.17 arises.

$$\text{rot rot } \vec{E} \quad = \quad \text{grad div } \vec{E} \quad - \quad \text{div grad } \vec{E} \qquad (2.3.4)$$

$$\begin{pmatrix} \frac{\delta^2 E_y}{\delta x \delta y} - \frac{\delta^2 E_x}{\delta y^2} - \frac{\delta^2 E_x}{\delta z^2} + \frac{\delta^2 E_z}{\delta x \delta z} \\ \frac{\delta^2 E_z}{\delta y \delta z} - \frac{\delta^2 E_y}{\delta z^2} - \frac{\delta^2 E_y}{\delta x^2} + \frac{\delta^2 E_x}{\delta y \delta x} \\ \frac{\delta^2 E_x}{\delta z \delta x} - \frac{\delta^2 E_z}{\delta x^2} - \frac{\delta^2 E_z}{\delta y^2} + \frac{\delta^2 E_y}{\delta z \delta y} \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + \frac{\delta^2 E_y}{\delta y \delta x} + \frac{\delta^2 E_z}{\delta z \delta x} \\ \frac{\delta^2 E_x}{\delta x \delta y} + \frac{\delta^2 E_y}{\delta y^2} + \frac{\delta^2 E_z}{\delta z \delta y} \\ \frac{\delta^2 E_x}{\delta x \delta z} + \frac{\delta^2 E_y}{\delta y \delta z} + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} - \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + \frac{\delta^2 E_x}{\delta y^2} + \frac{\delta^2 E_x}{\delta z^2} \\ \frac{\delta^2 E_y}{\delta x^2} + \frac{\delta^2 E_y}{\delta y^2} + \frac{\delta^2 E_y}{\delta z^2} \\ \frac{\delta^2 E_z}{\delta x^2} + \frac{\delta^2 E_z}{\delta y^2} + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} \qquad (2.4.2.17)$$

Equations 2.3.4 and 2.4.2.17 are the basis for all further calculations in this paper.

2.4.3 THE DERIVATION OF THE HERTZ WAVE (ELECTRICAL TRANSVERSAL WAVE)

With Equation 2.4.2.17, a statement about the nature of the electromagnetic wave can now be made. Equation 2.4.2.17 shows that there are three different elements that play a role in the interpretation of an electric wave. There are the transverse elements ($\frac{\delta^2 E_x}{\delta y^2}$, $\frac{\delta^2 E_x}{\delta z^2}$, $\frac{\delta^2 E_y}{\delta x^2}$, $\frac{\delta^2 E_y}{\delta z^2}$, $\frac{\delta^2 E_z}{\delta x^2}$, $\frac{\delta^2 E_z}{\delta y^2}$), the longitudinal elements ($\frac{\delta^2 E_x}{\delta x^2}$, $\frac{\delta^2 E_y}{\delta y^2}$, $\frac{\delta^2 E_z}{\delta z^2}$) and a combination of these two elements ($\frac{\delta^2 E_y}{\delta x \delta y}$, $\frac{\delta^2 E_z}{\delta x \delta z}$, $\frac{\delta^2 E_z}{\delta y \delta z}$, $\frac{\delta^2 E_x}{\delta x \delta y}$, $\frac{\delta^2 E_x}{\delta x \delta z}$, $\frac{\delta^2 E_y}{\delta y \delta z}$).

In order to do justice to the current interpretation of the electromagnetic wave, in relation to Equation 2.4.2.17, the following two assumptions must be made. On the one hand there must be no longitudinal parts and on the other hand there must be no combination of longitudinal wave part and transversal wave part. From this it follows that only the transverse components from Equation 2.4.2.17 can be considered as a basis for an interpretation of the electromagnetic wave in order to ultimately derive a Hertzian wave. This fact is shown in Equation 2.4.3.1. Equation 2.3.4 is used here for better orientation with Equation 2.4.3.1.

$$\text{rot rot } \vec{E} \quad = \quad \text{grad div } \vec{E} \quad - \quad \text{div grad } \vec{E} \qquad (2.3.4)$$

$$\begin{pmatrix} 0 - \frac{\delta^2 E_x}{\delta y^2} - \frac{\delta^2 E_x}{\delta z^2} + 0 \\ 0 - \frac{\delta^2 E_y}{\delta z^2} - \frac{\delta^2 E_y}{\delta x^2} + 0 \\ 0 - \frac{\delta^2 E_z}{\delta x^2} - \frac{\delta^2 E_z}{\delta y^2} + 0 \end{pmatrix} = \begin{pmatrix} 0 + 0 + 0 \\ 0 + 0 + 0 \\ 0 + 0 + 0 \end{pmatrix} - \begin{pmatrix} 0 + \frac{\delta^2 E_x}{\delta y^2} + \frac{\delta^2 E_x}{\delta z^2} \\ \frac{\delta^2 E_y}{\delta x^2} + 0 + \frac{\delta^2 E_y}{\delta z^2} \\ \frac{\delta^2 E_z}{\delta x^2} + \frac{\delta^2 E_z}{\delta y^2} + 0 \end{pmatrix} \qquad (2.4.3.1)$$

In a three-dimensional coordinate system with the coordinates x, y and z, Equation 2.4.3.1 fulfills the physical assumption that there is no wave component in the direction of propagation of the electric wave (longitudinal wave). This raises the question of how the wave actually moves in the direction of propagation, since there is only a laterally oscillating part of the wave? There is also no physically plausible explanation for the case in which the electric wave propagates in a vacuum. In order to be able to derive Equation 2.4.3 from Equation 2.4.3.1, the meta-vector $\vec{c}$ in Equation 2.2.7 is first replaced by the E-field vector $\vec{E}$ at this point. This creates Equation 2.4.3.2. In Equation 2.4.3, c^2 is the square of the speed of light.

$$\Delta \vec{E} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{E}}{\delta t^2} \tag{2.4.3}$$

$$\text{rot rot } \vec{c} = \text{grad div } \vec{c} - \text{div grad } \vec{c} = \text{grad div } \vec{c} - \Delta \vec{c} \tag{2.2.7}$$

$$\text{rot rot } \vec{E} = \text{grad div } \vec{E} - \text{div grad } \vec{E} = \text{grad div } \vec{E} - \Delta \vec{E} \tag{2.4.3.2}$$

Starting from Equation 2.4.3.2, Equation 2.4.3.3 can be described under the conditions from Equation 2.4.3.1, since term $\text{grad div}\, \vec{E}$ was set to zero there.

$$\text{rot rot } \vec{E} = \text{div grad } \vec{E} = \Delta \vec{E} \tag{2.4.3.3}$$

If the mathematical-physical expressions from Equation 2.4.3.1 are now inserted into Equation 2.4.3.3, Equation 2.4.3.4 results.

$$\Delta \vec{E} = \begin{pmatrix} 0 + \dfrac{\delta^2 E_x}{\delta y^2} + \dfrac{\delta^2 E_x}{\delta z^2} \\ \dfrac{\delta^2 E_y}{\delta x^2} + 0 + \dfrac{\delta^2 E_y}{\delta z^2} \\ \dfrac{\delta^2 E_z}{\delta x^2} + \dfrac{\delta^2 E_z}{\delta y^2} + 0 \end{pmatrix} \tag{2.4.3.4}$$

If the individual transversal parts of the vectorial components from Equation 2.4.3.4 are added and combined to $\dfrac{\delta^2 E_x}{\delta s_x^2}$, $\dfrac{\delta^2 E_y}{\delta s_y^2}$ and $\dfrac{\delta^2 E_z}{\delta s_z^2}$, Equation 2.4.3.5 results.

$$\Delta \vec{E} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta s_x^2} \\ \frac{\delta^2 E_y}{\delta s_y^2} \\ \frac{\delta^2 E_z}{\delta s_z^2} \end{pmatrix} = \begin{pmatrix} 0 + \frac{\delta^2 E_x}{\delta y^2} + \frac{\delta^2 E_x}{\delta z^2} \\ \frac{\delta^2 E_y}{\delta x^2} + 0 + \frac{\delta^2 E_y}{\delta z^2} \\ \frac{\delta^2 E_z}{\delta x^2} + \frac{\delta^2 E_z}{\delta y^2} + 0 \end{pmatrix} \tag{2.4.3.5}$$

In order to derive Equation 2.4.3 from Equation 2.4.3.5, the speed of light c must first be defined. Since the speed of light is a velocity, we can write it as a velocity vector, in Equation 2.4.3.6.

$$\vec{c} = \begin{pmatrix} \frac{\delta s_x}{\delta t} \\ \frac{\delta s_y}{\delta t} \\ \frac{\delta s_z}{\delta t} \end{pmatrix} \tag{2.4.3.6}$$

The speed of light c is currently defined in physics independently of the moving starting point and is therefore the same in all three spatial directions within a medium. This assumption becomes problematic when the electromagnetic wave propagates through a transition between two substances. At this point, however, a mathematical derivation of this problem is dispensed with, since this exceeds the objective of the scope of this elaboration. Here reference is only made to the substance in the vacuum.

The assumption that the speed of light is the same in all three spatial directions means that it can also be equated in all three spatial directions. Equation 2.4.3.7 follows from this.

$$\vec{c} = \begin{pmatrix} \frac{\delta s_x}{\delta t} \\ \frac{\delta s_y}{\delta t} \\ \frac{\delta s_z}{\delta t} \end{pmatrix} = \begin{pmatrix} \frac{\delta s}{\delta t} \\ \frac{\delta s}{\delta t} \\ \frac{\delta s}{\delta t} \end{pmatrix} \tag{2.4.3.7}$$

If the speed of light $\vec{c}$ is the same in all spatial directions, as described in Equation 2.4.3.7, it can also be assumed to be a constant. Equation 2.4.3.8 follows from this.

$$c = \frac{\delta s}{\delta t} \tag{2.4.3.8}$$

If the speed of light is now squared, the expression from Equation 2.4.3.9 results.

$$c^2 = \left(\frac{\delta s}{\delta t}\right) \cdot \left(\frac{\delta s}{\delta t}\right) = \frac{(\delta s)^2}{(\delta t)^2} = \frac{\delta s^2}{\delta t^2} \tag{2.4.3.9}$$

The notation from Equation 2.4.3.9 was chosen to prevent misunderstandings regarding a double derivation. Only the square of a derivative is described here.

If $\frac{c^2}{c^2}$ is now inserted into Equation 2.4.3.5, Equation 2.4.3.10 results. Since the speed of light c was defined as a constant, this can also be used in Equation 2.4.3.10 by offsetting the individual components there with $\frac{c^2}{c^2}$.

$$\Delta \vec{E} = \frac{c^2}{c^2} \cdot \begin{pmatrix} \frac{\delta^2 E_x}{\delta s_x^2} \\ \frac{\delta^2 E_y}{\delta s_y^2} \\ \frac{\delta^2 E_z}{\delta s_z^2} \end{pmatrix} = \frac{\frac{\delta s^2}{\delta t^2}}{\frac{\delta s^2}{\delta t^2}} \cdot \begin{pmatrix} \frac{\delta^2 E_x}{\delta s_x^2} \\ \frac{\delta^2 E_y}{\delta s_y^2} \\ \frac{\delta^2 E_z}{\delta s_z^2} \end{pmatrix} = \begin{pmatrix} \frac{\frac{\delta s^2}{\delta t^2}}{\frac{\delta s^2}{\delta t^2}} \cdot \frac{\delta^2 E_x}{\delta s_x^2} \\ \frac{\frac{\delta s^2}{\delta t^2}}{\frac{\delta s^2}{\delta t^2}} \cdot \frac{\delta^2 E_y}{\delta s_y^2} \\ \frac{\frac{\delta s^2}{\delta t^2}}{\frac{\delta s^2}{\delta t^2}} \cdot \frac{\delta^2 E_z}{\delta s_z^2} \end{pmatrix} \tag{2.4.3.10}$$

If the expressions δs_x^2 , δs_y^2 and δs_z^2 are now equated with δs^2 , they can be shortened against each other. However, setting these terms equal requires an adjustment of δs in the numerator and denominator of the individual components of $\Delta \vec{E}$. When this is done, Equation 2.4.3.11 results.

$$\Delta \vec{E} = \begin{pmatrix} \dfrac{\frac{\delta s^2}{\delta t^2}}{\frac{\delta s^2}{\delta t^2}} \cdot \dfrac{\delta^2 E_x}{\delta s_x^2} \\ \dfrac{\frac{\delta s^2}{\delta t^2}}{\frac{\delta s^2}{\delta t^2}} \cdot \dfrac{\delta^2 E_y}{\delta s_y^2} \\ \dfrac{\frac{\delta s^2}{\delta t^2}}{\frac{\delta s^2}{\delta t^2}} \cdot \dfrac{\delta^2 E_z}{\delta s_z^2} \end{pmatrix} = \begin{pmatrix} \dfrac{\frac{1}{\delta t^2}}{\frac{\delta s^2}{\delta t^2}} \cdot \dfrac{\delta^2 E_x}{1} \\ \dfrac{\frac{1}{\delta t^2}}{\frac{\delta s^2}{\delta t^2}} \cdot \dfrac{\delta^2 E_y}{1} \\ \dfrac{\frac{1}{\delta t^2}}{\frac{\delta s^2}{\delta t^2}} \cdot \dfrac{\delta^2 E_z}{1} \end{pmatrix} = \begin{pmatrix} \dfrac{1}{\frac{\delta s^2}{\delta t^2}} \cdot \dfrac{\delta^2 E_x}{\delta t^2} \\ \dfrac{1}{\frac{\delta s^2}{\delta t^2}} \cdot \dfrac{\delta^2 E_y}{\delta t^2} \\ \dfrac{1}{\frac{\delta s^2}{\delta t^2}} \cdot \dfrac{\delta^2 E_z}{\delta t^2} \end{pmatrix} \qquad (2.4.3.11)$$

If Equation 2.4.3.11 is simplified further, based on Equation 2.4.3.9, and the speed of light is again factored out as a constant from the individual components, Equation 2.4.3.12 arises.

$$\Delta \vec{E} = \frac{1}{\frac{\delta s^2}{\delta t^2}} \cdot \begin{pmatrix} \dfrac{\delta^2 E_x}{\delta t^2} \\ \dfrac{\delta^2 E_y}{\delta t^2} \\ \dfrac{\delta^2 E_z}{\delta t^2} \end{pmatrix} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{E}}{\delta t^2} \qquad (2.4.3.12)$$

Equation 2.4.3.12 corresponds to Equation 2.4.3 and is thus an expression for the electric wave equation.

$$\Delta \vec{E} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{E}}{\delta t^2} \qquad (2.4.3)$$

A Hertzian transverse electric wave is thus derived from Equation 2.4.2.17.

2.4.4 DERIVATION OF THE ELECTRICAL WAVE EQUATION AS A PURE LONGITUTINAL WAVE

In order to derive a pure longitudinal wave from the electrical wave equation, the transversal and combined wave components in Equation 2.4.2.17 must now be set to zero. Equation 2.3.4

is also written here and serves for better orientation with regard to Equation 2.4.2.17. Here, too, expression div grad $\vec{E}$ from Equation 2.3.4 is equivalent to $\Delta \vec{E}$.

$$\text{rot rot } \vec{E} \quad = \quad \text{grad div } \vec{E} \quad - \quad \text{div grad } \vec{E} \tag{2.3.4}$$

$$\begin{pmatrix} \frac{\delta^2 E_y}{\delta x \delta y} - \frac{\delta^2 E_x}{\delta y^2} - \frac{\delta^2 E_x}{\delta z^2} + \frac{\delta^2 E_z}{\delta x \delta z} \\ \frac{\delta^2 E_z}{\delta y \delta z} - \frac{\delta^2 E_y}{\delta z^2} - \frac{\delta^2 E_y}{\delta x^2} + \frac{\delta^2 E_x}{\delta y \delta x} \\ \frac{\delta^2 E_x}{\delta z \delta x} - \frac{\delta^2 E_z}{\delta x^2} - \frac{\delta^2 E_z}{\delta y^2} + \frac{\delta^2 E_y}{\delta z \delta y} \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + \frac{\delta^2 E_y}{\delta y \delta x} + \frac{\delta^2 E_z}{\delta z \delta x} \\ \frac{\delta^2 E_x}{\delta x \delta y} + \frac{\delta^2 E_y}{\delta y^2} + \frac{\delta^2 E_z}{\delta z \delta y} \\ \frac{\delta^2 E_x}{\delta x \delta z} + \frac{\delta^2 E_y}{\delta y \delta z} + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} - \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + \frac{\delta^2 E_x}{\delta y^2} + \frac{\delta^2 E_x}{\delta z^2} \\ \frac{\delta^2 E_y}{\delta x^2} + \frac{\delta^2 E_y}{\delta y^2} + \frac{\delta^2 E_y}{\delta z^2} \\ \frac{\delta^2 E_z}{\delta x^2} + \frac{\delta^2 E_z}{\delta y^2} + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} \tag{2.4.2.17}$$

As already mentioned, in Equation 2.4.2.17, the transversal and combined wave components are initially set to zero. It also follows from this that the div $\vec{E}$ cannot be assumed to be zero and therefore that there is an electric field density. Equation 2.4.4.1 describes these circumstances.

$$\begin{pmatrix} 0 - 0 - 0 + 0 \\ 0 - 0 - 0 + 0 \\ 0 - 0 - 0 + 0 \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + 0 + 0 \\ 0 + \frac{\delta^2 E_y}{\delta y^2} + 0 \\ 0 + 0 + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} - \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + 0 + 0 \\ 0 + \frac{\delta^2 E_y}{\delta y^2} + 0 \\ 0 + 0 + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} \tag{2.4.4.1}$$

If the expression div grad $\vec{E}$ from Equation 2.3.4 is now equated with the last term from Equation 2.4.4.1, Equation 2.4.4.2 results.

$$\text{div grad } \vec{E} = \Delta \vec{E} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta_x^2} + 0 + 0 \\ 0 + \frac{\delta^2 E_y}{\delta_y^2} + 0 \\ 0 + 0 + \frac{\delta^2 E_z}{\delta_z^2} \end{pmatrix} \tag{2.4.4.2}$$

The right-hand side of Equation 2.4.4.2 can now be simplified again to Equation 2.4.4.3 based on Equation 2.4.3.5. In contrast to Equation 2.4.3.5, Equation 2.4.4.3 does not describe transversal but only longitudinal wave components.

$$\Delta \vec{E} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta s_x^2} \\ \frac{\delta^2 E_y}{\delta s_y^2} \\ \frac{\delta^2 E_z}{\delta s_z^2} \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + 0 + 0 \\ 0 + \frac{\delta^2 E_y}{\delta y^2} + 0 \\ 0 + 0 + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} \qquad (2.4.4.3)$$

If the constant speed of light, as described in Equation 2.3.4.9, is now factored out of Equation 2.4.4.3 in the form $\frac{c^2}{c^2}$, Equation 2.3.4.12 arises.

$$c^2 = \left(\frac{\delta s}{\delta t}\right) \cdot \left(\frac{\delta s}{\delta t}\right) = \frac{(\delta s)^2}{(\delta t)^2} = \frac{\delta s^2}{\delta t^2} \qquad (2.3.4.9)$$

$$\Delta \vec{E} = \frac{1}{\frac{\delta s^2}{\delta t^2}} \cdot \begin{pmatrix} \frac{\delta^2 E_x}{\delta t^2} \\ \frac{\delta^2 E_y}{\delta t^2} \\ \frac{\delta^2 E_z}{\delta t^2} \end{pmatrix} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{E}}{\delta t^2} \qquad (2.3.4.12)$$

The detailed derivation of the Equations 2.3.4.9 and 2.3.4.12 can be found in the Equations 2.3.4.5 to 2.3.4.12. Since the derivation from Equation 2.4.4.3 is the same as from Equation 2.3.4.5, a new derivation will not be carried out at this point.

Equation 2.3.4.12 corresponds to Equation 2.4.3 and is thus again an expression for the electric wave equation.

$$\Delta \vec{E} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{E}}{\delta t^2} \qquad (2.4.3)$$

A longitudinal electric wave is thus derived from Equation 2.4.2.17.

2.4.5 DERIVATION OF THE ELECTRIC WAVE EQUATION AS A COMBINATION OF LONGITUTINAL WAVE AND TRANSVERSAL WAVE

Equations 2.3.4 and 2.4.2.17 are used again here as a starting point for the interpretation of the electric wave equation as a combination of longitudinal and transverse wave components.

$$\text{rot rot } \vec{E} \quad = \quad \text{grad div } \vec{E} \quad - \quad \text{div grad } \vec{E} \qquad (2.3.4)$$

$$\begin{pmatrix} \frac{\delta^2 E_y}{\delta x \delta y} - \frac{\delta^2 E_x}{\delta y^2} - \frac{\delta^2 E_x}{\delta z^2} + \frac{\delta^2 E_z}{\delta x \delta z} \\ \frac{\delta^2 E_z}{\delta y \delta z} - \frac{\delta^2 E_y}{\delta z^2} - \frac{\delta^2 E_y}{\delta x^2} + \frac{\delta^2 E_x}{\delta y \delta x} \\ \frac{\delta^2 E_x}{\delta z \delta x} - \frac{\delta^2 E_z}{\delta x^2} - \frac{\delta^2 E_z}{\delta y^2} + \frac{\delta^2 E_y}{\delta z \delta y} \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + \frac{\delta^2 E_y}{\delta y \delta x} + \frac{\delta^2 E_z}{\delta z \delta x} \\ \frac{\delta^2 E_x}{\delta x \delta y} + \frac{\delta^2 E_y}{\delta y^2} + \frac{\delta^2 E_z}{\delta z \delta y} \\ \frac{\delta^2 E_x}{\delta x \delta z} + \frac{\delta^2 E_y}{\delta y \delta z} + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} - \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + \frac{\delta^2 E_x}{\delta y^2} + \frac{\delta^2 E_x}{\delta z^2} \\ \frac{\delta^2 E_y}{\delta x^2} + \frac{\delta^2 E_y}{\delta y^2} + \frac{\delta^2 E_y}{\delta z^2} \\ \frac{\delta^2 E_z}{\delta x^2} + \frac{\delta^2 E_z}{\delta y^2} + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} \qquad (2.4.2.17)$$

Starting from Equation 2.4.2.17, only the combined wave components that are irrelevant to the expression $\text{div grad}\,\vec{E}$ are now eliminated from the equation, resulting in Equation 2.4.5.1.

However, these terms are interesting because they each have a longitudinal part and a transversal part. However, what role these play in the interpretation of an electromagnetic wave is not dealt with in this paper.

$$\begin{pmatrix} 0 - \frac{\delta^2 E_x}{\delta y^2} - \frac{\delta^2 E_x}{\delta z^2} + 0 \\ 0 - \frac{\delta^2 E_y}{\delta z^2} - \frac{\delta^2 E_y}{\delta x^2} + 0 \\ 0 - \frac{\delta^2 E_z}{\delta x^2} - \frac{\delta^2 E_z}{\delta y^2} + 0 \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + 0 + 0 \\ 0 + \frac{\delta^2 E_y}{\delta y^2} + 0 \\ 0 + 0 + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} - \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + \frac{\delta^2 E_x}{\delta y^2} + \frac{\delta^2 E_x}{\delta z^2} \\ \frac{\delta^2 E_y}{\delta x^2} + \frac{\delta^2 E_y}{\delta y^2} + \frac{\delta^2 E_y}{\delta z^2} \\ \frac{\delta^2 E_z}{\delta x^2} + \frac{\delta^2 E_z}{\delta y^2} + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} \qquad (2.4.5.1)$$

If the last term of Equation 2.3.4 is now equated with the last term of Equation 2.4.5.1, Equation 2.4.5.2 results.

$$\Delta \vec{E} = \text{div grad } \vec{E} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + \frac{\delta^2 E_x}{\delta y^2} + \frac{\delta^2 E_x}{\delta z^2} \\ \frac{\delta^2 E_y}{\delta x^2} + \frac{\delta^2 E_y}{\delta y^2} + \frac{\delta^2 E_y}{\delta z^2} \\ \frac{\delta^2 E_z}{\delta x^2} + \frac{\delta^2 E_z}{\delta y^2} + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} \tag{2.4.5.2}$$

If the electric wave is interpreted as a combined wave with a transverse and longitudinal wave component, Equation 2.4.5.2 shows that the change in the electric field $\vec{E}$ also has three components in all three spatial directions for all three vector components.
Here, too, the right-hand side of Equation 2.4.5.2 can be summarized again. Equation 2.4.5.3 arises.

$$\Delta \vec{E} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta s_x^2} \\ \frac{\delta^2 E_y}{\delta s_y^2} \\ \frac{\delta^2 E_z}{\delta s_z^2} \end{pmatrix} = \begin{pmatrix} (\frac{\delta^2 E_x}{\delta x^2}) + (\frac{\delta^2 E_x}{\delta y^2}) + (\frac{\delta^2 E_x}{\delta z^2}) \\ (\frac{\delta^2 E_y}{\delta x^2}) + (\frac{\delta^2 E_y}{\delta y^2}) + (\frac{\delta^2 E_y}{\delta z^2}) \\ (\frac{\delta^2 E_z}{\delta x^2}) + (\frac{\delta^2 E_z}{\delta y^2}) + (\frac{\delta^2 E_z}{\delta z^2}) \end{pmatrix} \tag{2.4.5.3}$$

Equation 2.4.5.3 states that the E-field can also change at an angle to the direction of propagation of the wave. This means that the electromagnetic wave, under the conditions from Equation 2.4.5.3, also has density states that propagate intermittently in the direction of propagation. The impact movement can therefore be accompanied by a transverse movement. The question that arises from this is what form the electromagnetic wave has in reality?
If density states within the electric field are assumed, then the electric wave must be interpreted as an interval-like change of density states. The result of this interval-like change in density states are alternating field lines that could be interpreted as vortices. It also shows that the changing density states are not limited to the periphery of an antenna, but move through space. This is an indication that there is a substance or medium in which this occurs.
Again the constant speed of light, as it is described in the Equation 2.3.4.9, is factored out in the form $\frac{c^2}{c^2}$ from the Equation 2.4.5.3, the Equation 2.3.4.12 arises again.

$$c^2 = \left(\frac{\delta s}{\delta t}\right) \cdot \left(\frac{\delta s}{\delta t}\right) = \frac{(\delta s)^2}{(\delta t)^2} = \frac{\delta s^2}{\delta t^2} \qquad (2.3.4.9)$$

$$\Delta \vec{E} = \frac{1}{\frac{\delta s^2}{\delta t^2}} \cdot \begin{pmatrix} \frac{\delta^2 E_x}{\delta t^2} \\ \frac{\delta^2 E_y}{\delta t^2} \\ \frac{\delta^2 E_z}{\delta t^2} \end{pmatrix} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{E}}{\delta t^2} \qquad (2.3.4.12)$$

At this point there is again the note that the detailed derivation of Equations 2.3.4.9 and 2.3.4.12 can be found in Equations 2.3.4.5 to 2.4.3.12. Since the derivation from Equation 2.4.5.3 is the same as from Equation 2.3.4.5, a new derivation is not used at this point either. Equation 2.3.4.12 corresponds to Equation 2.4.3 and is thus again an expression for the electric wave equation.

$$\Delta \vec{E} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{E}}{\delta t^2} \qquad (2.4.3)$$

An electrical wave is thus derived from Equation 2.4.2.17, which has both transverse and longitudinal components.

2.5 THE MAGNETIC WAVE EQUATION

At this point, the three possible magnetic wave types are not mathematically derived in detail, since the same mathematical framework conditions apply to the magnetic field as to the electric field. Accordingly, only the most important equations for the derivation of the magnetic wave are used here and vector $\vec{E}$ is replaced by vector $\vec{H}$. Equations 2.3.4 and 2.4.2.17 are the starting point for the description of the magnetic wave.

$$\text{rot rot } \vec{E} = \text{grad div } \vec{E} - \text{div grad } \vec{E} \qquad (2.3.4)$$

$$\begin{pmatrix} \frac{\delta^2 E_y}{\delta x \delta y} - \frac{\delta^2 E_x}{\delta y^2} - \frac{\delta^2 E_x}{\delta z^2} + \frac{\delta^2 E_z}{\delta x \delta z} \\ \frac{\delta^2 E_z}{\delta y \delta z} - \frac{\delta^2 E_y}{\delta z^2} - \frac{\delta^2 E_y}{\delta x^2} + \frac{\delta^2 E_x}{\delta y \delta x} \\ \frac{\delta^2 E_x}{\delta z \delta x} - \frac{\delta^2 E_z}{\delta x^2} - \frac{\delta^2 E_z}{\delta y^2} + \frac{\delta^2 E_y}{\delta z \delta y} \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + \frac{\delta^2 E_y}{\delta y \delta x} + \frac{\delta^2 E_z}{\delta z \delta x} \\ \frac{\delta^2 E_x}{\delta x \delta y} + \frac{\delta^2 E_y}{\delta y^2} + \frac{\delta^2 E_z}{\delta z \delta y} \\ \frac{\delta^2 E_x}{\delta x \delta z} + \frac{\delta^2 E_y}{\delta y \delta z} + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} - \begin{pmatrix} \frac{\delta^2 E_x}{\delta x^2} + \frac{\delta^2 E_x}{\delta y^2} + \frac{\delta^2 E_x}{\delta z^2} \\ \frac{\delta^2 E_y}{\delta x^2} + \frac{\delta^2 E_y}{\delta y^2} + \frac{\delta^2 E_y}{\delta z^2} \\ \frac{\delta^2 E_z}{\delta x^2} + \frac{\delta^2 E_z}{\delta y^2} + \frac{\delta^2 E_z}{\delta z^2} \end{pmatrix} \qquad (2.4.2.17)$$

If, in Equations 2.3.4 and 2.4.2.17, as already mentioned, the vector of the electric field $\vec{E}$ is replaced by the vector of the magnetic field $\vec{H}$, the two Equations 2.5.1 and 2.5.2 arise.

$$\text{rot rot } \vec{H} = \text{grad div } \vec{H} - \text{div grad } \vec{H} \qquad (2.5.1)$$

$$\begin{pmatrix} \frac{\delta^2 H_y}{\delta x \delta y} - \frac{\delta^2 H_x}{\delta y^2} - \frac{\delta^2 H_x}{\delta z^2} + \frac{\delta^2 H_z}{\delta x \delta z} \\ \frac{\delta^2 H_z}{\delta y \delta z} - \frac{\delta^2 H_y}{\delta z^2} - \frac{\delta^2 H_y}{\delta x^2} + \frac{\delta^2 H_x}{\delta y \delta x} \\ \frac{\delta^2 H_x}{\delta z \delta x} - \frac{\delta^2 H_z}{\delta x^2} - \frac{\delta^2 H_z}{\delta y^2} + \frac{\delta^2 H_y}{\delta z \delta y} \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 H_x}{\delta x^2} + \frac{\delta^2 H_y}{\delta y \delta x} + \frac{\delta^2 H_z}{\delta z \delta x} \\ \frac{\delta^2 H_x}{\delta x \delta y} + \frac{\delta^2 H_y}{\delta y^2} + \frac{\delta^2 H_z}{\delta z \delta y} \\ \frac{\delta^2 H_x}{\delta x \delta z} + \frac{\delta^2 H_y}{\delta y \delta z} + \frac{\delta^2 H_z}{\delta z^2} \end{pmatrix} - \begin{pmatrix} \frac{\delta^2 H_x}{\delta x^2} + \frac{\delta^2 H_x}{\delta y^2} + \frac{\delta^2 H_x}{\delta z^2} \\ \frac{\delta^2 H_y}{\delta x^2} + \frac{\delta^2 H_y}{\delta y^2} + \frac{\delta^2 H_y}{\delta z^2} \\ \frac{\delta^2 H_z}{\delta x^2} + \frac{\delta^2 H_z}{\delta y^2} + \frac{\delta^2 H_z}{\delta z^2} \end{pmatrix} \qquad (2.5.2)$$

The two Equations 2.5.1 and 2.5.2 will serve as the basis for the derivation of the three possible magnetic waves in the following calculations.

2.5.1 THE TRANSVERSAL MAGNETIC WAVE

Starting from Equation 2.5.2, all terms with longitudinal components are first deleted. Equation 2.5.1.1 results from this. Equation 2.5.1 is also used here for a better understanding of the individual components from Equation 2.5.1.1.

$$\text{rot rot } \vec{H} = \text{grad div } \vec{H} - \text{div grad } \vec{H} \qquad (2.5.1)$$

$$\begin{pmatrix} 0 - \frac{\delta^2 H_x}{\delta y^2} - \frac{\delta^2 H_x}{\delta z^2} + 0 \\ 0 - \frac{\delta^2 H_y}{\delta z^2} - \frac{\delta^2 H_y}{\delta x^2} + 0 \\ 0 - \frac{\delta^2 H_z}{\delta x^2} - \frac{\delta^2 H_z}{\delta y^2} + 0 \end{pmatrix} = \begin{pmatrix} 0 + 0 + 0 \\ 0 + 0 + 0 \\ 0 + 0 + 0 \end{pmatrix} - \begin{pmatrix} 0 + \frac{\delta^2 H_x}{\delta y^2} + \frac{\delta^2 H_x}{\delta z^2} \\ \frac{\delta^2 H_y}{\delta x^2} + 0 + \frac{\delta^2 H_y}{\delta z^2} \\ \frac{\delta^2 H_z}{\delta x^2} + \frac{\delta^2 H_z}{\delta y^2} + 0 \end{pmatrix} \qquad (2.5.1.1)$$

Analogous to the derivation of Equation 2.4.3.5, Equation 2.5.1.2 can now be derived from Equation 2.5.1.1.

$$\operatorname{div}\operatorname{grad}\vec{H} = \Delta\vec{H} = \begin{pmatrix} \dfrac{\delta^2 H_x}{\delta s_x^2} \\ \dfrac{\delta^2 H_y}{\delta s_y^2} \\ \dfrac{\delta^2 H_z}{\delta s_z^2} \end{pmatrix} = \begin{pmatrix} 0 + (\dfrac{\delta^2 H_x}{\delta y^2}) + (\dfrac{\delta^2 H_x}{\delta z^2}) \\ (\dfrac{\delta^2 H_y}{\delta x^2}) + 0 + (\dfrac{\delta^2 H_y}{\delta z^2}) \\ (\dfrac{\delta^2 H_z}{\delta x^2}) + (\dfrac{\delta^2 H_z}{\delta y^2}) + 0 \end{pmatrix} \tag{2.5.1.2}$$

If now, as in Equation 2.4.3.10, the constant speed of light as described in Equation 2.3.4.9 is factored out of Equation 2.5.1.2 in the form $\dfrac{c^2}{c^2}$, Equation 2.5.1.3 arises.

$$c^2 = (\frac{\delta s}{\delta t}) \cdot (\frac{\delta s}{\delta t}) = \frac{(\delta s)^2}{(\delta t)^2} = \frac{\delta s^2}{\delta t^2} \tag{2.3.4.9}$$

$$\Delta\vec{H} = (\frac{1}{(\dfrac{\delta s^2}{\delta t^2})}) \cdot \begin{pmatrix} (\dfrac{\delta^2 H_x}{\delta t^2}) \\ (\dfrac{\delta^2 H_y}{\delta t^2}) \\ (\dfrac{\delta^2 H_z}{\delta t^2}) \end{pmatrix} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{H}}{\delta t^2} \tag{2.5.1.3}$$

Equation 2.5.1.3 thus corresponds to Equation 2.4.4, which maps the magnetic wave equation.

$$\Delta\vec{H} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{H}}{\delta t^2} \tag{2.4.4}$$

A transverse magnetic wave would then be derived from Equation 2.5.2.

2.5.2 THE LONGITUTINAL MAGNETIC WAVE

Starting again from Equation 2.5.2, the following calculations are used to derive a longitudinal wave as a magnetic wave. Equation 2.5.1 is again used for better orientation for the individual components of Equation 2.5.2.

$$\text{rot rot } \vec{H} \quad = \quad \text{grad div } \vec{H} \quad - \quad \text{div grad } \vec{H} \qquad (2.5.1)$$

$$\begin{pmatrix} \frac{\delta^2 H_y}{\delta x \delta y} - \frac{\delta^2 H_x}{\delta y^2} - \frac{\delta^2 H_x}{\delta z} + \frac{\delta^2 H_z}{\delta x \delta z} \\ \frac{\delta^2 H_z}{\delta y \delta z} - \frac{\delta^2 H_y}{\delta z^2} - \frac{\delta^2 H_y}{\delta x^2} + \frac{\delta^2 H_x}{\delta y \delta x} \\ \frac{\delta^2 H_x}{\delta z \delta x} - \frac{\delta^2 H_z}{\delta x^2} - \frac{\delta^2 H_z}{\delta y^2} + \frac{\delta^2 H_y}{\delta z \delta y} \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 H_x}{\delta x^2} + \frac{\delta^2 H_y}{\delta y \delta x} + \frac{\delta^2 H_z}{\delta z \delta x} \\ \frac{\delta^2 H_x}{\delta x \delta y} + \frac{\delta^2 H_y}{\delta y^2} + \frac{\delta^2 H_z}{\delta z \delta y} \\ \frac{\delta^2 H_x}{\delta x \delta z} + \frac{\delta^2 H_y}{\delta y \delta z} + \frac{\delta^2 H_z}{\delta z^2} \end{pmatrix} - \begin{pmatrix} \frac{\delta^2 H_x}{\delta x^2} + \frac{\delta^2 H_x}{\delta y^2} + \frac{\delta^2 H_x}{\delta z^2} \\ \frac{\delta^2 H_y}{\delta x^2} + \frac{\delta^2 H_y}{\delta y^2} + \frac{\delta^2 H_y}{\delta z^2} \\ \frac{\delta^2 H_z}{\delta x^2} + \frac{\delta^2 H_z}{\delta y^2} + \frac{\delta^2 H_z}{\delta z^2} \end{pmatrix} \qquad (2.5.2)$$

If in Equation 2.5.2 all terms with transversal parts are deleted, Equation 2.5.2.1 results, analogous to Equation 2.4.4.1.

$$\begin{pmatrix} 0 - 0 - 0 + 0 \\ 0 - 0 - 0 + 0 \\ 0 - 0 - 0 + 0 \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 H_x}{\delta x^2} + 0 + 0 \\ 0 + \frac{\delta^2 H_y}{\delta y^2} + 0 \\ 0 + 0 + \frac{\delta^2 H_z}{\delta z^2} \end{pmatrix} - \begin{pmatrix} \frac{\delta^2 H_x}{\delta x^2} + 0 + 0 \\ 0 + \frac{\delta^2 H_y}{\delta y^2} + 0 \\ 0 + 0 + \frac{\delta^2 H_z}{\delta z^2} \end{pmatrix} \qquad (2.5.2.1)$$

Similar to Equation 2.4.4.2, Equation 2.5.2.2 can be derived from Equation 2.5.2.1.

$$\text{div grad } \vec{H} = \Delta \vec{H} = \begin{pmatrix} \frac{\delta^2 H_x}{\delta s_x^2} \\ \frac{\delta^2 H_y}{\delta s_y^2} \\ \frac{\delta^2 H_z}{\delta s_z^2} \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 H_x}{\delta x^2} + 0 + 0 \\ 0 + \frac{\delta^2 H_y}{\delta y^2} + 0 \\ 0 + 0 + \frac{\delta^2 H_z}{\delta z^2} \end{pmatrix} \qquad (2.5.2.2)$$

If the constant speed of light, as described in Equation 2.3.4.9, is now factored out of Equation 2.5.2.2 in the form $\frac{c^2}{c^2}$, Equation 2.5.1.3 arises.

$$c^2 = (\frac{\delta s}{\delta t}) \cdot (\frac{\delta s}{\delta t}) = \frac{(\delta s)^2}{(\delta t)^2} = \frac{\delta s^2}{\delta t^2} \qquad (2.3.4.9)$$

$$\Delta \vec{H} = (\frac{1}{(\frac{\delta s^2}{\delta t^2})}) \cdot \begin{pmatrix} (\frac{\delta^2 H_x}{\delta t^2}) \\ (\frac{\delta^2 H_y}{\delta t^2}) \\ (\frac{\delta^2 H_z}{\delta t^2}) \end{pmatrix} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{H}}{\delta t^2} \qquad (2.5.1.3)$$

Equation 2.5.1.3 corresponds to Equation 2.4.4, which maps the magnetic wave equation.

$$\Delta \vec{H} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{H}}{\delta t^2} \qquad (2.4.4)$$

A magnetic longitudinal wave would then be derived from Equation 2.5.2.

2.5.3 THE MAGNETIC WAVE AS A COMBINATION OF A LONGITUTINAL WAVE AND A TRANSVERSAL WAVE

Here, too, the derivation begins with the two Equations 2.5.1 and 2.5.2.

$$\text{rot rot } \vec{H} = \text{grad div } \vec{H} - \text{div grad } \vec{H} \qquad (2.5.1)$$

$$\begin{pmatrix} \frac{\delta^2 H_y}{\delta x \delta y} - \frac{\delta^2 H_x}{\delta y^2} - \frac{\delta^2 H_x}{\delta z} + \frac{\delta^2 H_z}{\delta x \delta z} \\ \frac{\delta^2 H_z}{\delta y \delta z} - \frac{\delta^2 H_y}{\delta z^2} - \frac{\delta^2 H_y}{\delta x^2} + \frac{\delta^2 H_x}{\delta y \delta x} \\ \frac{\delta^2 H_x}{\delta z \delta x} - \frac{\delta^2 H_z}{\delta x^2} - \frac{\delta^2 H_z}{\delta y^2} + \frac{\delta^2 H_y}{\delta z \delta y} \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 H_x}{\delta x^2} + \frac{\delta^2 H_y}{\delta y \delta x} + \frac{\delta^2 H_z}{\delta z \delta x} \\ \frac{\delta^2 H_x}{\delta x \delta y} + \frac{\delta^2 H_y}{\delta y^2} + \frac{\delta^2 H_z}{\delta z \delta y} \\ \frac{\delta^2 H_x}{\delta x \delta z} + \frac{\delta^2 H_y}{\delta y \delta z} + \frac{\delta^2 H_z}{\delta z^2} \end{pmatrix} - \begin{pmatrix} \frac{\delta^2 H_x}{\delta x^2} + \frac{\delta^2 H_x}{\delta y^2} + \frac{\delta^2 H_x}{\delta z^2} \\ \frac{\delta^2 H_y}{\delta x^2} + \frac{\delta^2 H_y}{\delta y^2} + \frac{\delta^2 H_y}{\delta z^2} \\ \frac{\delta^2 H_z}{\delta x^2} + \frac{\delta^2 H_z}{\delta y^2} + \frac{\delta^2 H_z}{\delta z^2} \end{pmatrix} \qquad (2.5.2)$$

Starting from Equation 2.5.2, all parts that do not correspond to term $\operatorname{div}\operatorname{grad}\vec{H}$ from Equation 2.5.1 are first deleted there. This results in Equation 2.5.3.1.

$$\begin{pmatrix} 0 - \frac{\delta^2 H_x}{\delta y^2} - \frac{\delta^2 H_x}{\delta z^2} + 0 \\ 0 - \frac{\delta^2 H_y}{\delta z^2} - \frac{\delta^2 H_y}{\delta x^2} + 0 \\ 0 - \frac{\delta^2 H_z}{\delta x^2} - \frac{\delta^2 H_z}{\delta y^2} + 0 \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 H_x}{\delta x^2} + 0 + 0 \\ 0 + \frac{\delta^2 H_y}{\delta y^2} + 0 \\ 0 + 0 + \frac{\delta^2 H_z}{\delta z^2} \end{pmatrix} - \begin{pmatrix} \frac{\delta^2 H_x}{\delta x^2} + \frac{\delta^2 H_x}{\delta y^2} + \frac{\delta^2 H_x}{\delta z^2} \\ \frac{\delta^2 H_y}{\delta x^2} + \frac{\delta^2 H_y}{\delta y^2} + \frac{\delta^2 H_y}{\delta z^2} \\ \frac{\delta^2 H_z}{\delta x^2} + \frac{\delta^2 H_z}{\delta y^2} + \frac{\delta^2 H_z}{\delta z^2} \end{pmatrix} \qquad (2.5.3.1)$$

Equation 2.5.3.2 can now be derived from Equation 2.5.3.1 in analogy to Equation 2.4.5.3.

$$\operatorname{div}\operatorname{grad}\vec{H} = \Delta \vec{H} = \begin{pmatrix} \frac{\delta^2 H_x}{\delta s_x^2} \\ \frac{\delta^2 H_y}{\delta s_y^2} \\ \frac{\delta^2 H_z}{\delta s_z^2} \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 H_x}{\delta x^2} + \frac{\delta^2 H_x}{\delta y^2} + \frac{\delta^2 H_x}{\delta z^2} \\ \frac{\delta^2 H_y}{\delta x^2} + \frac{\delta^2 H_y}{\delta y^2} + \frac{\delta^2 H_y}{\delta z^2} \\ \frac{\delta^2 H_z}{\delta x^2} + \frac{\delta^2 H_z}{\delta y^2} + \frac{\delta^2 H_z}{\delta z^2} \end{pmatrix} \qquad (2.5.3.2)$$

If the constant speed of light, as described in Equation 2.3.4.9, is now factored out of Equation 2.5.3.2 in the form $\frac{c^2}{c^2}$, Equation 2.5.1.3 arises.

$$c^2 = \left(\frac{\delta s}{\delta t}\right) \cdot \left(\frac{\delta s}{\delta t}\right) = \frac{(\delta s)^2}{(\delta t)^2} = \frac{\delta s^2}{\delta t^2} \qquad (2.3.4.9)$$

$$\Delta \vec{H} = \left(\frac{1}{\left(\frac{\delta s^2}{\delta t^2}\right)}\right) \cdot \begin{pmatrix} \left(\frac{\delta^2 H_x}{\delta t^2}\right) \\ \left(\frac{\delta^2 H_y}{\delta t^2}\right) \\ \left(\frac{\delta^2 H_z}{\delta t^2}\right) \end{pmatrix} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{H}}{\delta t^2} \qquad (2.5.1.3)$$

Equation 2.5.1.3 corresponds to Equation 2.4.4, which maps the magnetic wave equation.

$$\Delta \vec{H} = \frac{1}{c^2} \cdot \frac{\delta^2 \vec{H}}{\delta t^2} \tag{2.4.4}$$

A magnetic wave would then be derived as a combination of longitudinal wave and transverse wave from Equation 2.5.2.

2.5.4 COMBINATION OF THE ELECTRICAL AND MAGNETIC WAVE EQUATION

2.5.5 THE POYNTING WAVE

First, the Poynting vector $\vec{S}$ is explained here. This is formed from the cross product between the electric and magnetic fields and is shown in Equation 2.5.5.1.

$$\vec{S} = \vec{E} \times \vec{H} \tag{2.5.5.1}$$

If it is now assumed that the electric field $\vec{E}$ and the magnetic field $\vec{H}$ are at a ninety-degree angle to each other, as is the case with the Hertzian electromagnetic wave, it follows that in this case the Poynting vector in the direction of propagation of this wave, i.e. at a ninety-degree angle to the field lines of both fields.
Since the Poynting vector defines the density and the direction of the energy transport, there is also an energy wave in the transverse Hertzian wave that moves in the direction of propagation of the transverse Hertzian wave and has both density states and transports energy. Nikola Tesla described such a wave during a lecture on May 20, 1891, at Columbia College in New York and made several demonstrations in which he made Geißler tubes glow in free space.
If the two field values of the electric field $\vec{E}$ and the magnetic field $\vec{H}$ change during a specific time t, this can be described as a time derivative of the two field values. As a result, the Poynting vector also changes as a function of time. So if Equation 2.5.5.1 is used as the calculation basis for a Poynting energy wave, Equation 2.5.5.2 arises.

$$\Delta \vec{S} = \Delta(\vec{E} \times \vec{H}) = \frac{1}{c^2} \cdot \frac{\delta \vec{S}}{\delta t^2} = \frac{1}{c^2} \cdot \frac{\delta(\vec{E} \times \vec{H})}{\delta t^2} \tag{2.5.5.2}$$

In order to calculate a Poynting wave as an example, some calculation principles must first be defined. First of all, a Cartesian coordinate system is assumed below. The propagation direction of the electric and magnetic waves is the x-direction in the following calculation example, for a better understanding of the situation. Furthermore, a Hertzian wave, i.e. a transverse wave, is assumed. This means that the field lines of the electric and magnetic waves are at a ninety-degree angle to the direction of propagation of the two waves in the x-direction. In the following calculation example, in Equation 2.5.5.5, the electric field lines are in the y-direction and the magnetic field lines are in the z-direction. Both field sizes are dependent on time.

$$\frac{1}{c^2}\cdot\frac{\delta^2}{\delta t^2}\begin{pmatrix} S_x \\ 0 \\ 0 \end{pmatrix} = \frac{1}{c^2}\cdot\frac{\delta^2}{\delta t^2}\left(\begin{pmatrix} 0 \\ E_y \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix}\right) = \frac{1}{c^2}\cdot\frac{\delta^2}{\delta t^2}\begin{pmatrix} (E_y \cdot B_z) - (0 \cdot 0) \\ (0 \cdot 0) - (0 \cdot B_z) \\ (0 \cdot 0) - (E_y \cdot 0) \end{pmatrix} \qquad (2.5.5.5)$$

Equation 2.5.5.5 shows that the electromagnetic wave can also be described as a Poyntingian energy wave, the properties of which correspond to those of a longitudinal wave.
For the sake of completeness, Equation 2.5.5.5 is shown in Equation 2.5.5.6 in its general form.

$$\frac{1}{c^2}\cdot\frac{\delta^2}{\delta t^2}\begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \frac{1}{c^2}\cdot\frac{\delta^2}{\delta t^2}\left(\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}\right) = \frac{1}{c^2}\cdot\frac{\delta^2}{\delta t^2}\begin{pmatrix} (E_y \cdot B_z) - (E_z \cdot B_y) \\ (E_z \cdot B_x) - (E_x \cdot B_z) \\ (E_x \cdot B_y) - (E_y \cdot B_x) \end{pmatrix} \qquad (2.5.5.6)$$

Assuming that both electric and magnetic waves consist of longitudinal and transverse parts, this would also apply to Poynting's wave.

2.5.6 THE REINTERPRETATION OF THE ELECTROMAGNETIC WAVE EQUATION

In order to be able to correctly interpret the electromagnetic wave equation, the process by which this wave is generated must be understood. First of all, an electric dipole is assumed at this point. If the poles are designed as a sphere and have different electrical polarization, an electric field is formed between them. However, the field lines do not form at specific points, but on the entire surface of the two poles. If it is now assumed that the field lines form at a ninety-degree angle to the pole surface and connect both poles, semicircular to oval field lines arise between the poles. If the poles are now polarized alternately, i.e. an alternating voltage is applied, the field lines also change their flow direction alternately. In addition, the field polarization reversal is accompanied by an alternating weakening and strengthening of the field lines. This means that the total cross-sectional area in space, which is penetrated by the field lines, also changes alternately. If the measuring point is now parallel to the dipole and is at some distance, the electric wave and the magnetic wave can be regarded as transverse waves. However, if the measuring point is directly between the two poles of the dipole or in the immediate vicinity of one of the poles, the waves can be interpreted as longitudinal waves. Since, as was already shown mathematically in the elaboration "The Reinterpretation of the 'Maxwell Equations'" (Martin, 2021), the electric field as well as the magnetic field have density states, it can now be assumed that the field lines of both fields are directly coupled to these density states. This means that the potential difference within the two fields results in their field lines. From this follows the realization that the electromagnetic wave, by its nature, is a wave that moves through space with alternating field sources and field sinks. It should be noted here that for this assumption a location of the measuring point that is parallel to the field lines of the dipole can lead to the interpretation of a transversal as well as to the interpretation of a longitudinal wave.

If the Poynting vector is considered as a possible basis for the interpretation of the electromagnetic wave, this results in a longitudinal wave for the Hertzian transverse wave, which can be defined as a directed energy wave. This directed energy wave indicates a change in energy density over time in the direction of propagation of the Hertzian electromagnetic wave. One might call this an energy burst. However, the direction of propagation of this Poyntig wave alternates under the assumption that both the electric and the magnetic wave are not purely transverse waves. In any case, the Poynting wave can be used to transport energy.

3. DISCUSSION

1. It remains to be discussed whether the expression, $\operatorname{div}(\vec{B}) = 0$, is physically feasible since the mathematical requirement consists of Equation 2.1.4, $(\mathrm{Sp})(\operatorname{grad}\vec{B}) = \operatorname{div}(\vec{B})$. And if $\operatorname{div}(\vec{B}) = 0$ is admissible, what does this mean for Equation 3.1 and ultimately for the law of induction?

$$(\mathrm{Sp})(\operatorname{grad}\vec{B}) = \operatorname{div}(\vec{B}) = \frac{\delta B_x}{\delta x} + \frac{\delta B_y}{\delta y} + \frac{\delta B_z}{\delta z} = 0 \qquad (3.1)$$

2. What is the meaning of the expressions $\frac{\delta^2 E_y}{\delta x \; \delta y}$, $\frac{\delta^2 E_z}{\delta x \; \delta z}$, $\frac{\delta^2 E_z}{\delta y \; \delta z}$, $\frac{\delta^2 E_x}{\delta y \; \delta x}$, $\frac{\delta^2 E_x}{\delta z \; \delta x}$, $\frac{\delta^2 E_y}{\delta z \; \delta y}$, $\frac{\delta^2 H_y}{\delta x \; \delta y}$, $\frac{\delta^2 H_z}{\delta x \; \delta z}$, $\frac{\delta^2 H_z}{\delta y \; \delta z}$, $\frac{\delta^2 H_x}{\delta y \; \delta x}$, $\frac{\delta^2 H_x}{\delta z \; \delta x}$ and $\frac{\delta^2 H_y}{\delta z \; \delta y}$ from Equations 2.4.2.17 and 2.5.2 for the electromagnetic wave?

3. What impact would Poynting's wave have on the interpretation of Hertzian waves?

4. What does Equation 3.2 describe and under what conditions is it valid? $|\vec{S}|$ stands for the absolute value of the pointing vector from Equation 2.5.5.1.

$$|\vec{S}| \cdot \mathrm{e}^{(-j\omega t)} = |\vec{S}| \cdot (\cos(j\,\omega\, t) - \sin(j\,\omega\, t)) \tag{3.2}$$

$$\vec{S} = \vec{E} \times \vec{H} \tag{2.5.5.1}$$

5. What effect does Equation 3.3 have on the electromagnetic wave equation?

$$\vec{v} \; \mathrm{div}(\vec{B}) = \vec{j}_m \tag{3.3}$$

4. CONCLUSION

First, in this elaboration, a transversal wave was derived from Equations 2.4.2.17 and 2.5.2, as described by Heinrich Hertz. However, both equations also offered the possibility of a respective longitudinal wave. In the elaboration "The Reinterpretation of the 'Maxwell Equations'" (Martin, 2021) it was shown mathematically that $\mathrm{div}(\vec{B}) = \rho_m$ is a condition without which the law of induction cannot work. The expression $(\mathrm{Sp})(\mathrm{grad}\,\vec{B}) = \mathrm{div}(\vec{B})$ makes this connection since $(\mathrm{Sp})(\mathrm{grad}\,\vec{B})$ is the basis for $\frac{\delta \vec{B}}{\delta t}$. This results in the already described longitudinal wave for the wave equation. At this point it is assumed that the electromagnetic wave is not a purely transverse wave, but a combination of transverse and longitudinal waves.

It follows from the fact that the electromagnetic wave is a wave that can be described by alternating sources and sinks moving in space. These sources and sinks are then the cause of the field lines, both from the electric field and from the magnetic field.
Furthermore, the Poynting vector was used to derive a longitudinal wave based on the electromagnetic wave, which is suitable for energy transport. On May 20, 1891, Nikola Tesla demonstrated some experiments at Columbia College in New York. All in all, this means that the electromagnetic wave equation should be reinterpreted, since the described longitudinal waves may result in new possible applications both in technology and in other areas.

5. CONFLICTS OF INTEREST

The author(s) declare(s) that there is no conflict of interest regarding the publication of this article.

6. PROOF OF FINANCING

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7. BIBLIOGRAPHY:

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