The Role of Acceleration in the Twin Paradox

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Abstract

In this paper we discuss the role of the acceleration in solving the Twin Paradox of Special Relativity.

Key Words: special relativity, general relativity, twin paradox.

1 Introduction

In the well know the Twin Paradox in the Theory of Relativity, one twin (Alan) stays on earth while the other one (Bob) takes a rocket for an interstellar travel. When Bob returns home he will be younger then his brother due to the relativity effects. However, for the reciprocity of these effects (Bob is moving with respect to Alan $\Leftrightarrow$ Alan is moving with respect to Bob) Alan should also be younger then Bob. This is the paradox!
In this paper we will discuss the following statement:

The Twin Paradox can be explained by the fact that the travelling twin must markedly accelerate in at least three phases of the trip (beginning, direction change, and end), while the other will only experience negligible acceleration due to rotation and revolution of Earth. During the acceleration phases of space travel, time dilation is not symmetric.

Which is present on many web pages and which is a common way to summarise the solution to the paradox. In particular we will try to evaluate the effects of these accelerations quantitatively.

NOTE: We will use years as a unit for time an light-years as a unit for distance. In this units, the speet of light $c = 1$. Unit will be often omitted in the paper.

2 Solution of the Paradox in SR

In SR the paradox is often solved using the Minkowski diagram. The travel of Bob can be divided in two legs in both of which Bob is in an inertial reference frame:

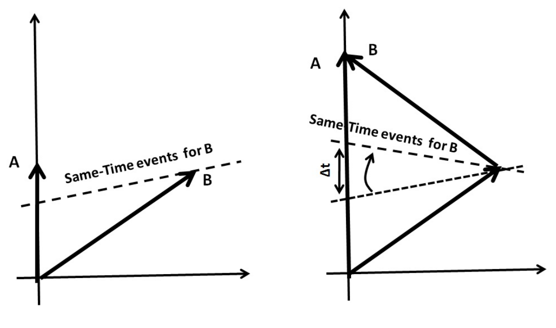

Figure 1: Solution in SR

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In both legs Bob will see Alan getting older faster and viceversa (for the reciprocity of the effects) however, when Bob inverts his direction, the line of contemporaneity for Bob will change so that for him a negligible amount of time will pass, while for Alan this phase will last an amount of time Δt of many years.

As a final result, although for Alan his brother Bob was ageing faster then him in the two legs, taking into account the time it took for his brother to turn (virtually not getting any older in the process), at the end of the trip, his brother Bob will be younger then him.

3 The Effect of the Acceleration in SR

We want to take into account the effect of acceleration. Alan sits comfortably in his inertial reference frame while his brother Bob takes a trip in a non inertial reference frame. According to Alan and to SR, in each of the two legs his brother Bob spends a proper time period $\Delta\tau_L$:

$$\Delta\tau_L = \frac{\Delta t_L}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \tag{1}$$

where Δt_L is the time experienced by Alan and v is the absolute value of the Speed of Bob in Alan inertial reference system.

When bob sets off, turns and arrives back home, he experiences three accelerations that should slow down his proper time. We may suppose that, since this effect is not reciprocal, at the end of the trip he should be the youngest brother in perfect numerical accord with the previous solution. Let us evaluate the proper time of Bob during the acceleration.

In SR the proper time period $\Delta\tau$ spent by an object undergoing a constant acceleration g is given by (see [1]):

$$\Delta\tau_g = \frac{c}{g} arcsinh\left(\frac{g\Delta t_g}{c}\right) \tag{2}$$

where the above equation is valid only in the case the object is accelerated from a null velocity to a velocity v for a period of time Δt_g with:

$$\Delta t_g = \frac{v}{g} \tag{3}$$

During the trip Bob undergoes two identical accelerations and two identical deceleration. Since the effect is symmetrical, the proper time period experienced by Bob during the acceleration and the deceleration is identical and therefore the contribution to the proper time experienced by Bob in the trip due to acceleration is is equal to $4\Delta\tau_g$.

To understand the impact of the acceleration on the final age of Bob, let us make a numerical example plugging some numbers in the above equations.

Input data:

$$\begin{cases} v &= 0.7 \\ \Delta t_L &= 1 \text{ (year)} \\ \Delta t_g &= 1/365 \text{ (1 day)} \end{cases} \tag{4}$$

We get:

$$\begin{cases} g &= 255.50 \\ \Delta\tau_L &= 1.4003 \text{ (year)} \\ \Delta\tau_g &= 0.0025545 \text{ (0.93 day)} \end{cases} \tag{5}$$

4 The Effect of the Acceleration in GR

Since in GR acceleration is equivalent to gravitational field, we may infer that we need to take into account GR for an exact solution of the Twin Paradox. However, we have seen that Bob can be accelerated and decelerated in a time which is negligible in Alan proper time with respect of the whole trip (one day in the example above). Moreover, since acceleration is equivalent to gravity, in this phase the proper time of Bob can only tick slower as in the case above where only SR is taken into account, and therefore this effect gives an additional contribution to the age of Alan negligible (only four days in the example above).

At the moment I struggle to find out how and if I need to take into account GR contribution. However, for the reason above this calculation is not necessary. I may add a proper analysis in a further issue of this paper.

5 Conclusions

From the analysis above we conclude that in the Twin Paradox, at east in the version where one twin flies away to a star and comes back, the effect of the accelerations is negligible. According to Alan, it takes a long time for Bob tu turn while for Bob the turn is most instantaneous. This is due to the fact that Bob line of contemporaneity rotates during the turn in the plane of the events and the effect is magnified by the fact that the two twins are far away.

As a final comment, given v we can always choose the acceleration/deceleration time Δt in such a way the required acceleration given by (3) makes the relativistic effects related to the acceleration negligible with respect to any reference frame, and after that make the two legs in which the speed is constant long enough to make Δt negligible with respect to them.

Different is the case where the two twins are close each other (i.e. bob flying in a circular orbit around Alan) where the acceleration may have a greater effect. But this is a different story!!!

I may analyse this case in a further version of the paper.

References

[1] Wikipedia. *Time dilation* - *https://en.wikipedia.org/wiki/Time_dilation*