Properties of Tiny Objects and Vast Things

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Abstract

Physics has yet to complete a catalog of properties of objects, complete a list of elementary particles, describe dark matter, and explain dark energy phenomena. This paper shows modeling that catalogs properties (such as charge and mass) and elementary particles (such as quarks and gluons). The catalog of properties includes known properties and suggests new properties. The catalog of elementary particles includes all known elementary particles and suggests new elementary particles. The modeling has bases in integer arithmetic and complements popular modeling that has bases in space-time coordinates. This paper shows applications that combine popular modeling, the expanded set of properties, and the expanded set of elementary particles. Applications describe dark matter, explain known ratios of dark matter effects to ordinary matter effects, point to possible resolutions for so-called tensions (between data and popular modeling) regarding dark energy phenomena, and suggest insight about galaxy evolution.

Keywords: elementary particles, dark matter, dark energy, galaxy formation, neutrino masses

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1. Introduction

1.1. Overview

This paper pursues the following goals.

1. Catalog properties of objects.
2. Catalog elementary particles.
3. Use a set of properties and a set of elementary particles to explain data that associate with the two-word term dark matter and to explain data that associate with the two-word term dark energy.

This paper intertwines the following modeling.

1. INFDA (or, inferences from data). INFDA has bases in observations of (or in data about) nature and in interpretations of observations. INFDA includes the following bases.
   (a) PRO (or, properties). PRO includes properties - such as charge, mass, and momentum - that provide bases for describing observations.
   (b) HPA (or, hypothesized attributes). HPA includes attributes - such as dark matter - that might provide bases for describing observations.
2. POST (or, popular space-time modeling). POST has bases in mathematics related to space-time coordinates. POST includes kinematics models, serves physics branches, and includes hypothesized attributes.
   (a) POST includes the following bases for KM (or, kinematics modeling).
      i. CM - Classical mechanics. CM includes the following.
         A. ND - Newtonian dynamics.
         B. SR - Special relativity.
         C. GR - General relativity.

ii. QM - Quantum mechanics. QM includes the following.
A. QF - Quantum field theory.

(b) POST serves the following physics branches, which collect, try to explain, and try to predict INFDA.
i. EP (or, elementary particles). EP POST includes the following collection of work.
A. SM (or, the elementary particle Standard Model). SM includes INFDA about EP. SM has bases in QF.
ii. CA (or, cosmology and astrophysics). CA POST includes the following collection of work.
A. CC (or, concordance cosmology). CC includes INFDA about stars, solar systems, black holes, galaxies, galaxy clusters, and so forth. CC has bases in ND, SR, and GR.
iii. OB (or, other branches). OB includes hadron physics, nuclear physics, atomic physics, optics, and so forth.

(c) POST includes HPA. The following trio associates with one set of HPA. POST does not provide specifications for dark matter or dark energy.
i. OM (or, ordinary matter). OM associates (approximately) with stuff that INFDA associates directly with observations of light.
ii. DM (or, dark matter). DM associates with INFDA that suggest more gravitational attraction between objects than the gravitational attraction that POST associates with OM.
iii. DE (or, dark energy). DE associates with INFDA that suggest (less gravitational attraction or even) gravitational repulsion between objects that POST associates with OM plus DM.

3. SAMO (or, single-attribute, multiple-objects modeling). SAMO is a subset of POST ND. SAMO has bases in one type of multipole-expansion mathematics.
4. SOMA (or, single-object, multiple-attributes modeling). SOMA has bases in integer mathematics (for which there is an associated type of multipole-expansion mathematics that differs from the type that underlies SAMO). Mathematics that underlies SOMA associates with the two-word term Diophantine equations. SOMA outputs a catalog of PRO (including charge and mass) of objects and a catalog of EP (including the electron, the Z boson, and all other known elementary particles).
(a) SOMA includes PRO that POST does not include. One PRO that SOMA includes and POST does not include is isomer. SOMA suggests that nature includes six isomers (or, near copies) of all EP except LRI (or, long-range interaction) elementary bosons. LRI elementary bosons include the (known) photon and the (might-be) graviton.

SOMA does not have direct bases in space-time coordinates. Mathematics underlying POST KM and mathematics underlying SOMA share the notion of degrees of freedom and share notions that associate with the three-word phrase isotropic harmonic oscillators. SOMA does not have direct bases in POST KM notions of tangent spaces to space-time spaces or in POST KM notions of phase spaces. POST KM regarding tangent spaces has bases in mathematics that associates with continuous coordinates. POST KM regarding phase spaces has bases in mathematics that associates with continuous coordinates. SOMA has bases in mathematics that associates with integers. SOMA points to PRO - such as velocity - that associate with POST KM regarding tangent spaces and with POST KM regarding phase spaces.

SAMO provides conceptual bridges between POST and SOMA. Each one of SAMO and SOMA has bases in multipole-expansion mathematics.

POST SM evolved based on proposals for new PRO and proposals for new EP.

SOMA proposes yet other new PRO (such as isomer) and yet other new EP. SOMA proposes specifications for DM and DE.

This paper shows that the combination of blending SOMA-proposed PRO with POST PRO and blending SOMA-proposed EP with POST EP suggests insight about EP INFDA and explains (otherwise seemingly unexplained) CA INFDA.

This paper points to possibilities for developing the following modeling.

1. NEST (or, new space-time modeling). NEST has bases in mathematics related to space-time coordinates. NEST embraces all POST that successfully describes INFDA. NEST embraces inputs from SOMA. Regarding INFDA, NEST improves on (or suggests bases for problems with) some not yet successful POST.
(a) NEST includes the POST bases for KM and the following new basis for KM.
i. EQ (or, extended quantum mechanics). If developed, EQ might enable modeling that is simpler than counterpart QM modeling. EQ might explain or predict INFDA that QM does not explain or predict. EQ would have bases in SOMA. EQ might need to have bases in some INFDA that POST QM explains or in some outputs from POST QM.

Table 1: Acronyms. This paper uses acronyms as singular nouns, plural nouns, and adjectives. The symbol † denotes the notion that one value of S pertains. The symbol ‡ denotes the notion that more than one value of S might pertain.

Acronym	Phrase
CA	Cosmology and astrophysics
CC	Concordance cosmology
CM	Classical mechanics
CMB	Cosmic microwave background radiation
DE	Dark energy
DM	Dark matter
EP	Elementary particle(s)
EQ	Extended quantum mechanics
GR	General relativity
HPA	Hypothesized attributes
IGM	Inter-galactic medium
INFDA	Inferences from data
IZDARP	Isomer-zero dark matter elementary particles
IZORDP	Isomer-zero ordinary matter elementary particles
KM	Kinematics modeling
LRI	Long-range interaction
ND	Newtonian dynamics
NEST	New space-time modeling
OB	Other branches (or, physics branches other than EP and CA)
OM	Ordinary matter
POST	Popular space-time modeling
PRO	Properties
QF	Quantum field theory
QM	Quantum mechanics
SAMO	Single-attribute (or, single-property), multiple-objects modeling
SL †	A field or an EP that associates with the spin-S LRI
SL ‡	A set of SL fields or of SL EP that associate with LRI
SM	(The) elementary particle Standard Model
SOMA	Single-object, multiple-attributes (or, multiple-properties) modeling
SR	Special relativity
TBD	To be determined

(b) NEST might add insight regarding at least the physics branches EP, CA, hadron physics, and atomic physics.

This paper proceeds as follows.

1. The introduction section provides context and perspective.
2. The methods section discusses aspects of SAMO and develops SOMA.
3. One results section deploys SOMA and POST to explain and predict EP INFDA that POST alone does not explain or predict. Relevant physics branches include EP and CA. Relevant HPA include OM and DM.
4. One results section deploys SOMA and POST to explain and predict CA INFDA that POST alone does not explain or predict. Relevant physics branches include CA. Relevant HPA include OM, DM, and DE.
5. One discussion section discusses some relationships that link INFDA, POST, and SOMA. Relevant physics branches include EP and CA. Relevant HPA include OM, DM, and DE.
6. One discussion section deploys SOMA to suggest aspects of NEST and EQ. Relevant physics branches include EP, CA, and some OB.
7. The concluding remarks section summarizes the paper.

Table 1 lists acronyms that this paper uses.

1.2. Information about POST

1.2.1. Electromagnetism, gravity, physics constants, and physics properties

POST discusses possibilities for relationships between electromagnetism and gravity. Reference [1] explores notions of a coupling between electromagnetism and gravity. Reference [2] and reference [3] discuss Einstein-Maxwell equations that suggest combining electromagnetic stress-energy tensors and the Einstein field equations, which have origins in modeling regarding gravitation. References [4] and [5] discuss gravitoelectromagnetism, which suggests similarities between gravity and electromagnetism.

Reference [6] and articles to which reference [6] alludes discuss, at least in the context of GR, possible relationships between mass and angular momentum.

Reference [7] discusses notions of repulsive components of gravity.

1.2.2. Kinematics models

POST QF models feature conditionally convergent mathematics. Reference [8] discusses the use of renormalization to cope with conditional convergence. Reference [9] discusses an approach to renormalization.

1.2.3. Elementary particles

Reference [10] provides an overview of POST EP and POST SM.

Reference [11] lists some types of modeling that POST has considered regarding trying to extend the elementary particle Standard Model, including trying to suggest elementary particles that people have yet to find. Reference [12] provides information about some of these types of modeling. References [13], [14], and [15] provide information about modeling and about experimental results. Reference [16] (including reviews numbered 86, 87, 88, 89, 90, and 94) provides other information about modeling and about experimental results.

Reference [17] suggests the notions of DM charges and DM photons. Reference [18] suggests that DM might include hadron-like particles.

Reference [19] suggests the notion of an inflaton field.

Reference [20] discusses the notion of a graviton.

Reference [21] discusses notions of sterile neutrinos and heavy neutrinos.

Reference [22] notes that POST QF suggests that massless elementary particles cannot have spins that exceed two.

A symmetry regarding Maxwell's equations suggests that nature might include magnetic monopoles. Reference [23] discusses theory. Reference [15] reviews modeling and experiments regarding magnetic monopoles. Reference [24] discusses a search - for magnetic monopoles - that did not detect magnetic monopoles.

Reference [13] reviews modeling and experiments regarding axions. Reference [13] also notes modeling that suggests that nature might include axions.

Reference [14] reviews modeling and experiments regarding leptoquarks.

Reference [21] discusses modeling and data about neutrino masses and neutrino oscillations.

References [25], [26], [27], and [28] discuss experimental tests of theories of gravity.

1.2.4. Cosmology and astrophysics

Reference [29] provides an overview of CC and related topics regarding general physics, DM, and EP. Reference [30] provides an overview of cosmology. References [31], [32], [33], and [34] review aspects of CC. Reference [35] discusses observational tests for cosmological models.

Reference [36] discusses possibilities leading up to a Big Bang.

References [37] and [32] discuss inflation.

Reference [38] discusses attempts to explain the rate of expansion of the universe.

POST suggests so-called tensions between cosmology models and cosmology data. Reference [39] discusses such tensions.

POST suggests that CC underestimates recent increases in the rate of expansion of the universe. References [33], [40], [41], [42], and [43] discuss relevant notions. Reference [33] suggests that possible POST resolutions regarding such an underestimate might focus on phenomena early in the history of the universe.

Reference [44] discusses POST notions of possible types of DM.

Reference [45] suggests the following notions regarding DM. Most DM comports with notions of cold dark matter. Models that associate with the two-word term modified gravity might pertain; but - to the extent that the models suggest long-range astrophysical effects - such models might prove problematic. POST suggests limits on the masses of basic DM objects. Observations suggest small-scale challenges to the notion that all DM might be cold dark matter. People use laboratory techniques to try to detect DM. People use astrophysical techniques to try to infer properties of DM. (Reference [46] discusses astrophysical and cosmological techniques.)

Reference [47] discusses galaxy formation and evolution, plus contexts in which galaxies form and evolve. Reference [47] discusses parameters by which POST classifies and describes galaxies. Reference [47] seems not to preclude galaxies that have few OM stars. Reference [47] seems not to preclude galaxies that have little OM.

Reference [48] suggests that CC modeling might not adequately explain gravitational interactions between neighboring galaxies.

1.2.5. SAMO

Reference [49] discusses multipole expansions regarding POST ND electrostatics and the PRO of charge.

Reference [50] discusses a multipole expansion regarding POST ND gravitation and the PRO of mass.

Reference [51] discusses multipole expansions regarding acoustics.

1.3. Themes for some opportunities that this paper addresses

This paper suggests opportunities to expand physics abilities to successfully explain and predict INFDA.

This paper also suggests opportunities to broaden, unite, and simplify physics techniques for successfully explaining and predicting INFDA.

Regarding physics techniques, this paper addresses questions such as the following questions.

1. To what extent might it be possible to unite newer aspects of POST and older aspects of POST? The following provides an example.
   (a) To what extent might NEST CM reflect aspects of POST QM that include fermion aspects? For example, to what extent might NEST CM notions of potentials handle POST QM aspects such as the notion - for fermions - of Pauli exclusion? Here, POST QM uses notions of wave functions that are antisymmetric under the exchange of fermions. POST CM does not include wave functions.
2. To what extent might NEST QM provide alternatives to POST QM reliance on conditionally convergent mathematics? The following provides an example.
   (a) To what extent might NEST QM calculate anomalous magnetic moments without using POST QF techniques?
3. To what extent might NEST CM incorporate successful aspects of POST CM GR into NEST CM ND and NEST CM SR? The following notions provide motivation to do so.
   (a) POST GR might be incompatible with the SOMA notion of isomers.
   (b) POST GR might be incompatible with POST QF.

This paper reflects opportunities that associate with co-evolution of INFDA, SOMA, and NEST.

Regarding co-evolution, INFDA and POST co-evolved. For example, INFDA and POST co-evolved to posit a list of elementary particles. The list expanded over time. This paper includes notions that SOMA and NEST posit. Posits by SOMA and NEST might have similarities to assumptions that POST posits. Some posits by SOMA and NEST lead to explanations for aspects (such as a list of all known elementary particles) that POST posits. Further work, within or beyond SOMA and NEST, might lead to explanations for posits that SOMA and NEST make.

2. Methods

2.1. SAMO

POST deploys SAMO to characterize electrostatic and gravitational potentials that associate with systems of objects. Reference [49] discusses multipole expansions regarding electrostatic potentials that associate with (possibly complicated) distributions of objects that have nonzero charge. Reference [50] discusses a multipole expansion regarding gravitational potentials that associate with (possibly complicated) spatial distributions of objects that have nonzero mass. Regarding acoustics, reference [51] discusses cases in which the words monopole, dipole, and quadrupole associate with various numbers of similar speakers, with the geometric arrangement of the speakers, and with the relative phases of the sounds that each speaker emits.

SAMO can associate with the following notions.

- ND (or, modeling based on Newtonian dynamics) pertains.

- The objects do not move.
- Modeling associates with the mutual rest frame of the objects.
- Modeling associates with one (and not more than one) property (such as charge or mass).
- Modeling suggests an approximate characterization of a potential. The characterization features a sum of terms, with each term including a factor of the form that equation (1) shows. r is the distance from some point (which may be a so-called center of property, such as the center of mass). $n_{\Gamma'}$ is a nonnegative integer. Each term might include a factor that varies based on angular coordinates. Along an axis (or, straight line) that associates with a choice of angular coordinates, the approximation degrades as r decreases.

$$r^{-n_{\Gamma'}} \tag{1}$$

- The word monopole associates with $n_{\Gamma'} = 1$. The word dipole associates with $n_{\Gamma'} = 2$. The word quadrupole associates with $n_{\Gamma'} = 3$. The word octupole associates with $n_{\Gamma'} = 4$. The one-element term 16-pole can associate with $n_{\Gamma'} = 5$. And so forth.

2.2. From SAMO toward SOMA

POST includes properties that associate with motion.

Each one of POST and SOMA has bases in the notion that nonzero velocity can associate with three degrees of freedom (that do not associate with a constraint of zero velocity) and with a new property.

The property of charge-current (or, current of charge) associates with charge that moves.

SOMA associates the one-element term one-some with a static (or, non-moving) property and the one-element term three-some with a dynamic property that associates with linear motion of a static property. The SOMA (and POST) one-word term momentum associates with moving mass or moving energy.

SOMA associates the one-element term four-some with the combination of a one-some static property and a three-some for which the three-some property is the dynamic counterpart of the one-some static property.

POST includes the property of angular momentum. POST considers that - compared to zero angular momentum - nonzero angular momentum associates with three new degrees of freedom. (For example, one degree of freedom can associate with a magnitude of angular momentum and two degrees of freedom can associate with an axis of rotation.) SOMA echoes the POST degrees of freedom. SOMA considers angular momentum to be a one-some property.

SOMA considers that three-some properties can point to one-some properties. For example, the three-some property of charge-current points to the one-some property of magnetic moment. For each of charge-current and magnetic moment, three new (compared to for just static charge) degrees of freedom pertain. For charge-current, the three degrees of freedom associate with linear velocity. For magnetic moment, the three degrees of freedom can associate with angular velocity.

In POST ND for a point-like object and a one-some property that associates with a specific value of $n_{\Gamma'}$, the three-some property associates with $n_{\Gamma'} + 1$. For example, if the object has nonzero charge (which is a one-some property), the word monopole pertains regarding the potential that associates with the one-some property. The word dipole pertains regarding the potential that associates with the three-some property charge-current. Also, the word dipole pertains regarding the potential that associates with the (new) one-some property (magnetic moment) that associates with the (original) three-some property (charge current).

POST includes modeling - based on three spatial derivatives with respect to time - that features the series position, velocity, and acceleration. For an object, velocity associates with a first application (regarding position) of the derivatives. Acceleration associates with a second application (regarding position) of the derivatives. SOMA does not embrace the notion that acceleration might be a three-some partner to a one-some velocity. The following sentences provide an explanation. SOMA (and also some aspects of POST) considers that the object models (not completely as an independent object but) as part of a system that includes objects that generate the forces that lead to nonzero acceleration. A notion of (at least a partial) loss of identity pertains regarding the object.

In POST QF, notions of fields tend to supplant notions of objects. Fields - in effect - observe each other. A field pervades the physical space that associates with the domain of the mathematical coordinates that provide the basis for modeling.

POST associates the notion of a spin-1 boson - the photon - with the electromagnetic field. POST includes the notion that one can model a photon in terms of integer units of circular polarization. (This discussion de-emphasizes modeling that has bases in linear polarization. This discussion does not run counter to the notion that - for POST - modeling based on linear polarization can pertain. POST QM includes means for computing amplitudes that associate with linear polarization from amplitudes that associate with circular polarization.) One unit of circular polarization associates with an angular momentum of magnitude $\hbar$. A photon associates with two modes. One mode associates with a nonnegative integer number of units of left-circular polarization. One mode associates with a nonnegative integer number of units of right-circular polarization. In POST, the two modes model as being independent of each other. For example, two units of left-circular polarization and one unit of right-circular polarization do not combine to net to one unit of left-circular polarization.

SOMA has a basis in mathematics that associates with a notion of adding and subtracting unequal integer units of circular polarization. For example, mathematically, subtracting one unit of right-circular polarization from two units of left-circular polarization yields one unit of left-circular polarization.

For such a sum for which there is more than one term, the individual terms do not directly associate with fields (such as the electromagnetic field). The sum can associate with a field (such as the electromagnetic field).

SOMA associates positive sums with left-circular polarization and associates negative sums with right-circular polarization. (This choice - instead of a choice in which positive sums associate with right-circular polarization and negative sums associate with left-circular polarization - is arbitrary. Making the choice affects choices of words and does not affect results from modeling.)

A sum that totals to plus one associates with electromagnetism and the left-circular polarization mode for electromagnetic fields. A sum that totals to minus one associates with electromagnetism and the right-circular polarization mode for electromagnetic fields. A sum that totals to plus two associates with gravitation and the left-circular polarization mode for gravitational fields. A sum that totals to minus two associates with gravitation and the right-circular polarization mode for gravitational fields.

SOMA posits that a mathematical basis for SOMA needs to embrace the following notions.

- Modeling should point to PRO (including electromagnetic properties such as charge and gravitational properties such as mass) that associate with POST KM.
- For a four-some for which POST associates the word scalar (as opposed to vector or tensor) with the one-some property, SOMA should not be incompatible with the notion that POST might associate the SR notion of four-vector with the four-some. (SOMA needs to be compatible with various POST KM, including SR.)
- Modeling should not necessarily depend on the POST QM notion of quantized units of excitations for (for example) electromagnetic fields. (SOMA needs to be compatible with various POST KM, including ND and SR.)
- Uses of space-time coordinates should not necessarily apply. (SOMA needs to be compatible with various POST KM.)
- Extending modeling to reflect three new degrees of freedom should be straightforward.

2.3. Mathematical bases for SOMA and physical interpretations of SOMA

Equation (2) shows a term in which k is a positive integer and s_k can be one of minus one, zero, or plus one.

$$ks_k \tag{2}$$

SOMA associates the integer k with the POST notion of $k\hbar$ units of angular momentum. SOMA associates $s_k = +1$ with the POST notion of left-circular polarization and with mathematics that associates with the three-element term one-dimensional harmonic oscillator. SOMA associates $s_k = -1$ with the POST notion of right-circular polarization and with mathematics that associates with the three-element term one-dimensional harmonic oscillator.

Equation (3) shows a sum of terms of the form that equation (2) shows. K_n denotes a set of relevant values of k. (An integer k appears no more than once in such a sum.)

$$s = \sum_{k \in K_n} ks_k \tag{3}$$

Regarding sums of the form that equation (3) shows, the symbol k_{max} denotes the largest value of k for which $|s_k| = 1$.

Per reference [52], for integers n for which $n \geq 2$, mathematics associates $SU(n)$ symmetry with the ground state of an n-dimensional isotropic harmonic oscillator. SOMA uses the expression gen(*group*) to denote the number of generators of the group *group*. Mathematics provides, for integers $n \geq 2$, the result $\text{gen}(SU(n)) = n^2 - 1$.

For each value of k, SOMA considers that relevant mathematics associates with a two-dimensional isotropic harmonic oscillator that associates with the two one-dimensional harmonic oscillators that associate, respectively, with $s_k = +1$ and $s_k = -1$. Mathematics associates $SU(2)$ symmetry with the ground state of the two-dimensional isotropic harmonic oscillator. SOMA associates the three generators of the $SU(2)$ group with three degrees of freedom.

Mathematics associates a $U(1)$ symmetry with a one-dimensional harmonic oscillator. Mathematics provides that $\text{gen}(U(1)) = 1$.

SOMA associates $s_k = 0$ with mathematics that associates with the three-element term one-dimensional harmonic oscillator. SOMA associates the one generator of the $U(1)$ group that associates with an s_k value of $s_k = 0$ with one degree of freedom.

Equation (4) defines Σ.

$$\Sigma \equiv |s| \tag{4}$$

For electromagnetism, equation (3) and the equation $\Sigma = 1$ provide a touch point between POST and SOMA. SOMA posits that, regarding POST, $s = +1$ associates with the left-circular polarization mode of the electromagnetic field. SOMA posits that, regarding POST, $s = -1$ associates with the right-circular polarization mode of the electromagnetic field. SOMA posits that solutions that associate with $\Sigma = 1$ associate with electromagnetic properties of objects.

For each solution to equation (3), there is exactly one other solution that features a sign reversal for each non-zero s_k. SOMA uses the one-element term solution-pair to denote such a pair of solutions.

SOMA associates the symbol 1L with the electromagnetic field. POST includes the notion that an excitation - by an object - of the electromagnetic field associates with the set (that is relevant to the object) of electromagnetic properties for which the object exhibits nonzero values. SOMA posits that each one of the properties associates with one solution-pair. If the set of relevant solution-pairs includes more than one solution-pair, SOMA posits that excitations do not carry information sufficient for an observation (which associates with a de-excitation of the electromagnetic field) to specify just one solution-pair as being associated with the excitation.

For $K_n = \{1\}$, SOMA associates the three degrees of freedom that associate with $s_1 = \pm 1$ with three spatial degrees of freedom. The three degrees of freedom associate with the POST notion of potentials that can associate with non-moving charges and electric fields.

Compared to $K_n = \{1\}$, $K_n = \{1, 2\}$ associates with adding three degrees of freedom. SOMA posits that the three new degrees of freedom can associate with POST notions of moving charge and potentials that associate with magnetic fields.

SOMA associates with two cases regarding $K_n = \{1, 2\}$ and moving charge. For each case, $1 = \Sigma = |s| = |-1 + 2|$ for each of the two solutions that associate with the relevant solution-pair. (One of the two solutions associates with $s_1 = -1$ and $s_2 = +1$. The other one of the two solutions associates with $s_1 = +1$ and $s_2 = -1$. Regarding details regarding solution-pairs, this paper generally shows the solution for which $s_{k_{max}} = +1$ and usually does not show the solution for which $s_{k_{max}} = -1$.)

For the first case, the three additional (compared to $K_n = \{1\}$) degrees of freedom associate with (linear) velocity. Here, regarding POST SR, charge (which associates with $K_n = \{1\}$) associates with one (temporal) component of a four-vector. Charge-current (which associates with $K_n = \{1, 2\}$) associates with three (spatial) components of the four-vector. SOMA associates the one-element term one-some with the property (charge) and the one-element term three-some with the current (charge-current) of charge.

POST KM does not necessarily require the use of SR. In general, SOMA associates the notion of one-some with an intrinsic property (such as charge). In general, SOMA associates the notion of three-some with an extrinsic property (such as velocity).

For the second case, the three additional (compared to $K_n = \{1\}$) degrees of freedom associate with angular velocity. Here, one-sum use of the solution-pair associates with the property of magnetic moment. POST might consider that a three-some notion of (linear) velocity (of an object that has non-zero magnetic moment) can pertain.

SOMA associates with the notion that - for a property to which SOMA points - POST terminology can vary based on a choice of KM. For example, regarding one-some use of the SOMA solution-pair for which $K_n = \{2\}$, the word mass can pertain regarding POST ND and the word energy can pertain regarding POST GR.

Equation (5) shows notation that SOMA associates with solution-pairs. The letter g is a convenience (regarding notation) that this paper chooses based on the notions that (for $\Sigma = 1$) one might think of the two-word term gamma rays and that (for $\Sigma = 2$) one might think of the word gravity. The symbol Γ denotes a list - in ascending order - of the positive integers k for which $|s_k| = 1$.

$$\Sigma \mathrm{g} \Gamma \tag{5}$$

For example, for a solution-pair for which one-some use associates with magnetic moment, the expression 1g1‘2 pertains regarding ΣgΓ. (This paper uses the symbol ‘ to separate integers in lists.)

Technically, Γ is not a set. Γ is a list of elements that are members of a set K_n. SOMA uses the notation $k \in \Gamma$ to denote the notion that k appears in the list Γ.

In SOMA, n_Γ denotes the number of members of a set K_n (and, equivalently, the number of elements in a list Γ that associates with a set K_n). SOMA associates the word monopole with $n_\Gamma = 1$. SOMA associates the word dipole with $n_\Gamma = 2$. SOMA associates the word quadrupole with $n_\Gamma = 3$. And so forth. (Table 2 illustrates such associations.)

SOMA uses symbols of the form Σg’ to denote solution-pairs for which the integer $\Sigma \in \Gamma$ (or, for which Σ appears in the list Γ). For example, 1g1‘2 associates with 1g’. Symbols of the form Σg” denote solution-pairs for which the integer Σ does not appear in the list Γ. For example, 3g1‘2 associates with 3g”. (Here, one solution associates with $s = +1 + 2 = +3$.)

Regarding equation (1), $n_{\Gamma'}$ associates with n_Γ that associate with Σg’ solution-pairs. $n_{\Gamma'}$ does not associate with n_Γ that associate with Σg” solution-pairs. SOMA associates the symbol $n_{\Gamma''}$ with n_Γ that associate with Σg” solution-pairs.

SOMA uses the word cascade (or, the two-word phrase cascade step) to describe results of processes that add - to one Γ - one new integer and thereby produce a new Γ. For a cascade step that starts from a solution-pair $\Sigma_1 \mathrm{g} \Gamma_1$ and that produces a solution-pair $\Sigma_2 \mathrm{g} \Gamma_2$, SOMA limits (regarding the cascade) resulting solution-pairs to solution-pairs for which Σ_2 matches Σ_1. For one original solution-pair, more than one cascade solution-pair might pertain.

SOMA also uses the word cascade to refer to a network of solution-pairs that cascade (from each other) based on multiple cascade steps that ensue from one solution-pair. The solution-pair 1g1‘2 associates with a first step in a cascade that starts with the solution-pair 1g1. One-some use of the 1g1‘2 solution-pair associates with the property of magnetic moment. A next cascade step provides the 1g1‘2‘4 solution-pair. Three-some use of the 1g1‘2‘4 solution-pair associates with the motion of the property of magnetic moment. One-some use of the 1g1‘2‘4 solution-pair can associate - for the planet Earth - with the notion that the axis of rotation does not align with the axis of the magnetic field. The direction of the axis of the magnetic field rotates with a period of one day. This rotation associates with a (new) PRO of the planet Earth.

To the extent that SOMA can underlie POST SR, a solution-pair $\Sigma_1 \mathrm{g} \Gamma_2$ that cascades in one step from a solution-pair $\Sigma_1 \mathrm{g} \Gamma_1$ dilutes the potential that associates with the solution-pair $\Sigma_1 \mathrm{g} \Gamma_1$. One-some aspects related to $\Sigma_1 \mathrm{g} \Gamma_1$ equals 1g1 and three-some aspects related to $\Sigma_1 \mathrm{g} \Gamma_2$ equals 1g1‘2 provide an example. As one increases the velocity (and hence the charge-current) of an object that contributes to the electromagnetic field, observers sense an increase in charge. Yet, the electric-field component of the potential does not change. In effect, the magnetic-field component of the potential dilutes the effect of the charge that observers sense.

Generalization leads to the following notions.

1. One-some uses of 1g’ solution-pairs that further cascade associate with electromagnetic properties of objects.
2. One-some uses of 2g’ solution-pairs that further cascade associate with gravitational properties of objects. For example, regarding POST modeling the following notions pertain. (The three-word phrase some aspects of leaves open the possibility that other solution-pairs can also associate with the notion that follows the three-word phrase some aspects of.)
   (a) 2g2 associates with mass (for example, regarding ND) or energy (for example, regarding GR) and with attraction between objects. (For GR, 2g2 associates with one on-diagonal component of the stress-energy tensor.)

(b) 2g2'4 associates with rotation and with dilution of attraction between objects. (For GR, 2g2'4 associates with - some aspects of - rotational frame-dragging; with - some aspects of - three on-diagonal components of the stress-energy tensor, and with - some aspects of - gravitationally repulsive pressure.)
(c) 2g1'2'3 associates with attraction between objects. (For GR, 2g1'2'3 associates with - some aspects of - the off-diagonal components of the stress-energy tensor.)
(d) 2g' monopole, quadrupole, and 16-pole solution-pairs associate with attraction between objects.
(e) 2g' dipole and octupole solution-pairs associate with dilution of attraction between objects.
(f) For a pair of similar objects that always move away from each other, the dominating gravitational effects transit (over time) all or a portion of the following sequence: 16-pole attraction, octupole repulsion, quadrupole attraction, dipole repulsion, and monopole attraction. Generally, a pair of neighboring less-massive objects transits all or a portion of the sequence more rapidly than a pair of neighboring more-massive objects.

3. Quadrupole and higher-order-pole solution-pairs associate with interactions between fields.
4. One-some uses of monopole solution-pairs associate with POST notions of scalar properties. For 1g', one-some use of 1g1 associates with charge. For 2g', one-some use of 2g2 associates with rest energy (for GR) or with mass (for ND).
5. Three-some uses of dipole solution-pairs associate with POST notions of velocities and with currents of scalar properties.

2.4. SOMA notions regarding isomers of elementary particles

For a POST ND potential that would associate with r^0, no force would pertain.

SOMA posits that the oscillator pair that associates with $s_0 = \pm 1$ associates with three generators and three so-called isomer-pairs. (Regarding SOMA based on terms ΣgΓ, only positive values of k associate with lists Γ and sets K_n.)

Each ΣgΓ solution-pair associates with two solutions. SOMA associates the one-element term left-solution with one of the two solutions. SOMA associates the one-element term right-solution with the other one of the two solutions.

SOMA uses the acronym LRI to abbreviate the two-element term long-range interaction. LRI fields associate with ΣgΓ solution-pairs for which $\Sigma \geq 1$. The electromagnetic field associates with $\Sigma = 1$. The gravitational field associates with $\Sigma = 2$. SOMA posits relevance for LRI fields for which $\Sigma = 3$ and $\Sigma = 4$.

Per table 6, SOMA posits that elementary particles associate with solution-pairs for which $\Sigma = 0$.

SOMA posits that each LRI field associates with a zero-mass elementary boson. SOMA associates the symbol 1L with the electromagnetic field and with the photon, the symbol 2L with the gravitational field and with the would-be graviton, the symbol 3L with a possible carrier of $\Sigma = 3$ LRI, and the symbol 4L with a possible carrier of $\Sigma = 4$ LRI. For a symbol SL, the symbol S denotes the POST notion of spin (as in the POST expression $S(S+1)\hbar^2$).

SOMA posits the relevance of six isomers of the set of all elementary particles except LRI elementary bosons. Here, six equals three (as in the number of isomer-pairs) times two (as in the number of solutions in a solution-pair).

Within each isomer-pair the following notions pertain. One isomer associates with the POST notions (for nonzero mass, nonzero spin elementary particles) of left-handedness for particles and right-handedness for antiparticles. The other isomer associates with the POST notions (for nonzero mass, nonzero spin elementary particles) of right-handedness for particles and left-handedness for antiparticles.

The oscillator that associates with $s_0 = 0$ associates with one generator and the notion of not being associated with just one specific isomer. SOMA posits that LRI fields and instances of LRI elementary bosons are not specific to just one isomer.

SOMA names the isomers with one-element terms - isomer-zero, isomer-one, ..., and isomer-five. The three isomer-pairs associate, respectively, with isomer-zero and isomer-three, isomer-one and isomer-four, and isomer-two and isomer-five. The notion of left-handed particles (in the context of the three-word phrase particle and antiparticle) associates with each one of isomer-zero, isomer-two, and isomer-four. The notion of right-handed particles (in the context of the three-word phrase particle and antiparticle) associates with each one of isomer-one, isomer-three, and isomer-five.

The masses of counterpart non-LRI elementary particles do not vary between isomers.

- For each one of isomer-zero and isomer-three, the flavour of the lowest-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the highest-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the intermediate-mass charged lepton equals the remaining quark flavour.
- For each one of isomer-one and isomer-four, the flavour of the lowest-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the intermediate-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the highest-mass charged lepton equals the remaining quark flavour.
- For each one of isomer-two and isomer-five, the flavour of the intermediate-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the highest-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the lowest-mass charged lepton equals the remaining quark flavour.

SOMA posits that POST associates with isomer-zero. For example, POST non-LRI elementary particles that have nonzero mass and nonzero spin associate with left-handedness.

POST does not associate with the other five isomers that SOMA includes.

SOMA suggests that - generally - effects that associate with the five non-isomer-zero isomers measure as effects of POST notions of DM. SOMA suggests that effects that associate with some isomer-zero instances of as-yet-unfound elementary particles also measure as effects of POST notions of DM.

The SOMA notion of six isomers provides a basis for explaining INFDA, including ratios of DM effects to OM effects. Per discussion (in this paper) that cites reference [53], SOMA notions regarding matches between flavours of charged leptons and flavours of quarks point to the notion that SOMA is not necessarily incompatible with observations that pertain to the Bullet Cluster collision of two galaxy clusters.

2.5. SOMA notions regarding the strong interaction

POST ND associates each one of the electromagnetic potential and the gravitational potential - that a non-zero mass, nonzero charge object generates - with a spatial dependence that is proportional to r^{-1}.

Regarding the strong interaction, POST QF includes the notion of asymptotic freedom and an associated (approximate) potential that is proportional to r^{+1}.

SOMA posits that $k = -1$ associates with the strong interaction. (Regarding SOMA based on terms $\Sigma \mathrm{g}\Gamma$, only positive values of k associate with lists Γ and sets K_n.)

The oscillator pair that associates with $s_{-1} = \pm 1$ associates with three generators. SOMA associates the three generators with the three POST color charges - red, blue, and green.

The oscillator that associates with $s_{-1} = 0$ associates with one generator and the notion of no (or, white or clear) color charge.

2.6. Details regarding SOMA solution-pairs

Table 2 alludes to all $s = \sum_{k \in K_n} (ks_k)$ expressions for which $1 \leq k \leq k_{max} \leq 4$.

SOMA includes solution-pairs for which integers k for which $k \geq 5$ pertain. For each of those solution-pairs, $k_{max} \geq 5$ pertains. In general, the following notions pertain.

SOMA posits that each relevant solution-pair comports with equation (6).

$$1 \in \Gamma \text{ or } 2 \in \Gamma \text{ or } 3 \in \Gamma \text{ or } 4 \in \Gamma \tag{6}$$

For each solution-pair $\Sigma \mathrm{g}\Gamma$, equation (7) defines k_{n_0}.

$$k_{n_0} \equiv \max\{k | 1 \leq k \leq 4 \text{ and } k \in \Gamma\} \tag{7}$$

For each solution-pair $\Sigma \mathrm{g}\Gamma$, equation (8) computes n_0.

$$n_0 = \text{ the number of } k \text{ for which } 1 \leq k \leq k_{n_0} \text{ and } k \notin \Gamma\} \tag{8}$$

Equation (6) and equation (8) imply that the range $0 \leq n_0 \leq 3$ pertains regarding n_0.

For $n_\Gamma \geq 4$, each one of some combinations of Γ and Σ associates with more than one solution-pair. For a combination of Γ and Σ that associates with more than one solution-pair, equation (9) shows a symbol that SOMA uses.

$$\Sigma \mathrm{g}\Gamma \mathrm{x} \tag{9}$$

Table 2: $\Sigma = |s| = |\sum_{k \in K_n}(ks_k)|$ solution-pairs for which $1 \leq k \leq k_{max} \leq 4$. The columns labeled $1 \cdot s_1$ through $4 \cdot s_4$ show contributions that associate with terms of the form ks_k. Regarding table 2, the integer n_0 equals the number of k for which $1 \leq k \leq k_{max} \leq 4$ and $s_k = 0$. The integer n_Γ equals the number of k for which $1 \leq k$ and k appears in the list Γ. The number $n_{\Sigma g \Gamma}$ equals $2^{n_\Gamma - 1}$ and states the number of solution-pairs. The column for which the one-element label is SOMA-pole associates mathematically with the number of solution-pairs. For a row for which exactly one solution-pair pertains, the column shows the word monopole. For a row for which exactly two solution-pairs pertain, the column shows the word dipole. For a row for which exactly four solution-pairs pertain, the column shows the word quadrupole. For a row for which exactly eight solution-pairs pertain, the column shows the word octupole. For the case of octupole, each one of $\Sigma = 2$ and $\Sigma = 4$ associates with two solution-pairs. Regarding $\Sigma = 2$, $|-1+2-3+4| = 2 = |-1-2-3+4|$. Regarding $\Sigma = 4$, $|-1-2+3+4| = 4 = |+1+2-3+4|$.

k_{max}	Γ	$1 \cdot s_1$	$2 \cdot s_2$	$3 \cdot s_3$	$4 \cdot s_4$	Σ	n_0	n_Γ	$n_{\Sigma g \Gamma}$	SOMA-pole
1	1	± 1	-	-	-	1	0	1	1	Monopole
2	2	0	± 2	-	-	2	1	1	1	Monopole
2	1‘2	± 1	± 2	-	-	1,3	0	2	2	Dipole
3	3	0	0	± 3	-	3	2	1	1	Monopole
3	1‘3	± 1	0	± 3	-	2,4	1	2	2	Dipole
3	2‘3	0	± 2	± 3	-	1,5	1	2	2	Dipole
3	1‘2‘3	± 1	± 2	± 3	-	0,2,4,6	0	3	4	Quadrupole
4	4	0	0	0	± 4	4	3	1	1	Monopole
4	1‘4	± 1	0	0	± 4	3,5	2	2	2	Dipole
4	2‘4	0	± 2	0	± 4	2,6	2	2	2	Dipole
4	3‘4	0	0	± 3	± 4	1,7	2	2	2	Dipole
4	1‘2‘4	± 1	± 2	0	± 4	1,3,5,7	1	3	4	Quadrupole
4	1‘3‘4	± 1	0	± 3	± 4	0,2,6,8	1	3	4	Quadrupole
4	2‘3‘4	0	± 2	± 3	± 4	1,3,5,9	1	3	4	Quadrupole
4	1‘2‘3‘4	± 1	± 2	± 3	± 4	0,2,2,4,4,6,8,10	0	4	8	Octupole

2.7. Reaches that associate with SOMA solution-pairs

SOMA uses the word instance and the word reach to describe aspects of the extent to which LRI solution-pairs associate with interactions within and between isomers.

SOMA posits that equation (10) pertains for each LRI solution-pair. The positive integer n_I denotes a number of instances of a solution-pair. The positive integer R_I denotes the reach - in number of isomers - that associates with one instance of the solution-pair.

$$n_I R_I = 6 \tag{10}$$

INFDA and POST suggest that, to a first approximation, DM appears - to OM - to be electromagnetically dark. SOMA posits that, to a first approximation, each isomer appears - to each other isomer - to be electromagnetically dark. For the solution-pair 1g1, SOMA posits that $n_I = 6$ and $R_I = 1$. SOMA points to six instances of the PRO of charge.

INFDA and POST suggest that gravity associates with interactions between OM and DM and interactions between DM and DM. For the solution-pair 2g2, SOMA posits that $n_I = 1$ and $R_I = 6$. SOMA points to one instance of the PRO of mass.

SOMA posits - based in part on information that table 2 provides - the following extrapolations. (Discussion related to equation (8) pertains.) For $n_0 = 0$, $R_I = 1$ pertains. For $1 \leq n_0 \leq 3$, equation (11) pertains.

$$R_I = \text{gen}(SU(7))/\text{gen}(SU(2n_0 + 1)) \tag{11}$$

Thus, SOMA posits the following reaches for one-some uses of LRI solution-pairs - $R_I = 1$ for $n_0 = 0$, $R_I = 6$ for $n_0 = 1$, $R_I = 2$ for $n_0 = 2$, and $R_I = 1$ for $n_0 = 3$.

SOMA posits that the R_I for a three-some use of an LRI solution-pair equals the R_I for the one-some use of the solution-pair from which the three-some solution-pair cascades in one step. (Otherwise, the notion of moving property might not comport with the notion of property.)

Equation (12) shows SOMA notation for the notion that a reach of R_I associates with each instance of a solution-pair $\Sigma g \Gamma$.

$$\Sigma(R_I)g\Gamma \tag{12}$$

SOMA extends the use of the notation (that equation (12) shows) to include $\Sigma(R_I)\Phi\Gamma$, in which Φ associates with a notion of a so-called family of elementary particles. (Table 6 and discussion related to table 6 provide details regarding families of elementary particles.) Φ =L associates with LRI fields. Other than for Φ =L, SOMA posits that $R_I = 1$.

2.8. Strengths of gravitational interactions

SOMA posits that an excitation of a field (such as the 2L field - or, the gravitational field) encodes knowledge of the instances of the properties that associate with the excitation. In effect, the field carries that knowledge.

SOMA points to the following notions about gravitational interactions between an object A and an object C. Here, the symbol m_A denotes the mass of the object A and the symbol m_C denotes the mass of the object C. The symbol $f_{A,i}$ denotes the fraction of m_A that associates with isomer-i. The symbol $f_{C,i}$ denotes the fraction of m_C that associates with isomer-i. This discussion de-emphasizes the role of LRI fields in binding object A into an object and the role of LRI fields in binding object C into an object.

The following paragraphs discuss some specific cases.

For interactions between the Sun (as an object A) and a photon (as an object C) that isomer-zero stuff emitted, $f_{A,0} = 1$, $f_{A,i} = 0$ for each of the other five isomers, $f_{C,0} = 1$, and $f_{C,i} = 0$ for each of the other five isomers. SOMA does not suggest concerns regarding POST that is based on GR.

For interactions between the Sun (as an object A) and an OM planet (as an object C), $f_{A,0} = 1$, $f_{A,i} = 0$ for each of the other five isomers, $f_{C,0} = 1$, and $f_{C,i} = 0$ for each of the other five isomers. SOMA does not suggest concerns regarding POST that is based on GR.

For interactions between the Sun (as an object A) and a one-isomer planet (as an object C), results regarding components - of 2L - for which $R_I \neq 6$ can vary by the isomer that associates with object C. For example, the Sun rotates and the reach of an instance of a one-some use of the solution-pair 2g2‘4 is two isomers. If the one-isomer planet associates with isomer-zero or isomer-three, the one-some 2g2‘4 component of 2L (that associates with object A) affects the trajectory that associates with the orbit of object C. If the one-isomer planet associates with isomer-one, isomer-two, isomer-four, or isomer-five, the one-some 2g2‘4 component of 2L (that associates with object A) does not affect the trajectory that associates with the orbit of object C.

The following paragraphs and equations point to notation and notions regarding general cases.

For a one-some use of a 2L component (or, of a solution-pair 2gΓ) - associated with gravity caused by object A - for which the reach is R_I, the symbol F_{R_I} denotes a factor such that equation (13) provides a factor that pertains regarding the strength of the interaction between object A and object C.

$$F_{R_I} m_A m_C \tag{13}$$

For a reach R_I of six, equation (14) pertains.

$$F_6 = (\sum_{0 \leq i \leq 5} f_{A,i})(\sum_{0 \leq i \leq 5} f_{C,i}) = 1 \tag{14}$$

For a reach R_I of two, equation (15) pertains.

$$F_2 = \sum_{i=0,1,2} ((f_{A,i} + f_{A,i+3})(f_{C,i} + f_{C,i+3})) \leq 1 \tag{15}$$

For a reach R_I of one, equation (16) pertains.

$$F_1 = \sum_{0 \leq i \leq 5} (f_{A,i} f_{C,i}) \leq 1 \tag{16}$$

For a case in which each $f_{A,i} = 1/6$ and each $f_{C,i} = 1/6$, equation (17) pertains.

$$F_{R_I} = 6/R_I \tag{17}$$

For a case in which $f_{A,0} = 1$, each other $f_{A,i} = 0$, $f_{C,0} = 1$, and each other $f_{C,i} = 0$, equation (18) pertains.

$$F_{R_I} = 1, \text{ for } R_I = 1, 2, \text{ or } 6 \tag{18}$$

The following paragraphs discuss some specific cases.

For interactions between two neighboring non-overlapping galaxies (one as an object A and one as an object C), some modeling might assume that each $f_{A,i} \approx 1/6$ and each $f_{C,i} \approx 1/6$. 2g2 associates with a reach of six. One-some use of 2g2‘4 associates with a reach of two. One-some use of 2g1‘2‘3 associates with a reach of one. SOMA points to the notion that POST (including POST GR) would not necessarily be adequately accurate.

Regarding gravitationally-based observations pertaining to events in which a pair of small-mass black holes merge to form one black hole, SOMA suggests that two sets of signatures might pertain. One set would associate with mergers for which the merging black holes associate with just one isomer-pair. The other set would associate with mergers for which each incoming black hole associates with an isomer-pair with which the other incoming black hole does not associate.

For a general case of a point-like object C interacting with the 2L field that associates with an object A, object C senses all 2gΓ solution-pair components that associate with the 2L field that associates with object A. The weighting that associates with any one one-some solution-pair associates with the geometric factor of the pole (monopole, dipole, or so forth) that associates with the one-some solution-pair, with an orientation factor that associates with a tensor-like notion (scalar for monopole, vector for dipole, and so forth), and with an isomer composition factor F_{R_I}. (SOMA uses the word weighting to avoid possible conflation with POST notions such as probability and amplitude. This paper does not operationally define the one-word term weighting.) For POST ND, the geometric factor associates with r^{-n_Γ}. Likely, effects that associate with one geometric factor or with two geometric factors dominate compared to effects that associate with other geometric factors.

2.9. Conservation of and symmetries that associate with Σg'-related properties

POST includes the notion of conservation of charge. For SOMA one-some use of 1g1, there are six instances of 1g1. SOMA posits that - in effect - each isomer associates with its own instance of the property of charge. Conservation of charge pertains for each isomer.

POST includes the notion of conservation of energy. For SOMA one-some use of 2g2, there is one instance of 2g2. SOMA posits that conservation of energy pertains for the combination of the six isomers and the set of LRI elementary particles.

Equation (19) shows notation that SOMA uses to describe a reach that includes all six isomers and all SL (or, all LRI) phenomena.

$$R_I = 6\uplus \tag{19}$$

POST includes the notion of conservation of momentum. For SOMA three-some use of 2g2‘4, there is one instance of 2g2‘4. (Three-some use of 2g2‘4 inherits its reach from one-some use of 2g2.) SOMA posits that conservation of momentum associates with three-some use of 2g2‘4 and $R_I = 6\uplus$.

POST includes the notion of conservation of angular momentum. Based on its relationship to three-some use of 2g2‘4, one might expect that one-some use of 2g2‘4 associates with angular momentum. SOMA suggests that - for one-some use of 2g2‘4 - $n_0 = 2$ associates with three instances of 2g2‘4 and with three isomer-pairs. Conservation of angular momentum would need to embrace angular momentum that reflects phenomena that associate with SL as well as phenomena that associate with stuff based on isomers of non-SL elementary particles. SOMA posits that - in effect - the degrees of freedom that associate with $k = 0$ erase some degrees of freedom that associate with $n_0 = 2$ and leave (regarding angular momentum) an effective result (regarding calculating reaches and numbers of instances) of $n_0 = 1$. SOMA posits that conservation of angular momentum associates with one-some use of 2g2‘4 and $R_I = 6\uplus$.

SOMA suggests relevance for one-some use of the 3g3 solution-pair and related three-some use of the 3g3‘6 solution-pair. There are three instances of 3g3. Each instance associates with its own isomer-pair. Per discussion regarding table 6, SOMA posits that three-some use of the 3g3‘6 solution-pair associates with interactions in which a 3L boson transforms into two somewhat similar elementary fermions. One of the fermions associates with left-solution. The other fermion associates with right-solution. SOMA suggests that conservation of net-left-minus-right (as in the number of left-handed elementary particles minus the number of right-handed elementary particles) pertains - for each one of the three isomer-pairs - regarding fermion elementary particles. SOMA suggests that - in association with the notion that $6 \in \Gamma$ for 3g3‘6 - an approximate conservation of flavour symmetry pertains. Conservation of flavour pertains for interactions mediated by single elementary bosons. Conservation of flavour does not necessarily pertain for interactions mediated by pairs of W bosons. (POST QF associates interactions mediated by pairs of W bosons with weak-interaction CP symmetry violation. Here, C symmetry associates with notions of charge conjugation - or charge reversal. P symmetry associates with notions of parity reversal. CP symmetry associates with notions of combined charge conjugation and parity reversal.)

SOMA suggests relevance for one-some use of the 4g4 solution-pair and related three-some use of the 4g4‘8 solution-pair. There are six instances of 4g4. Each instance associates with its own isomer. Per discussion regarding table 6, SOMA posits that three-some use of the 4g4‘8 solution-pair associates with interactions in which a 4L boson transforms into two somewhat similar elementary bosons. To the extent that either one of the elementary bosons into which a 4L boson transforms associates with an isomer (that is, is not an SL elementary boson), SOMA suggests that conservation of isomer pertains and that the other one of the elementary bosons into which the 4L boson transforms associates with the same isomer. SOMA suggests that - in association with the notion that $8 \in \Gamma$ for 4g4'8 - an approximate conservation of net-left-minus-right symmetry pertains. Conservation of net-left-minus-right might not necessarily pertain for interactions - that involve elementary bosons - mediated by Higgs bosons.

2.10. SOMA components and SOMA reaches for long-range interactions

Table 3 shows cascades that associate with Σg' solution-pairs for which $1 \leq \Sigma \leq 4$ and $1 \leq k_{max} \leq 8$.

Table 3 points to PRO that associate with INFDA. The PRO to which table 3 alludes associate with Σg' solution-pairs for which $1 \leq n_\Gamma \leq 2$.

Table 3 points to some PRO that POST associates with conservation laws. This paper provides (via discussion related to equation (43)) a second pointer to conservation of charge. Per POST, equation (43) associates with conservation of charge. Discussion in this paper points to conservation of weak isospin and conservation of color charge. Per POST, equation (44) associates with conservation of weak isospin and equation (45) associates with conservation of color charge. Table 3 points to a new conservation law - conservation of fermion net-left-minus-right. Conservation of fermion net-left-minus-right pertains for each one of the three isomer-pairs. Fermion net-left-minus-right for LRI fields and LRI elementary particles is zero. (LRI fields associate with bosons, such as the photon.) Conservation of fermion net-left-minus-right pertains universally.

2.11. SOMA bases for modeling events that affect fields

SOMA posits that interactions between fields associate with Σg" solution-pairs for which $n_{\Gamma''} \geq 3$.

Table 4 shows some aspects of some cascades that associate with some Σg" solution-pairs for which $n_{\Gamma''} \geq 3$, $1 \leq \Sigma \leq 3$, and $1 \leq k_{max} \leq 8$.

For cases in which a multi-component object participates in an event that excites a field and the event associates with a one-some use of a Σg" solution-pair for which $n_{\Gamma''} \geq 3$, SOMA suggests that the following statements pertain.

1. For a one-some solution-pair for which $5 \notin \Gamma$ and $7 \notin \Gamma$, KM modeling does not necessarily need to associate with the notion of a change of state within the object. KM modeling might associate with the notion of a change (regarding the object) in a three-some property such as momentum.
2. For a one-some solution-pair for which $5 \in \Gamma$ and $7 \notin \Gamma$, KM modeling needs to associate with the notion of a change of state within the object.
3. For a one-some solution-pair for which $5 \notin \Gamma$ and $7 \in \Gamma$, KM modeling needs to associate with the notion of a change of state within the object and with the notion that the change of state associates with an interaction with a second object that was not originally part of the (original) object.

De-excitations (by entities that make observations) - of fields - associate with generating data that provide bases for INFDA.

2.12. Properties that associate with fields that associate with SOMA solution-pairs

Table 5 lists sets of solution-pairs and notes properties for which excitations or de-excitations of the fields that associate with the solution-pairs lead to data that lead to INFDA.

3. Results - Elementary particles

3.1. A catalog of elementary particles

SOMA uses the following notions to catalog elementary particles. A symbol of the form $S\Phi$ associates with a so-called family of elementary particles. Each elementary particle associates with one family. Each family associates with one of one, three, or eight elementary particles. For a family, the value S denotes the spin (in units of $\hbar$) for each elementary particle in the family. S associates with the POST expression $S(S+1)\hbar^2$ that associates with angular momentum. Regarding POST, known values of S include 0, 0.5, and 1. For each one of the numbers 0, 0.5, 1, 2, 3, and 4, SOMA points to at least one possible elementary particle for which S would equal that number. The symbol Φ associates with a symbol of the form X_Q, in which X is a capital letter and Q is the magnitude of charge (in units of $|q_e|$, in which q_e denotes the charge of an electron) for each particle in the family. For cases for which $Q = 0$, SOMA omits - from the symbols for families - the symbol Q.

Table 3: Cascades that associate with Σg' solution-pairs for which $1 \leq \Sigma \leq 4$ and $1 \leq k_{max} \leq 8$. The column with the one-element label one-some shows one-some solution-pairs. The column with the one-element label three-some shows three-some solution-pairs that cascade in one step from the one-some solution-pairs. The symbol † alludes to the notion that the one-some solution-pairs do not cascade from other one-some solution-pairs that the table shows. The symbol ‡ alludes to the notion that the solution-pairs appear more than once in the column that lists three-some solution-pairs. The acronym NNC (for the three-word phrase no next cascade) associates with the notion that - for the one-some solution-pairs - no three-some solution-pairs pertain. SOMA assumes that each one of NNC and 6 is a member of a one-some Γ associates with the notion that one-some solution-pairs are not relevant regarding SOMA. (Per discussion related to table 6, SOMA suggests that - regarding elementary particles - one-some use of solution-pairs for which $6 \in \Gamma$ associates with fermion elementary particles and does not associate with boson elementary particles.) The next-to-rightmost column designates rows that show SOMA-relevant four-somes. (A one-some solution-pair can associate with more than one four-some.) Here, the symbol SL associates with a known or would-be elementary particle and the value of S equals Σ. The rightmost column shows the reach for each SOMA-relevant four-some. The two columns with the one-word label property show properties that associate with conservation laws. The symbol ⊎ abbreviates the notation $R_I = 6⊎$. The one-element symbol F: associates with the word fermion. The one-element symbol B: associates with the word boson.

†	One-some	Property	Three-some	Property	SL	R_I
†	1g1	Charge	1g1‘2	-	1L	1
-	1g1‘2	-	1g1‘2‘4	-	1L	1
-	1g1‘2‘4	-	1g1‘2‘4‘8, 1g1‘2‘4‘6x	-	1L	6
-	1g1‘2‘4‘8	-	1g1‘2‘4‘6‘8x ‡	-	1L	6
-	1g1‘2‘4‘6‘8x ‡	-	NNC	-	-	-
-	1g1‘2‘4‘6x	-	1g1‘2‘4‘6‘8x ‡	-	-	-
†	1g1‘4‘6	-	1g1‘4‘6‘8	-	-	-
-	1g1‘4‘6‘8	-	1g1‘2‘4‘6‘8x ‡	-	-	-
†	2g2	Energy ⊎	2g2‘4	Momentum ⊎	2L	6
-	2g2‘4	Angular momentum ⊎	2g2‘4‘8	-	2L	2
-	2g2‘4‘8	-	NNC	-	-	-
†	2g1‘2‘3	-	2g1‘2‘3‘4x, 2g1‘2‘3‘6	-	2L	1
-	2g1‘2‘3‘4x	-	2g1‘2‘3‘4‘8x	-	2L	1
-	2g1‘2‘3‘4‘8x	-	2g1‘2‘3‘4‘6‘8x ‡	-	2L	1
-	2g1‘2‘3‘4‘6‘8x ‡	-	NNC	-	-	-
-	2g1‘2‘3‘6	-	2g1‘2‘3‘6‘8x	-	-	-
-	2g1‘2‘3‘6‘8x	-	2g1‘2‘3‘4‘6‘8x ‡	-	-	-
†	3g3	F: Net-left-minus-right	3g3‘6	F: Flavour	3L	2
-	3g3‘6	-	NNC	-	-	-
†	3g2‘3‘4	-	3g2‘3‘4‘8, 3g2‘3‘4‘6	-	3L	6
-	3g2‘3‘4‘8	-	3g2‘3‘4‘6‘8 ‡	-	3L	6
-	3g2‘3‘4‘6	-	3g2‘3‘4‘6‘8 ‡	-	-	-
-	3g2‘3‘4‘6‘8 ‡	-	NNC	-	-	-
†	4g4	B: Net-left-minus-right	4g4‘8	-	4L	1
-	4g4‘8	-	NNC	-	4L	1
†	4g1‘2‘3‘4x	-	4g1‘2‘3‘4‘6x	-	4L	1
-	4g1‘2‘3‘4‘6x	-	4g1‘2‘3‘4‘6‘8x ‡	-	-	-
-	4g1‘2‘3‘4‘6‘8x ‡	-	NNC	-	-	-
†	4g1‘2‘3‘4‘8x	-	4g1‘2‘3‘4‘6‘8x ‡	-	4L	1

Table 4: Some aspects of some cascades that associate with some Σg" solution-pairs for which $n_{\Gamma''} \geq 3$, $1 \leq \Sigma \leq 3$, and $1 \leq k_{max} \leq 8$. The column with the one-element label one-some shows one-some solution-pairs. The column with the one-element label three-some shows three-some solution-pairs. The symbol † alludes to the notion that the one-some solution-pair does not cascade from other one-some solution-pairs that the table shows. The rightmost column shows the reach for each four-some. The column with the one-word label event points to phenomena that associate with excitations or de-excitations of LRI fields. Regarding rows for which $\Sigma = 1$ (except for the first row for which $\Sigma = 1$ and for the last row for which $\Sigma = 1$), table 4 suggests events that associate with atomic physics. For each row for which $\Sigma = 1$, other events might pertain. Each row in table 4 might associate with events for which the interacting objects are not atoms or directly related to atoms. TBD denotes the three-word phrase to be determined. For items that table 4 lists, SOMA suggests - per discussion related to equation (55) - that $7 \in \Gamma$ associates with notions of external influences. For example, regarding the electron cloud in an atom, one-some use of 1g2'4'7 associates with adding or subtracting an electron. Regarding the electron cloud in an atom, one-some use of 1g2'4'7'8 associates with an interaction with the spin of the nucleus of the atom. Regarding one-some uses of solution-pairs, the symbol ‡ denotes the notion that $5 \notin \Gamma$ and $7 \notin \Gamma$.

†	One-some	Event	Three-some	SL	R_I
†	1g4'5'8	A star radiates or absorbs light.	1g4'5'6'8	1L	1
†	1g2'4'7	An atom adds or subtracts an electron.	1g2'4'7'8, 1g2'4'6'7	1L	2
†	1g2'4'5	An atomic electron transits to a new principal energy.	1g2'4'5'8, 1g2'4'5'6	1L	2
-	1g2'4'5'8	An atomic fine-structure change occurs.	1g2'4'5'6'8x	1L	2
-	1g2'4'7'8	An atomic hyperfine transition occurs.	1g2'4'6'7'8x	1L	2
†	1g2'3'4 ‡	TBD	1g2'3'4'8, 1g2'3'4'6	1L	6
†	2g1'3'4 ‡	TBD	2g1'3'4'8, 2g1'3'4'6	2L	6
†	2g1'4'5	TBD	2g1'4'5'8, 2g1'4'5'6	2L	2
†	2g3'4'5	TBD	2g3'4'5'8, 2g3'4'5'6	2L	2
†	2g4'5'7	TBD	2g4'5'7'8, 2g4'5'6'7	2L	1
†	3g1'2'4 ‡	TBD	3g1'2'4'8, 3g1'2'4'6	3L	6

Table 5: Solution-pairs and the INFDA that associate with the solution-pairs. The one-element notation sol-pairs abbreviates the one-element term solution-pairs. The leftmost column points to one or more solution-pairs. (For each one of the last nine rows, the notion of boson elementary particle pertains and table 6 provides further information.) The column with the one-letter label S provides information about the range of spins. (For the first of the two 0g" rows, table 6 provides further information. For the second of the two 0g" rows, discussion related to equation (55) provides further information.) The range of n_Γ pertains for one-some solution-pairs. The column with the one-element label 5,7 provides information about the extent - regarding solution-pairs - to which $5 \in \Gamma$ and the extent to which $7 \in \Gamma$. The word no associates with the notion that neither of the two numbers (5 and 7) appears in Γ. The number 5 associates with the notion that the number 5 appears in Γ. The number 7 associates with the notion that the number 7 appears in Γ. The word yes associates with the notion that both of the two numbers (5 and 7) can appear in Γ. The next-to-rightmost column notes INFDA about which - in effect - changes in the states of the fields contribute data. The symbol NF associates with the notion that some solution-pairs associate with components of fields and not directly with the notion of field. The symbol † denotes an association with three-some solution-pairs (and not directly with one-some solution-pairs). Regarding 2L and POST GR, the symbol ⋯ alludes to - but does not mention - off-diagonal properties that associate with the stress-energy tensor. The rightmost column differentiates - regarding POST QM QF - the notion of anomalous property (for example, anomalous magnetic moment) from the notion of nominal property (for example, nominal magnetic moment).

Sol-pairs	S	n_Γ	5,7	Fields associate with measuring ⋯	Properties that events measure
Sg'	S≥ 1	≥ 1	No	(NF)	Nominal
Sg"	S≥ 1	2	No	(NF)	Anomalous
Sg"	S≥ 1	≥ 3	No	Principal states of multi-object systems	-
Sg"	S≥ 1	≥ 3	5	Internal (spin) states of multi-object systems	-
Sg"	S≥ 1	≥ 3	7	States of multi-object systems (in larger contexts)	-
0g"	-	≥ 3	No	The presence of elementary particles	-
0g"	-	≥ 4	Yes	Angular momentum states of multi-object systems	-
1Z, $1W_1$	S= 1	3	No	Weak-isospin of elementary particles	-
0H	S= 0	4	No	Masses of (some) elementary particles	-
0I	S= 0	4	No	Isomer	-
SJ	S≥ 1	≥ 5	No	Sameness of (two or more) fermions	-
SG	S≥ 1	≥ 5	No	Color charge of elementary fermions	-
1L	S= 1	5	No	Charge, magnetic moment, ⋯	-
2L	S= 2	6	No	Energy, momentum†, angular momentum, ⋯	-
3L	S= 3	7	No	F: net-left-minus-right, F: flavour†	-
4L	S= 4	8	No	B: net-left-minus-right	-

POST QF includes the notion that observations of elementary particles associate with interactions between fields. Per discussion above, SOMA associates one-some uses of SOMA-quadrupole solutions with interactions between fields. More generally, SOMA associates one-some uses of SOMA-higher-than-dipole solutions with interactions between fields.

SOMA posits that families of elementary particles associate with one-some uses of solution-pairs that result from cascades from the solution-pairs 0g1‘3‘4 and 0g1‘2‘3. For each one of the solution-pair 0g1‘3‘4 and the solution-pair 0g1‘2‘3, the word quadrupole pertains and the solution-pair associates with 0g”.

SOMA posits the following statements regarding properties of elementary particles.

- Boson or fermion? - A one-some use of a solution-pair for which $6 \notin \Gamma$ associates with bosons. A one-some use of a solution-pair for which $6 \in \Gamma$ associates with fermions. Regarding one-some uses of solution-pairs, equation (20) pertains. (The symbol $\Leftrightarrow$ abbreviates the four-word phrase if and only if.)

$$\text{The object is a fermion} \;\Leftrightarrow\; 6 \in (\text{one-some})\Gamma \tag{20}$$

- Charge or no charge? - A one-some use of a solution-pair for which $4 \notin \Gamma$ associates with a magnitude of charge that equals the magnitude of the charge of an electron. A one-some use of a solution-pair for which $4 \in \Gamma$ associates with zero charge. Equation (21) pertains.

$$Q = 0 \Leftrightarrow 4 \in \Gamma \tag{21}$$

- Magnitudes of quark charges? Each quark family associates with one-some use of one solution-pair that associates with $Q = 1$ and one-some use of one solution-pair that associates with $Q = 0$. Each quark associates with a Q of $(2/3) = (2/3)1 + (1/3)0$ or with a Q of $(1/3) = (1/3)1 + (2/3)0$.

- Mass or no mass? - For bosons, a one-some use of a solution-pair for which $8 \notin \Gamma$ associates with positive mass and a one-some use of a solution-pair for which $8 \in \Gamma$ associates with zero mass. For fermions, a one-some use of a solution-pair associates with positive mass. Equation (22) pertains. The symbol m associates with the PRO of mass.

$$m = 0 \Leftrightarrow (6 \notin \Gamma \text{ and } 8 \in \Gamma) \tag{22}$$

- Magnitude of spin? - For bosons, a one-some use of a solution-pair associates with the spin that equation (23) computes. For fermions, a one-some use of a solution-pair associates with the spin that equation (24) computes.

$$S = |n_{\Gamma''} - 4| \tag{23}$$

$$S = |n_{\Gamma''} - 4.5| \tag{24}$$

- Number of fermion flavours? - For elementary fermions, each one-some use of a solution-pair associates with $6 \in \Gamma$ and with three flavours.

Table 6 catalogs solution-pairs that might associate with elementary particles.

SOMA posits the following statements regarding transformations between elementary particles.

- Any elementary boson that associates with three-some use of a solution-pair for which $8 \in \Gamma$ and $6 \notin \Gamma$ can transform (or, regarding some POST terminology, decay) into a pair of elementary bosons that are similar to each other. For example, a Z boson can transform into two photons. The only elementary boson for which there is no three-some use of a solution-pair for which $8 \in \Gamma$ and $6 \notin \Gamma$ is the W boson. The W boson does not transform into two (hypothetical) elementary particles for which each of the two produced elementary particles would associate with $Q = 0.5$.

- Any elementary boson that associates with three-some use of a solution-pair for which $6 \in \Gamma$ can transform (or, regarding some POST terminology, decay) into a pair of elementary fermions. For example, a Z boson can transform into two elementary fermions that are antiparticles to each other. The W boson can transform into a pair of fermions (for example, an electron and a neutrino). The W boson is the only elementary boson that does not transform into two elementary fermions that are antiparticles to each other.

Table 6: Solution-pairs that might associate with elementary particles. The one element symbol 1-somes abbreviates the term one-somes. The one element symbol 3-somes abbreviates the term three-somes. For each item in the first column or the fifth column, each of the integers is a member of a relevant Γ and the relevant Γ does not include other integers. The symbol σ denotes a positive integer. The symbol n_{EP} denotes the number of elementary particles. A notation of the form n_σ denotes that - for each relevant value of σ - n_{EP} elementary particles pertain. For each one of some rows in table 6, three-some solutions cascade to become one-some solutions for the families to which the rightmost column alludes. For σL elementary particles, the letter L abbreviates the two-element phrase long-range interaction. For σL elementary particles, the range of σ includes at least the integer one (which associates with the photon) and the integer two (which associates with the would-be graviton). For each one of σJ, σG, and σL, discussion related to equation (54) addresses the notion of a maximal value for σ. SOMA posits that σJ elementary particles associate with Pauli repulsion. Regarding the first column in the table and the fifth column in the table, the symbol $+8\hat{}$ denotes $+8$ for $\sigma = 1$, $-8+16$ for $\sigma = 2$, $-8-16+32$ for $\sigma = 3$, and so forth. The symbol $+16\hat{}$ denotes $+16$ for $\sigma = 1$, $-16+32$ for $\sigma = 2$, $-16-32+64$ for $\sigma = 3$, and so forth. The symbol $+32\hat{}$ denotes $+32$ for $\sigma = 1$, $-32+64$ for $\sigma = 2$, $-32-64+128$ for $\sigma = 3$, and so forth. The acronym TBD abbreviates the three-word phrase to be determined. The symbol ‡ associates with the use - for a pair of boson families σJ and σG - of the same set of one-some solution-pairs and the same set of three-some solution-pairs. For a pair of boson families σJ and σG, there are two three-some solution-pairs for which $6 \in \Gamma$. SOMA posits that - across a physics-relevant pair of boson families σJ and σG - there are nine (or, 3×3) elementary particles.

$0 = \ldots$, re 0gΓ 1-somes	Families	n_{EP}	Names	$0 = \ldots$, re 0gΓ 3-somes	Cascades to
$\vert -1-3+4 \vert$	1Z	1	Z	$\vert +1-2-3+4 \vert$; $\vert -1-3-4+8 \vert$; $\vert +1-3-4+6 \vert$.	0H; 0I; 0.5N.
$\vert -1-2+3 \vert$	1W$_1$	1	W	$\vert +1-2-3+4 \vert$; $\vert -1-2-3+6 \vert$.	0H; 0.5C$_1$.
$\vert +1-2-3+4 \vert$	0H	1	Higgs	$\vert +1-2-3-4+8 \vert$; $\vert -1+2-3-4+6 \vert$.	1J, 1G; 0.5Q, 0.5M.
$\vert -1-3-4+8 \vert$	0I	1	Aye	$\vert +1-2-3-4+8 \vert$; $\vert -1-3-4-8+16 \vert$; $\vert -1+3-4-6+8 \vert$.	1J, 1G; 1L; 0.5R.
$\vert +1-3-4+6 \vert$	0.5N	3	Neutrinos	$\vert -1+3-4-6+8 \vert$; $\vert -1+2-3-4+6 \vert$.	0.5R; 0.5Q, 0.5M.
$\vert -1-2-3+6 \vert$	0.5C$_1$	3	Charged leptons	$\vert -1+3-4-6+8 \vert$; $\vert -1+2-3-4+6 \vert$.	0.5R; 0.5Q, 0.5M.
$\vert -1+3-4-6+8 \vert$	0.5R	3	Arcs	$\vert -1-2-3+4-6+8 \vert$, $\vert +1-2+3-4-6+8 \vert$.	-
$\vert -1+2-3-6+8 \vert$; $\vert -1+2-3-4+6 \vert$	0.5Q$_{y/3}$, $y = 1$ or 2	6	Quarks	$\vert -1-2-3+4-6+8 \vert$, $\vert +1-2+3-4-6+8 \vert$.	-
$\vert -1+2-3-4+6 \vert$	0.5M	3	Heavy neutrinos	$\vert -1-2-3+4-6+8 \vert$, $\vert +1-2+3-4-6+8 \vert$.	-
$\vert +1-2-3-4+8\hat{} \vert$ ‡	σJ	1_σ	Jay ($\sigma = 1$). TBD ($\sigma > 1$).	$\vert +1-2-3-4-8+16\hat{} \vert$; $\vert +1-2+3-4-6+8\hat{} \vert$, $\vert -1-2-3+4-6+8\hat{} \vert$.	$(\sigma+1)$J, $(\sigma+1)$G; -.
$\vert +1-2-3-4+8\hat{} \vert$ ‡	σG	8_σ	Gluons ($\sigma = 1$). TBD ($\sigma > 1$).	$\vert +1-2-3-4-8+16\hat{} \vert$; $\vert +1-2+3-4-6+8\hat{} \vert$, $\vert -1-2-3+4-6+8\hat{} \vert$.	$(\sigma+1)$J, $(\sigma+1)$G; -.
$\vert -1-3-4-8+16\hat{} \vert$	σL	1_σ	Photon ($\sigma = 1$). Graviton ($\sigma = 2$). TBD ($\sigma > 2$).	$\vert +1-2-3-4-8+16\hat{} \vert$; $\vert -1+3-4-6-8+16\hat{} \vert$, $\vert -1-3-4-8-16+32\hat{} \vert$.	$(\sigma+1)$J, $(\sigma+1)$G; -; $(\sigma+1)$L.

3.2. Relationships among properties of boson elementary particles

SOMA posits that equation (25) pertains regarding the masses of the nonzero-mass elementary bosons.

$$(m_{\mathrm{W}})^2 : (m_{\mathrm{Z}})^2 : (m_{\mathrm{Higgs}})^2 :: 7 : 9 : 17 \tag{25}$$

Equation (25) is not inconsistent with INFDA. Based on INFDA that reference [54] provides, the following notions pertain. The most accurately known of the three masses is m_{Z}. Based on the nominal value of m_{Z}, the nominal value (that equation (25) suggests) for m_{Higgs} is within 0.5 experimental standard deviations of m_{Higgs}. Based on the nominal value of m_{Z}, the nominal value (that equation (25) suggests) for m_{W} is within 3.6 experimental standard deviations of m_{W}. Based on INFDA that reference [55] provides, the following notions pertain. Based on the nominal value of m_{Z}, the nominal value (that equation (25) suggests) for m_{W} is within 1.1 experimental standard deviations of m_{W}. (Reference [55] does not provide INFDA about m_{Higgs}.) Based on the nominal value of m_{Z} that reference [55] suggests and on INFDA that reference [54] provides about m_{Higgs}, the nominal value that equation (25) suggests for m_{Higgs} is within 0.5 experimental standard deviations of m_{Higgs}.

SOMA suggests that equation (25) points to possible insight regarding - and a possible extension to - the POST notion of the weak mixing angle.

For each elementary boson, equation (26) and equation (27) define, respectively, the integer l_{ms} and the number j_m. The symbol Q denotes the magnitude (in units of the magnitude $|q_e|$ of the charge - q_e - of the electron) of the charge of the elementary boson. The symbol m' denotes the mass (in units of $m_{\mathrm{Z}}/3$) of the elementary boson.

$$l_{ms} = 0, \text{ if } m' = 0; \; l_{ms} = -1, \text{ if } m' > 0 \tag{26}$$

$$(j_m)^2 \equiv (m')^2 + S^2 + Q(Q+1) + l_{ms} \tag{27}$$

For each elementary boson to which table 6 alludes, SOMA suggests that j_m is an integer. For each known elementary boson, the notion that j_m is an integer is not inconsistent with INFDA.

3.3. Relationships among properties of fermion elementary particles

Regarding charged leptons, SOMA suggests a link between the strength of electromagnetism and the strength of gravity.

Equation (28) and equation (29) define, respectively, β' and β. m_τ denotes the mass of the tau particle (which is a charged lepton). m_e denotes the mass of the electron (which is a charged lepton). The right-hand side of equation (29) is the ratio of the electrostatic repulsion between two electrons to the gravitational attraction between the two electrons. The ratio does not depend on the distance between the two electrons.

$$\beta' \equiv m_\tau / m_e \tag{28}$$

$$(4/3) \cdot (\beta^2)^6 = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2) \tag{29}$$

Based on INFDA, $\beta \approx 3477.1891 \pm 0.0226$. (Reference [54] provides the relevant underlying INFDA.) The standard deviation associates almost entirely with the standard deviation for G_N, the gravitational constant.

Equation (30) shows an equality that SOMA posits.

$$\beta' = \beta \tag{30}$$

Equation (31) results from equation (30). The standard deviation associates almost entirely with the standard deviation for G_N, the gravitational constant.

$$m_{\tau,\text{ calculated}} \approx 1776.8400 \pm 0.0115 \text{ MeV}/c^2 \tag{31}$$

Equation (31) comports with INFDA. More than eight standard deviations fit within one INFDA standard deviation from the INFDA nominal value for m_τ.

SOMA suggests a formula that might approximately link the masses of all elementary fermions.

Equation (32) defines $m(l_m, l_q)$ and has bases in the equations that immediately follow equation (32). Equation (33) defines the fine-structure constant. Equation (38) has bases in trying to fit INFDA.

Table 7: Approximate values of $\log_{10}(m(l_m, l_q)/m_e)$ for known charged fermion elementary particles. Regarding flavour, this table generalizes, based on POST terminology that associates with charged leptons and with neutrinos. For example, POST uses the term electron-neutrino. In table 7, the symbol l_f numbers the three flavours. The "l_f (0.5C$_1$)" terms pertain for fermions in the 0.5C$_1$ family. The symbol 0.5Q$_{>0}$ denotes the pair 0.5Q$_{1/3}$ and 0.5Q$_{2/3}$. The "l_f (0.5Q$_{>0}$)" terms pertain for quarks (or, elementary particles in the two families 0.5Q$_{2/3}$ and 0.5Q$_{1/3}$). l_m is an integer parameter. The domain $-6 \leq l_m \leq 18$ might have relevance regarding modeling. Q denotes the magnitude of charge, in units of $|q_e|$. The family 0.5C$_1$ associates with $Q = 1$. The family 0.5Q$_{2/3}$ associates with $Q = 2/3$. The family 0.5Q$_{1/3}$ associates with $Q = 1/3$. Regarding table 7, $l_q = 3Q$ pertains. Regarding the rightmost four columns, items show $\log_{10}(m(l_m, l_q)/m_e)$ and - for particles that nature includes - the name of an elementary fermion. For each † case, no particle pertains. Each number in the column with the label $Q = 1/2$ equals the average of the number in the $Q = 2/3$ column and the number in the $Q = 1/3$ column. The notion of geometric mean pertains regarding the mass of the $Q = 2/3$ particle and the mass of the $Q = 1/3$ particle. Regarding each † case, equation (32) provides the number $m(l_m, l_q)$.

l_f (0.5C$_1$)	l_f (0.5Q$_{>0}$)	l_m	$Q = 1$	$Q = 2/3$	$Q = 1/2$	$Q = 1/3$
1 (Electron)	1 (Up, Down)	0	0.00 Electron	0.63 Up	0.80 †	0.97 Down
-	2 (Charm, Strange)	1	1.23 †	3.40 Charm	2.83 †	2.26 Strange
2 (Mu)	3 (Top, Bottom)	2	2.32 Muon	5.53 Top	4.72 †	3.91 Bottom
3 (Tau)	-	3	3.54 Tau	-	-	-

$$m(l_m, l_q) \equiv m_e \cdot (\beta^{1/3})^{l_m + (j''_{l_m})d''} \cdot (\alpha^{-1/4})^{g(l_q)\cdot(1+l_m) + j'_{l_q} d'(l_m)} \tag{32}$$

$$\alpha = ((q_e)^2/(4\pi\varepsilon_0))/(\hbar c) \tag{33}$$

$$j''_{l_m} = 0, +1, 0, -1 \text{ for, respectively, } l_m \bmod 3 = 0, 1, 3/2, 2; \text{ with } 3/2 \bmod 3 \equiv 3/2 \tag{34}$$

$$d'' = (2 - (\log(m_\mu/m_e)/\log(\beta^{1/3}))) \approx 3.840613 \times 10^{-2} \tag{35}$$

$$g(l_q) = 0, 3/2, 3/2, 3/2, 3/2, \text{ for, respectively, } l_q = 3, 2, 3/2, 1, 0 \tag{36}$$

$$j'_{l_q} = 0, -1, 0, +1, +3 \text{ for, respectively, } l_q = 3, 2, 3/2, 1, 0 \tag{37}$$

$$d'(0) \sim 0.324,\ d'(1) \sim -1.062,\ d'(2) \sim -1.509 \tag{38}$$

$$d'(l_m) = 0 \text{ for } l_m \leq -1 \text{ and for } l_m \geq 3 \tag{39}$$

Table 7 shows information about properties of known charged fermion elementary particles. (Reference [54] provides the data that underlies table 7.) Regarding similar tables for each one of isomer-one, isomer-two, isomer-four, and isomer-five, SOMA posits (per table 8) that the values of l_f that table 7 shows for the charged leptons are not appropriate. For example, for isomer-two, the l_f values in the leftmost column would be 3 (for the row for which - for quarks - $l_f = 1$), blank (for the row for which - for quarks - $l_f = 2$), 1 (for the row for which - for quarks - $l_f = 3$), and 2 (for the remaining row).

For each charged elementary fermion except the top quark, equation (32) suggests a mass that is within one experimental standard deviation of the nominal mass that reference [54] reports. Reference [54] alludes to three estimates for the mass of the top quark. Equation (32) provides a mass (for the top quark) that is within 4.4 deviations below the nominal mass that associates with direct measurements, within 4.3 upward standard deviations above the nominal mass that associates with cross-section measurements, and within 1.6 standard deviations below the nominal mass that associates with the four-element phrase pole from cross-section measurements.

The count of independent irrational numbers input into the above calculations of nine fermion elementary particle masses is seven. For example, the list consisting of m_e, m_μ, β, α, $d'(0)$, $d'(1)$, and $d'(2)$ includes seven irrational numbers.

SOMA suggests neutrino masses.

Reference [16] suggests that INFDA point to the notion that the sum of the three neutrino rest energies is at least approximately 0.06 eV and not more than approximately 0.12 eV. Reference [56] discusses data and modeling regarding upper bounds for the sum of the masses of the three neutrinos. Reference [57] discusses a lower bound of 0.06 eV, an upper bound of 0.15 eV, and a possible upper bound of 0.12 eV.

Reference [21] discusses the notion of neutrino mass mixing. Reference [54] suggests that an upper bound might be approximately 0.10 eV.

Neutrinos associate with $Q = 0$. SOMA posits that some $m(l_m, 0)$ solutions associate with neutrino masses. For $l_m \leq -1$ and for $l_m \geq 3$, no quarks pertain and SOMA posits that $d'(l_m) = 0$.

Equation (40) shows a result from equation (32).

$$mc^2 = m(-4,0)c^2 \approx 3.448 \times 10^{-2} \text{ eV} \tag{40}$$

SOMA points to the following two possibilities.

1. $mc^2 = m(-4,0)c^2 \approx 3.448 \times 10^{-2}$ eV pertains for each of the three neutrinos.
2. $mc^2 = m(-4,0)c^2 \approx 3.448 \times 10^{-2}$ eV pertains for each of two neutrinos. For one neutrino, one of $m(-6,0)c^2 \approx 4.2 \times 10^{-6}$ eV and $m(-5,0)c^2 \approx 4.4 \times 10^{-4}$ eV might pertain.

SOMA suggests that interactions that associate with 2g' solution-pairs conserve mass but do not necessarily conserve flavour. SOMA suggests that interactions that associate with 3g' solution-pairs conserve flavour but do not necessarily conserve mass. SOMA suggests that these notions regarding conservation of properties might associate with the POST notion that mass eigenstates for neutrinos do not necessarily equal flavor eigenstates for neutrinos.

POST INFDA suggest notions that associate with the two-word term neutrino oscillations and with the two-word term neutrino mixing. SOMA suggests interactions - between neutrinos and the environments through which neutrinos pass - that might explain neutrino oscillations and POST notions of mass-mixing. Examples include interactions intermediated by the would-be jay boson and (per table 4) interactions that associate with events that associate with one-some use of the 3g1‘2‘4 solution-pair. Even if all three neutrino flavours associate with the same mass, SOMA might explain INFDA that POST interprets as suggesting differences between neutrino masses.

SOMA suggests masses for the might-be zero-charge analogs to quarks.

SOMA suggests that the three flavors of arcs might associate respectively with the following rest energies - $m(0,0)c^2 \approx 10.7$ MeV, $m(1,0)c^2 \approx 6.8$ MeV, and $m(2,0)c^2 \approx 102$ MeV. (Per discussion related to equation (53), SOMA suggests that the three flavors of arcs might associate with somewhat different rest energies.)

SOMA might provide insight about the masses of the might-be heavy neutrinos.

SOMA suggests (but does not necessarily require) that the rest energies of the heavy neutrinos equal or exceed $m(6,0)c^2 \approx 2.5 \times 10^9$GeV. Bases for this suggestion include the following. The range $-6 \leq l_m \leq -4$ associates with masses for neutrinos. The range $-3 \leq l_m \leq -1$ might associate (for some modeling purposes) with right-handedness. The range $0 \leq l_m \leq 3$ associates with masses for charged elementary fermions and with masses for the might-be arc elementary fermions. The range $3 \leq l_m \leq 5$ might associate (for some modeling purposes) with right-handedness. The range $6 \leq l_m \leq 8$ might associate with masses for heavy neutrinos. Each one of the rest energies $m(6,0)c^2 \approx 2.5 \times 10^9$GeV, $m(7,0)c^2 \approx 2.7 \times 10^{11}$GeV, and $m(8,0)c^2 \approx 2.1 \times 10^{13}$GeV comports with INFDA limits. (References [58] and [59] discuss INFDA limits.)

3.4. Differences - between isomers - regarding properties of fermion elementary particles

If the stuff that associates with each of the five all-dark-matter isomers evolved similarly to the stuff that associates with isomer-zero, SOMA suggestions regarding DM might not adequately comport with observations regarding the Bullet Cluster collision of two galaxy clusters. (Discussion that cites reference [53] provides more information.)

SOMA uses the symbol l_i to number the isomers. The notion of isomer-l_i pertains.

Per discussion (including discussion regarding table 7) above, regarding each l_i that is at least one, SOMA posits that the elementary particles in isomer-l_i match - with respect to mass - the elementary particles in isomer-zero.

For modeling regarding flavours (and not - for $0 \leq l_i \leq 5$ - for modeling regarding masses), SOMA associates the quarks in isomer-l_i with three values of l_m. The values are $3l_i + 0$, $3l_i + 1$, and $3l_i + 2$. (Table 7 shows the associations for $l_i = 0$.) Across the six isomers, quarks associate with each value of l_m that is in the range $0 \leq l_m \leq 17$. Regarding quarks and flavours, SOMA posits that - within isomer-l_i - flavour 1 associates with $l_m = 3l_i$, flavour 2 associates with $l_m = 3l_i + 1$, and flavour 3 associates with $l_m = 3l_i + 2$.

Aspects of table 7 point to the notion that means for matching flavours and masses for charged leptons do not match means for matching flavours and masses for quarks. For charged leptons, isomer-zero does not have a charged lepton that associates with $l_m = 1$ and does have a charged lepton that associates

Table 8: Matches between masses and flavours, for isomers of charged elementary fermions. The symbol $0.5\mathrm{Q}_{>0}$ denotes the pair $0.5\mathrm{Q}_{1/3}$ and $0.5\mathrm{Q}_{2/3}$. As in table 7, here the symbol l_f numbers the three flavours.

Isomer	l_m ($0.5\mathrm{Q}_{>0}$)	Respective l_f ($0.5\mathrm{Q}_{>0}$)	l_m ($0.5\mathrm{C}_1$)	Respective l_f ($0.5\mathrm{C}_1$)
0	$0, 1, 2$	1,2,3	$0, 2, 3$	1,2,3
1	$3, 4, 5$	1,2,3	$3, 5, 6$	3,1,2
2	$6, 7, 8$	1,2,3	$6, 8, 9$	2,3,1
3	$9, 10, 11$	1,2,3	$9, 11, 12$	1,2,3
4	$12, 13, 14$	1,2,3	$12, 14, 15$	3,1,2
5	$15, 16, 17$	1,2,3	$15, 17, 18$	2,3,1

with $l_m = 3$. SOMA posits that - for each l_i - a charged lepton associates with each of $l_m = 3l_i + 0$, $l_m = 3l_i + 2$, and $l_m = 3l_i + 3$.

SOMA posits that - for each isomer-l_i such that $1 \leq l_i \leq 5$ - the charged-lepton flavour that associates with $l_m = 3(l_i)+0$ equals the flavour that associates with the isomer-(l_i-1) charged lepton that associates with the same value of l_m and - thus - with $l_m = 3(l_i - 1) + 3$. SOMA posits that, across the six isomers, one cyclical order pertains regarding flavours for charged leptons.

Table 8 shows, for isomers of charged elementary fermions, matches between masses and flavours.

3.5. Possibilities for conversions between dark matter and ordinary matter

The following notions point to possibilities for conversions between DM and OM. Regarding recent times, such conversions might be at least one of improbable and hard to detect (except possibly via experiments and precise measurements).

Conversions between isomers might occur based on interactions mediated by LRI fields. For each one of 1L, 2L, and 3L, table 4 shows at least one one-some solution-pair for which both $R_I \geq 2$ and there is a three-some solution-pair for which $6 \in \Gamma$. Examples of such solution-pairs include 1g2‘3‘4, 2g1‘3‘4, and 3g1‘2‘4. An isomer-zero pair of elementary fermions for which one fermion is the antiparticle of the other fermion could annihilate to excite an LRI field. That excitation could de-excite to produce one isomer-zero left-handed fermion elementary particle and one isomer-three right-handed fermion elementary particle. (For elementary fermions, table 3 notes that conservation of net-left-minus-right pertains regarding isomer-pairs and does not necessarily pertain regarding individual isomers.) Equation (41) symbolizes results of such an excitation and de-excitation.

$$FLH_{l_i=0} + FRH_{l_i=0} \rightarrow FLH_{l_i=0} + FRH_{l_i=3} \tag{41}$$

Early in the history of the universe, effects that associate with gravity and 2g1‘3‘4 might have catalyzed baryon asymmetry. Recently, cross-isomer conversions might associate with effects of high-energy photons and 1g2‘3‘4.

Some conversions between DM stuff and OM stuff might involve isomer-zero DM. Conversions between isomer-zero heavy neutrinos and isomer-zero leptons might be possible. Isomer-zero heavy neutrinos would measure as DM. Isomer-zero leptons measure as OM. Conversions between hadron-like particles that contain 0.5R particles and hadron particles (such as neutrons) might be possible. Hadron-like particles that contain 0.5R particles would measure as DM. Hadron particles (such as neutrons) measure as OM.

4. Results - Cosmology and astrophysics

4.1. Eras in the history of the universe

Reference [60] discusses POST notions regarding cyclic cosmology. SOMA includes the possibility that the present universe arose from an implosion of energy. SOMA does not yet consider either aspects that may have created the energy that would have imploded or whether the present universe might eventually implode.

POST CC points to three eras in the rate of expansion of the universe. The eras feature, respectively, rapid expansion; continued expansion, with the rate of expansion decreasing; and continued expansion, with the rate of expansion increasing.

SOMA suggests using the notion of eras regarding the separating from each other of clumps - that, today, POST would consider to be large - of stuff. Examples of such clumps might include galaxy clusters and even larger clumps. SOMA suggests (per discussion above) that, for a pair of similar objects that

Table 9: Eras regarding the rate of separating of large clumps. The rightmost two columns suggest eras. (Table 10 discusses aspects that associate with each one of some eras.) In table 9, the one-element term 1-some abbreviates one-some. In table 9, subsequent rows associate with later eras. The word inflation (or, the two-word term inflationary epoch) names the era that associates with the third row in the table. Regarding eras that would precede inflation, SOMA points to the possibility for the two eras that the table discusses. One-some solution-pair 0g1‘2‘3‘4‘8 associates with the jay boson. CC suggests inflation and the next two eras. Regarding inflation, POST hypothesizes this era. POST suggests that the inflationary epoch started about 10^{-36} seconds after the Big Bang. POST suggests that the inflationary epoch ended between 10^{-33} seconds after the Big Bang and 10^{-32} seconds after the Big Bang. Possibly, no direct evidence exists for the inflationary epoch. INFDA supports the notions of the two billions-of-years eras. TBD denotes to be determined. The symbol † denotes a possible association between the relevant era and some POST notions of a Big Bang. The leftmost four columns describe phenomena that SOMA suggests as noteworthy causes for the eras. Generally, a noteworthy cause associates with notions of acceleration. Generally, an era associates with a range of velocities. The symbol $\rightarrow$ associates with the notion that a noteworthy cause may gain prominence before an era starts.

Force	1-some solution-pairs	SOMA-pole	R_I	$\rightarrow$	Rate of separating	Duration
Attractive	2g1‘2‘3‘4‘8x	16-pole	6	$\rightarrow$	Is negative	TBD
Repulsive	0g1‘2‘3‘4‘8	-	1	$\rightarrow$	Turns positive †	TBD
Repulsive	2g1‘2‘3‘4x	Octupole	1	$\rightarrow$	Increases rapidly	Fraction of a second
Attractive	2g1‘2‘3	Quadrupole	1	$\rightarrow$	Decreases	Billions of years
Repulsive	2g2‘4	Dipole	2	$\rightarrow$	Increases	Billions of years
Attractive	2g2	Monopole	6	$\rightarrow$	Would decrease	-

always move away from each other, the dominating gravitational effects transit (over time) all or a portion of the following sequence: 16-pole attraction, octupole repulsion, quadrupole attraction, dipole repulsion, and monopole attraction.

Table 9 discusses eras in the rate of separating of large clumps. (Reference [36] discusses possibilities that might lead to a Big Bang. References [61], [19], [37], and [62] discuss the possible inflationary epoch. References [63], [64], [65], and [66] provide data and discussion about the two multi-billion-years eras. Reference [38] discusses attempts to explain the rate of expansion of the universe.)

Table 10 suggests details regarding eras to which table 9 alludes.

SOMA posits that one-some uses of solution-pairs 2g1‘2‘3‘4x and 2g2‘4 associate with CC notions of DE pressures.

SOMA suggests that some SOMA notions regarding eras that start with and follow the inflationary epoch do not necessarily depend significantly on SOMA notions regarding eras that might precede the inflationary epoch.

4.2. Baryon asymmetry

The two-word term baryon asymmetry associates with the POST INFDA notion that - regarding isomer-zero stuff - there are many more left-handed (or matter) fermion elementary particles than right-handed (or antimatter) fermion elementary particles. POST suggests that baryon asymmetry arose early in the history of the universe.

Discussion related to equation (41) points to a means that may have produced baryon asymmetry. Possibly, POST KM notions of lasing pertained regarding relevant excitations of LRI fields. SOMA suggests that processes leading to baryon asymmetry led to isomer-three stuff having fewer left-handed fermion elementary particles than right-handed fermion elementary particles.

This paper does not address the topic of the extent to which steps leading to a predominance in isomer-zero of left-handed elementary particles - and not to a predominance of right-handed elementary particles - have a basis (other than random chance) either in nature or in modeling.

4.3. The evolution of stuff that associates with dark matter isomers

4.3.1. Notions that are common to all six isomers

SOMA associates the symbol IZORDP with all elementary particles except 0.5M and 0.5R fermion elementary particles and SL boson elementary particles. IZORDP abbreviates the five-element phrase isomer-zero ordinary matter elementary particles. SOMA associates the symbol IZDARP with the 0.5M and 0.5R fermion elementary particles. IZDARP abbreviates the five-element phrase isomer-zero dark matter elementary particles. IZDARP associates with the notion that - regarding isomer-zero - these particles (and hadron-like particles made from just 0.5R and SG particles) measure as being DM and do not measure as being OM.

SOMA posits that each one of the six isomers associates with an instance of IZORDP and an instance of IZDARP.

Table 10: Details regarding eras regarding the rate of separating of large clumps. Table 9 discusses the eras. SOMA does not necessarily specify the elementary fermions for which isomers form during the era that associates with the two-word phrase is negative. To the extent that the first significant appearance of most known elementary particles occurs during or just after the inflationary era, SOMA suggests that isomers of at least one of 0.5M and 0.5R form during the era that associates with the two-word phrase is negative. 2g" solution-pairs (such as one to which table 4 alludes) for which $R_I = 6$ might associate with gravitational production of pairs of elementary fermions and with a notion of approximately equal production across the six isomers.) The symbol † associates with some aspects for which the involvement of 0.5M or 0.5R might pertain.

Rate of separating	Note
Is negative	Possibility: 2g1‘2‘3‘4‘8x and their compacting of "some form of energy" lead to conditions suitable for the universe to form and evolve. Possibility: The value of six for R_I (for 2g1‘2‘3‘4‘8x) associates with setting up a system for which roughly equal creation of isomers pertains. Possibility: Isomers of some fermion elementary particles and of 1J form. † Possibility: The following interactions might characterize this era. For each interaction, the net circular polarization for each of before and after the interaction might be zero. † Presumably, the formation of gluons (or, 1(1)G) could associate with the formation of arcs (or, 0.5(1)R)). • 2(6)g1‘2‘3‘4‘8x + 2(6)g1‘2‘3‘4‘8x → 0.5(1)M + 0.5(1)M. • 2(6)g1‘2‘3‘4‘8x + 2(6)g1‘2‘3‘4‘8x → 0.5(1)R + 0.5(1)R. • 2(6)g1‘2‘3‘4‘8x + 2(6)g1‘2‘3‘4‘8x → 1(1)J + 1(1)J. Possibility: The six isomers of the relevant elementary fermions populate approximately equally. † Possibility: Some clumps of relevant elementary fermion stuff serve - eventually - as seeds for galaxies. †
Turns positive	0g1‘2‘3‘4‘8 associates with the 1J (or, jay) boson. The jay boson associates with the notion of Pauli repulsion. Possibility: 1J bosons stop the implosion of stuff that features relevant elementary fermion particles. † Possibility: Isomers of 0I form. The following interaction might characterize this era. Here, the net circular polarization for each of before and after the interaction might be two. 1(1)J + 1(1)J → 2(1)g1‘2‘3‘4x + 0(1)I. Possibility: The six isomers of 0I populate approximately equally. Possibility: Aspects of this era associate with notions of a Big Bang.
Increases rapidly	Some CC modeling suggests that inflatons provide a major component of stuff. Possibility: The following interaction might characterize this era. Here, the net circular polarization for each of before and after the interaction might be two. 0(1)I + 2(1)g1‘2‘3‘4x → 0(1)I + 2(1)g1‘2‘3‘4x.
Decreases	-
Increases	-
Would decrease	This paper does not try to explore the possibility that (or to estimate a time at which) a transition - for the largest observable objects - from repulsion based on 2g2‘4 to attraction based on 2g2 might occur.

SOMA uses the two-element term isomer-l_i stuff to denote objects (including hadron-like particles, atom-like objects, and stars) that associate with the isomer-l_i set of IZORDP particles and IZDARP particles. SOMA uses the three-element term isomer-l_i IZORDP stuff to denote objects that contain just isomer-l_i IZORDP elementary particles. SOMA uses the three-element term isomer-l_i IZDARP stuff to denote objects that contain just isomer-l_i IZDARP elementary particles.

SOMA suggests that - at least after the inflationary epoch - 0.5R-based stuff consists of hadron-like particles. Each 0.5R-based-stuff hadron-like particle includes gluons and at least two arcs. SOMA does not suggest an extent to which 0.5R-based stuff might form primordial black holes. SOMA does not necessarily suggest that a two-or-three-hadron hadron-like particle can include both at least one quark and at least one arc.

SOMA does not suggest an extent to which 0.5M-based stuff might form primordial black holes.

4.3.2. The evolution of isomer-1, isomer-2, isomer-4, and isomer-5 IZORDP stuff

Here, SOMA uses the two-word term alt isomer-to designate an isomer other than isomer-zero and isomer-three.

For each one of the six isomers, a charged baryon that includes exactly three flavour 3 quarks is more massive than the counterpart zero-charge baryon that includes exactly three flavour 3 quarks. (For example, the hadron that includes just two tops and one bottom has a larger total mass than does the hadron that includes just one top and two bottoms.)

Per table 7 and table 8, alt isomer flavour 3 charged leptons are less massive than isomer-zero flavour 3 charged leptons. When flavour 3 quark states are much populated (and based on interactions mediated by W bosons), the stuff that associates with an alt isomer converts more charged baryons to zero-charge baryons than does the stuff that associates with isomer-zero. Eventually, regarding the stuff that associates with the alt isomer, interactions that entangle multiple W bosons result in the stuff that associates with the alt isomer having more neutrons and fewer protons than does the stuff that associates with isomer-zero. The sum of the mass of a proton and the mass of an alt isomer flavour 1 charged lepton exceeds the mass of a neutron. Compared to isomer-zero neutrons, alt isomer neutrons scarcely decay. The IGM (or, intergalactic medium) that associates with the alt isomer scarcely interacts with itself via electromagnetism.

4.3.3. The evolution of isomer-3 IZORDP stuff

The following two possibilities pertain. In one possibility, the evolution of isomer-three IZORDP stuff parallels the evolution of OM (or, isomer-zero IZORDP stuff). In a second possibility, the evolution of isomer-three IZORDP stuff does not parallel the evolution of OM (or, isomer-zero IZORDP stuff). The second possibility might associate with - for example - a difference in handedness - with respect to charged leptons or with respect to W bosons - between isomer-three and isomer-zero.

This paper nominally assumes that the evolution of isomer-three IZORDP stuff parallels the evolution of OM (or, isomer-zero IZORDP stuff).

4.4. Tensions - among data and models - regarding large-scale phenomena

SOMA suggests means to resolve tensions - between INFDA and POST - regarding the rate of expansion of the universe, regarding large-scale clumping of matter, and regarding gravitational interactions between neighboring galaxies.

4.4.1. The rate of expansion of the universe

Table 9 and table 10 discuss possible and known eras in the history of the universe.

POST CC underestimates - for the second multi-billion-years era - increases in the rate of expansion of the universe. (References [40], [41], [42], [43], [67], [68], [69], and [70] provide further information.) Reference [71] suggests that the notion that DM is similar to OM might help resolve the relevant tension.

SOMA suggests the following explanation for such underestimates.

When using modeling based on GR, POST CC might try to extend the use of an equation of state (or the use of a cosmological constant) that works well regarding early in the first multi-billion-years era. Regarding that time, SOMA suggests dominance by attractive effects that associate with one-some use of the 2g1‘2‘3 component of gravity. The notion of a reach of one pertains. The symbol 2(1)g1‘2‘3 pertains. SOMA suggests that - later in the first multi-billion-years era - repulsive effects that associate with one-some use of 2(2)g2‘4 become significant. Dominance by 2(2)g2‘4 pertains by the time the second multi-billion-years era starts. However, POST use of an equation of state that has roots in the time period in which 2(1)g1‘2‘3 dominates might - at best - extrapolate based on a notion of 2(1)g2‘4 (and not based on a notion of 2(2)g2'4). POST would underestimate the strength of the key gravitational driver - of expansion - by a factor of two.

SOMA points - conceptually - to the following possible remedy.

POST CC might change (regarding the stress-energy tensor or the cosmological constant) the aspects that would associate with repulsion and the 2g2'4 component of gravity. The contribution - to the pressure - that associates with one-some use of 2g2'4 needs to double (compared to the contribution that would associate with one-some use of 2(1)g2'4).

4.4.2. Large-scale clumping of matter

POST CC overestimates large-scale clumping of matter - OM and DM. (References [72], [73], [74], and [43] provide data and discussion.)

SOMA suggests that POST CC modeling associates with a repulsive component - 2(1)g2'4 - of gravity. SOMA suggests that 2(2)g2'4 pertains. (That is, for each instance of 2g2'4, a reach of two isomers pertains.) The additional (compared to CC modeling) repulsion might explain the overestimating that POST CC suggests.

4.4.3. Effects - within galaxies - of the gravity associated with nearby galaxies

POST CC might not account for some observations about effects - within individual galaxies - of the gravity associated with nearby galaxies. (Reference [48] provides further information.)

SOMA suggests that POST CC modeling associates with a repulsive component - 2(1)g2'4 - of gravity. SOMA suggests that 2(2)g2'4 pertains. The additional (compared to CC modeling) repulsion might explain at least some aspects of the INFDA that reference [48] discusses.

4.5. Formation and evolution of galaxies

4.5.1. Mechanisms regarding the formation and evolution of galaxies

Reference [75] suggests that galaxies form around early clumps of stuff. Reference [75] associates the word halo with such clumps.

Table 9 suggests that single-isomer stuff - such as stuff that features 0.5R particles - forms as early as during an era in which one-some solution-pairs 2g1'2'3'4'8x - which associate with attraction - dominate regarding prototype large clumps. Smaller-scale clumps might form before larger-scale clumps. Effects that associate with the one-some solution-pair 2g1'2'3 - which is attractive - might contribute to the formation of smaller-scale clumps. The reach that associates with 2g1'2'3 is one isomer.

SOMA suggests that each one of many early halos associates with one isomer. SOMA associates with such early halos the three-element term one-isomer original clump. Clumping occurs based on gravitational effects. Differences - between the evolution of stuff associating with any one of isomer-zero and isomer-three and the evolution of stuff associating with any one of isomers one, two, four, and five are not necessarily significant regarding this gravitationally based clumping. The six isomers might form such clumps approximately equally.

Table 11 discusses SOMA suggestions regarding the formation and evolution of a galaxy for which a notion of a one-isomer original clump pertains.

Presumably, some galaxies form based on two or more clumps, for which all of the clumps associate with just one isomer. Possibly, some galaxies form based on two or more clumps, for which some clumps associate with isomers that are not the same as the isomers that associate with some other clumps.

4.5.2. Aspects regarding the evolution of galaxies

Table 11 suggests three eras regarding the evolution of galaxies. The first era associates with the first two rows in table 11. The second era associates with the 2g2 attractive force that associates with the third row in table 11. The third era associates with collisions between and mergers of galaxies.

Some galaxies do not exit the first era and do not significantly collide with other galaxies.

Some galaxies result from aspects associating with the 2g2 attractive force that associates with the third row in table 11. Here, this paper discusses three cases. (Mixed cases and other cases might pertain.)

- Each one of some era-one galaxies does not collide with other galaxies. Such a galaxy accumulates (via 2g2 attraction) stuff associating with various isomers that have representation in nearby IGM (or, intergalactic medium). The galaxy becomes an era-two galaxy. The galaxy might include stuff that significantly associates with as many as five isomers.

Table 11: Stages and other information regarding the evolution of a galaxy for which a notion of a one-isomer original clump pertains. The table suggests stages, with subsequent rows associating with later stages. The next to rightmost column describes aspects of the stage. The leftmost four columns in the table describe a component of 2L that is a noteworthy cause for the stage. (Regarding phenomena that associate with gravitation, table 11 echoes aspects - including aspects regarding attraction and repulsion - that table 9 shows.) The one-element term 1-some abbreviates one-some. The symbol $\rightarrow$ associates with the notion that a noteworthy cause may gain prominence before a stage starts. Table 11 associates with a scenario in which a galaxy forms based on one original clump and initially does not significantly collide with other galaxies. The galaxy might retain some stuff that associates with the repelled isomer. The rightmost column in table 11 suggests terminology regarding the evolution of galaxies. (A galaxy can include stuff from more than one earlier galaxy.)

Force	1-some solution-pair	SOMA-pole	R_I	$\rightarrow$	Stage	Aspects of the stage	Era
Attractive	2g1'2'3	4	1	$\rightarrow$	1	A one-isomer original clump forms.	First
Repulsive	2g2'4	2	2	$\rightarrow$	2	The original clump repels (some) stuff that associates with the isomer that associates with the original clump and (most) stuff that associates with one other isomer.	First
Attractive	2g2	1	6	$\rightarrow$	3	The original clump attracts stuff that associates with the four not-repelled isomers and stuff that associates with the isomer that associates with the original clump.	Second
Attractive	2g2	1	6	$\rightarrow$	4	Another galaxy subsumes the original clump and might subsequently merge with yet other galaxies.	Third

- Each one of some era-two galaxies merges (via 2g2 attraction) mainly just with galaxies that feature the same five isomers. The galaxy that merged, in effect, loses it status of being a galaxy. The resulting larger object is an era-two galaxy. The galaxy might include stuff that significantly associates with as many as five isomers.

- Each one of some era-one or era-two galaxies merges (via 2g2 attraction) with other galaxies. The galaxy that merged, in effect, loses its status of being a galaxy. The resulting larger object is an era-three galaxy. The galaxy might include stuff that significantly associates with as many as six isomers.

4.6. Explanations for ratios of dark matter effects to ordinary matter effects

4.6.1. Nominal explanations

Table 12 provides explanations for ratios - that pertain to galaxies - of DM effects to OM effects. (References [76] and [77] provide data and discussion. Reference [76] influenced the choice - that this paper reflects - of a time range to associate with the word early. Regarding the combination of 0^+:1 and later, references [78], [79], [80], [81], [82], [83], and [84] provide data and discussion. Reference [85] discusses a galaxy that might have started as containing mostly OM. Regarding observed DM galaxies, references [75], [86], [87], and [88] provide data and discussion. Current techniques might not be capable of observing early DM galaxies. References [89] and [90] suggest, regarding galaxy clusters, the existence of clumps of DM that might be individual galaxies. Extrapolating from results that references [75] and [91] discuss regarding ultrafaint dwarf galaxies that orbit the Milky Way galaxy might suggest that the universe contains many DM:OM $1 : 0^+$ later galaxies. Reference [92] discusses a trail of galaxies for which at least two galaxies have little DM. Reference [92] suggests that the little-dark-matter galaxies result from a collision that would have some similarities to the Bullet Cluster collision. Regarding galaxies for which DM:OM ratios of $\sim$4:1 pertain, references [93] and [94] provide data and discussion. Regarding later galaxies for which DM:OM ratios of 5^+:1 pertain, reference [75] provides data and discussion. References [95] and [96] provide data about collisions of galaxies.)

Table 13 provides explanations for observed ratios - that pertain to larger-than-galaxies-scale stuff - of DM effects to OM effects. (Reference [54] provides data and discussion regarding densities of the universe. References [97], [98], [99], and [100] provide data and discussion regarding galaxy clusters.)

Table 12: Explanations for ratios - that pertain to galaxies - of DM effects to OM effects. DM:OM denotes a ratio of DM effects to OM effects. Inferences of DM:OM ratios come from interpreting data. Regarding galaxies, the notion of early associates with observations that pertain to galaxies that POST associates with (or, would, if people could detect the galaxies, associate with) high redshifts. High might associate with $z > 7$ and possibly with smaller values of z. Here, z denotes redshift. The word later associates with the notion that observations pertain to objects later in the history of the universe. The two-element phrase DM galaxy denotes a galaxy that contains much less OM than DM. Possibly, people have yet to directly detect early DM galaxies. Table 11 provides information about the explanations.

Objects	DM:OM	Examples	Explanation
Some early galaxies	$0^+ : 1$	Reported	OM original clump. Stage 1 or 2.
Some later galaxies	$0^+ : 1$	Reported	OM original clump. Stage 1 or 2.
Some early galaxies	$1 : 0^+$	No known reports	DM-isomer(s) original clump. Stage 1 or 2.
Some later galaxies	$1 : 0^+$	Reported	DM-isomer(s) original clump. Stage 1 or 2.
Some later galaxies	$\sim 4 : 1$	Reported	Non-isomer-three original clump. Stage 3.
Many later galaxies	$5^+ : 1$	Reported	Any-isomer(s) original clump(s). Stage 4.

Table 13: Explanations for observed ratios - that pertain to larger-than-galaxies-scale stuff - of DM effects to OM effects. DM:OM denotes a ratio of DM effects to OM effects. Inferences of DM:OM ratios come from interpreting data. The symbol IZDARP abbreviates the five-word phrase isomer-zero dark matter elementary particles. The symbol IZORDP abbreviates the five-word phrase isomer-zero ordinary matter elementary particles.

Aspect	DM:OM	Comment
Densities of the universe	$5^+ : 1$	IZDARP stuff that associates with isomer-zero through isomer-five associates with the plus in DM:OM $5^+ : 1$. IZORDP stuff that associates with isomer-one through isomer-five associates with the five in DM:OM $5^+ : 1$. IZORDP stuff that associates with isomer-zero associates with the one in DM:OM $5^+ : 1$.
Some galaxy clusters	$5^+ : 1$	SOMA posits that galaxy clusters (that have not collided with other galaxy clusters) associate with DM:OM ratios that are similar to DM:OM ratios for densities of the universe.

Table 14 lists ratios - that pertain to light that dates to about 400,000 years after the Big Bang - of observed effects to effects that POST estimates. (Reference [101] provides data and discussion regarding the amount of cosmic optical background. References [102], [103], and [104] provide data and discussion regarding absorption of CMB.)

The following two paragraphs provide SOMA explanations for the observations to which table 14 alludes.

The three-word phrase cosmic optical background associates with now nearly-optical light remaining from early in the universe. An observation inferred twice as much light as POST CC expected based on CC. POST CC suggests that atomic transitions produced the radiation that today measures as cosmic (optical and microwave) background radiation. SOMA suggests that isomer-one, isomer-two, isomer-four, and isomer-five stuff did not result in much stuff that is similar to isomer-zero atoms. SOMA suggests that isomer-three stuff evolved similarly to isomer-zero stuff. Across four types of changes in atomic energy levels, table 4 alludes to 1L-producing events that associate with $R_I \geq 2$. SOMA explains the two-to-one reported-to-expected ratio regarding the cosmic optical background. Isomer-zero stuff produced half of the observed light. Isomer-three stuff produced half of the observed light.

The four-element phrase some absorption of CMB associates with the notion that POST CC measured some specific depletion of CMB (or, cosmic microwave background radiation) and inferred twice as

Table 14: Ratios - that pertain to light that dates to about 400,000 years after the Big Bang - of observed effects to effects that POST estimates. The three-word phrase cosmic optical background associates with radiation that - recently - measures as optical radiation or as close (with respect to wavelengths) to optical radiation. The acronym CMB associates with radiation that - recently - measures as cosmic microwave background radiation. DM:OM denotes a ratio of DM effects to OM effects that this paper posits.

Aspect	Reported : Expected	Measurement	Posited DM:OM
Amount of cosmic optical background	2 : 1	One reported measurement	1 : 1
Some absorption of CMB	2 : 1	One reported measurement	1 : 1

much depletion as POST CC expected based solely on hyperfine interactions with OM hydrogen atoms. (Reference [102] has a basis in overall measurements of CMB and does not have a basis in an estimate - from CC - of how much CMB nature might have produced.) Possibly, half of the depletion associates with DM effects. Hyperfine interactions associate with the notion of atomic transitions within principal energy levels. SOMA suggests (per table 4) that isomer-three hydrogen-like atoms account for the half of the absorption for which isomer-zero (or, OM) hydrogen atoms do not account.

4.6.2. A possible alternative explanation for DM:OM ratios of $5^+ : 1$

SOMA suggests a possible variation - regarding explanations - for DM:OM ratios of $5^+ : 1$.

Here, SOMA uses the term alt isomer to refer to isomer-one, isomer-two, isomer-four, and isomer-five. Here, SOMA assumes that evolution of alt isomer stuff deviates - compared to the evolution of isomer-zero stuff - early enough that (nominally) isomer-zero high-energy photons produce alt isomer stuff significantly more copiously than (nominally) alt isomer photons produce isomer-zero stuff.

This variation might help account for the plus in each one of the DM:OM ratios of $5^+ : 1$, even if nature does not include arc (or, 0.5R) fermion elementary particles and does not include heavy neutrino (or, 0.5M) fermion elementary particles.

5. Discussion - POST and SOMA

5.1. Elementary particles that POST suggests and INFDA has yet to include

SOMA might provide insight regarding the existence and properties of some elementary particles that POST hypothesizes and INFDA does not yet point to.

5.1.1. Axion

POST suggests the possibility for an axion, - a zero-spin, nonzero-mass elementary particle. POST suggests that axions might decay into photons. POST suggests two possible needs that the axion might fulfill.

One possible need is for an elementary particle that would measure as DM. SOMA does not need to include an axion to explain INFDA about DM.

The other possible need is to resolve the so-called strong CP problem (which associates with POST QF quantum chromodynamics). POST QF includes a notion that might associate with the breaking of CP symmetry (or, symmetry with respect to charge conjugation and parity reversal) by the strong interaction. In SOMA, gluons associate with two three-some solution-pairs for which $6 \in \Gamma$. Possibly, mathematically, the notion of two such solution-pairs points to a possibility for - in effect - decoupling C transformations from P transformations. This paper does not discuss the extent to which, mathematically, such a decoupling might associate with the POST suggestion that the strong interaction might associate with violation of CP symmetry. INFDA has yet to point to examples of such symmetry breaking.

SOMA does not necessarily point to either of the two POST would-be needs for an axion elementary particle.

POST suggests that - under some circumstances - axions might convert into photons. SOMA suggests that observations that POST might associate with effects of axions might instead associate with the difference between the SOMA notion of 1(6)g1‘2‘4 and POST notions that SOMA associates with notions of 1(1)g1‘2‘4. Also, observations that POST might associate with effects of axions might instead associate with interactions involving jay (or, 1J) bosons or aye (or, 0I) bosons.

INFDA has yet to point to needs for POST or SOMA to include axion elementary particles.

5.1.2. Magnetic monopoles

Table 2 seems not to suggest a 1L interaction with a monopole other than an electric monopole. SOMA does not suggest a property that would associate with a magnetic monopole.

5.1.3. Right-handed W bosons

Reference [105] discusses a fraction of decays - of OM top quarks for which the decay products include W bosons - that might produce right-handed W bosons. The fraction, f_+, is 3.6×10^{-4}. Reference [16] provides a confidence level of 90 percent that the rest energy of a would-be W_R (or, right-handed W boson) exceeds 715 GeV. Reference [106] provides other information.

SOMA suggests that W_R bosons associate only with isomers one, three, and five. SOMA suggests possibilities for inter-isomer interactions and conversions.

Aspects of SOMA might approximately reproduce the above result that SM modeling suggests.

Aspects related to equation (32) suggest values of calculated masses that do not associate with masses of known or suggested elementary particles. For example, SOMA does not suggest that $m(5,3)$ associates with the inertial mass of an isomer-one charged lepton. However, perhaps such mass-like quantities associate with some measurable aspects of nature. For charged leptons and $0 \leq l_i \leq 4$ and $0 \leq l'_f \leq 3$, $m(3(l_i+1)+l'_f,3) = \beta m(3(l_i+0)+l'_f,3)$. One might conjecture that isomer-zero observations of some aspects of isomer-one phenomena associate with notions of non-inertial mass-like quantities that are β times the inertial masses for isomer-zero elementary particles (and that are β times inertial masses for the counterpart isomer-one elementary particles).

Based on notions of scaling that might calculate non-inertial mass-like quantities, SOMA might suggest that $f_+ \sim e^{(\beta^{-1})} - 1 \approx \beta^{-1} \approx 2.9 \times 10^{-4}$. This estimate might not be incompatible with results that reference [105] discusses. A notion of $m_{\text{non-inertial,W}_R\text{isomer one}}c^2 = \beta m_{\text{W}}c^2 \approx 2.8 \times 10^5$ GeV might pertain. Here, the notion of a non-inertial mass-like quantity might associate with INFDA that associate with interactions that associate with 1L or 1W_1. The interactions do not necessarily associate directly with 2L.

5.2. *Interactions involving the jay boson*

SOMA discusses interactions that involve jay bosons.

5.2.1. *Interactions - before or during inflation - that involve jay bosons*

SOMA considers interactions in which two jay bosons move in parallel, interact, and produce one aye boson plus something else. Generally, SOMA assumes that conservation of angular momentum pertains. Here, SOMA assumes that one can de-emphasize angular momentum that is not intrinsic to the relevant elementary particles. SOMA considers two cases. In the first case, the two jay bosons have the same (one of either right-circular or left-circular) polarization. Conservation of angular momentum allows an outgoing combination of one 2L particle and one 0I particle. The de-emphasizing of non-intrinsic angular momentum might - in effect - preclude producing one 1L particle and one 0I particle. In the second case, one jay boson has left-circular polarization and the other jay boson has right-circular polarization. Conservation of angular momentum allows the production of two 0I particles. The de-emphasizing of non-intrinsic angular momentum might - in effect - preclude producing one 1L particle and one 0I particle.

The two cases might comport with the notion that gravitation can be significant during inflation. The two cases might comport with the notion that jay bosons form before aye bosons form. (Table 10 provides relevant information.)

The two cases might comport with a POST notion that electromagnetism might become significant essentially only after inflation.

5.2.2. *Pauli repulsion*

POST includes the notion that two identical fermions cannot occupy the same state. Regarding POST QM, one notion is that repulsion between identical fermions associates with overlaps of wave functions. Another POST QM notion features wave functions that are anti-symmetric with respect to the exchange of two identical fermions.

SOMA might be compatible with such aspects of POST and, yet, not necessitate - for kinematics modeling - the use of wave functions. QM based on jay bosons might suffice. CM based on potentials that would associate with effects of jay bosons might suffice.

QM or CM based on jay bosons might suggest that the prevention of two identical fermions from occupying the same state might associate with, in effect, interactions - mediated by jay bosons - that try to change aspects related to the fermions. Notions of changing a spin orientation might pertain. For elementary fermions, notions of changing a flavour might pertain.

5.2.3. *Pauli crystals*

Reference [107] reports detection of Pauli crystals. SOMA suggests that modeling based on the notion of jay bosons might help explain relevant phenomena.

5.2.4. *Energy levels in positronium*

Reference [108] discusses the transition - between two states of positronium - characterized by the expression that equation (42) shows.

$$2^3S_1 \rightarrow 2^3P_0 \tag{42}$$

Four standard deviations below the nominal observed value of the energy that associates with the transition approximately equals four standard deviations above the nominal value of the energy that POST suggests.

SOMA notions regarding jay bosons might explain the might-be discrepancy regarding positronium. Compared to POST QF, a new notion of virtual charge exchange or a new notion of virtual flavour change might pertain.

To the extent that POST QF does not suffice, SOMA related to the jay boson might help to close the gap between observation and modeling.

5.3. Constraints regarding dark matter

5.3.1. Aspects related to collisions of pairs of galaxy clusters

Reference [53] discusses the Bullet Cluster collision of two galaxy clusters.

POST INFDA suggests two general types of trajectories for stuff. Most DM - from either one of the clusters - exits the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. Also, OM stars - from either cluster - exit the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. However, OM IGM (or, intergalactic medium) - from either cluster - lags behind the cluster's OM stars and DM. POST suggests that the OM IGM interacted electromagnetically with the other cluster's OM IGM, as well as gravitationally with the other cluster.

SOMA comports (regarding each cluster) with the POST INFDA interpretation, with one possible exception. SOMA suggests that isomer-three IGM interacts electromagnetically and follows trajectories that are consistent with OM IGM trajectories.

At least three possibilities arise.

For one possibility, per table 4, the light that POST INFDA associates with OM IGM might include light that associates with OM IGM and light that associates with isomer-three IGM.

For one possibility, POST INFDA considers that isomer-three IGM measures as dark matter and POST INFDA does not adequately report (or otherwise account for) lagging isomer-three IGM.

For one possibility, isomer-three IGM follows trajectories that are consistent with other DM trajectories.

SOMA suggests that POST INFDA may not be sufficient to rule out each one of the first two possibilities and to rule out a combination of the first two possibilities.

SOMA notions of DM are not necessarily incompatible with constraints - that have bases in observations of collisions of galaxy clusters - regarding DM.

5.3.2. Aspects related to cosmological models

Reference [75] summarizes some POST thinking about constraints on DM and about notions of DM. Reference [75] notes that CDM (or, cold dark matter) might comport well with various models. Some POST CC associates with the one-element term ΛCDM. Reference [75] notes that POST has yet to determine directly whether nature includes CDM stuff. The article notes that POST CC considers that notions of SIDM (or, self-interacting dark matter) might be appropriate regarding nature. POST CC also uses other terms, such as the three-word term warm dark matter, to note possible attributes of DM. For example, reference [109] suggests that notions of WDM (or, warm dark matter) might reduce discrepancies between data regarding clustering within galaxies and modeling that associates with CDM.

Notions such as SIDM and WDM arise from POST that differs from SOMA. SOMA is reluctant to try to closely associate SOMA notions with POST terms such as SIDM or WDM. (SOMA suggests that isomer-zero 0.5R-based stuff, isomer-zero 0.5M stuff, and all stuff associating with isomers one, two, four, and five might comport with some notions of CDM. SOMA suggests that the remaining DM stuff - or, isomer-three IZORDP stuff - might associate with some notions of WDM and with some notions of SIDM.)

SOMA notions of DM are not necessarily incompatible with constraints - that have bases in POST CC - regarding DM.

5.4. Some phenomena that associate with galaxies

5.4.1. Some quenching of star formation

Some galaxies seem to stop forming stars. (Reference [110] and reference [111] discuss examples.) Such quenching might take place within three billion years after the Big Bang, might associate with a lack of hydrogen atoms, and might (per reference [111]) pertain to half of the galaxies that associate with the notion of a certain type of galaxy.

SOMA suggests that some such quenching might associate with repulsion that associates with 2(2)g2'4. Quenching might associate with galaxies for which original clumps featured isomer-zero stuff or isomer-three stuff.

5.4.2. Some stopping of the accrual of matter

Reference [112] discusses a galaxy that seems to have stopped accruing both OM and DM about four billion years after the Big Bang.

The galaxy that reference [112] discusses might (or might not) associate with the notion of significant presence early on of one of isomers zero and three, one of isomers one and four, and one of isomers two and five. Such early presences might associate with a later lack of nearby stuff for the galaxy to accrue.

5.4.3. Aspects regarding stellar stream GD-1 in the Milky Way galaxy

Data regarding stellar stream GD-1 suggest the possibility of effects from a yet-to-be-detected non-ordinary-matter clump - in the Milky Way galaxy - with a mass of 10^6 to 10^8 solar masses. (References [113] and [114] provide data and discussion regarding the undetected object. Reference [114] cites reference [115] and reference [116].) SOMA suggests that the undetected object might be a clump of DM.

5.5. Modeling that might point to a phase change regarding the universe

POST includes the two-word term phase change and sometimes suggests that the notion of a phase change might pertain early in the history of the universe. Possibly, such a notion of phase change associates with - regarding 2L fields - a loss of significance (relative to one-some uses of 2gΓ solution-pairs for which $8 \notin \Gamma$ pertains for 2gΓ) for one-some uses of 2gΓ solution-pairs for which $8 \in \Gamma$ for 2gΓ.

5.6. Gauge symmetries

Equation (43), equation (44), and equation (45) show POST Gauge symmetries.

$$\text{Electromagnetic interaction: } U(1) \tag{43}$$

$$\text{Weak interaction: } SU(2) \times U(1) \tag{44}$$

$$\text{Strong interaction: } SU(3) \tag{45}$$

SOMA notions regarding degrees of freedom might lead - as follows - to a possible echo of POST Gauge symmetries.

Per table 6, for each INFDA elementary boson, one one-some solution-pair pertains and - regarding three-some solution-pairs for which $6 \in \Gamma$ - either one solution-pair pertains or two solution-pairs pertain. Each such three-some solution-pair cascades from the associated one-some solution-pair via the addition - to the Γ for the one-some solution-pair - of the number six to the list Γ.

POST associates - with each other - the electromagnetic interaction and the photon. SOMA associates - with the photon - one three-some solution-pair for which $6 \in \Gamma$. POST and SOMA associate - with the photon - two circular polarization modes. POST and SOMA consider that modeling for excitation and de-excitation of a mode has bases in mathematics that associates with a one-dimensional harmonic oscillator. For one mode, SOMA associates excitations with the harmonic oscillator that associates with $s_6 = -1$ and associates a $U(1)$ symmetry with the oscillator that associates with $s_6 = +1$. For the other mode, SOMA associates excitations with the harmonic oscillator that associates with $s_6 = +1$ and associates a $U(1)$ symmetry with the oscillator that associates with $s_6 = -1$. For each mode and for the overall notion of photon, SOMA posits that a $U(1)$ symmetry pertains.

POST associates - with each other - the strong interaction and the gluons. SOMA associates - with the gluons - two three-some solution-pairs for which $6 \in \Gamma$. SOMA posits that - across the two three-some solution-pairs, four one-dimensional harmonic oscillators have relevance. (Four equals two - as in the number of three-some solution-pairs for which $6 \in \Gamma$ - times two - as in a number of oscillators that associate with one three-some solution-pair.) POST and SOMA associate - with each gluon - two circular polarization modes. For each mode, SOMA posits that one harmonic oscillator associates with excitation and de-excitation and that an $SU(3)$ symmetry associates with the remaining three harmonic oscillators.

POST associates the weak interaction with the W and Z bosons. SOMA associates - with each of the W boson and Z boson - one three-some solution-pair for which $6 \in \Gamma$. The three-some solution-pair for which $6 \in \Gamma$ for the W boson differs from the three-some solution-pair for which $6 \in \Gamma$ for the Z boson. For the weak interaction, SOMA posits that - across the two three-some solution-pairs, four one-dimensional harmonic oscillators have relevance. POST and SOMA associate - with each of the W boson and the Z boson, three modes (or, angular momentum states). For each excitation mode, SOMA posits that one one-dimensional oscillator associates with excitation or de-excitation and that the other three one-dimensional oscillators associate with symmetry. For the Z boson, $4 \in \Gamma$, equation (21) pertains, and $2 \notin \Gamma$. For the Z boson, SOMA associates two of the symmetry-related one-dimensional oscillators with s_2 and $SU(2)$ symmetry and associates the other one symmetry-related one-dimensional oscillator with s_6 and $U(1)$ symmetry. For the W boson, $2 \in \Gamma$, equation (21) pertains, and $4 \notin \Gamma$. For the W boson, SOMA associates two of the symmetry-related one-dimensional oscillators with s_4 and $SU(2)$ symmetry and associates the other one symmetry-related one-dimensional oscillator with s_6 and $U(1)$ symmetry. SOMA posits that - regarding each excitation or de-excitation - $SU(2) \times U(1)$ symmetry pertains.

POST associates - for the weak interaction - the notion of a broken symmetry with the $SU(2)$ aspect of equation (44). SOMA posits that a notion of a broken symmetry regarding the SOMA $SU(2)$ aspect of $SU(2) \times U(1)$ symmetry associates with charge and with equation (21).

Possibly, the above three symmetries that SOMA posits associate - respectively - with the equation (43), equation (44), and equation (45) POST Gauge symmetries.

The above possible symmetries might associate aspects of SOMA with POST notions of Gauge symmetries. This paper does not explore relationships - between details of SOMA and details of POST - that might describe such an association.

5.7. Notions regarding numbers of dimensions and regarding fermion excitations

Regarding excitations of boson fields, POST QM includes notions of modeling based on harmonic oscillator mathematics and an associated potential that is proportional to r^{+2}.

SOMA posits that $k = -2$ associates with modeling based on harmonic oscillator mathematics. (Regarding SOMA based on terms $\Sigma g\Gamma$, only positive values of k associate with lists Γ and sets K_n.)

The oscillator pair that associates with $s_{-2} = \pm 1$ associates with three generators and three degrees of freedom. SOMA notes the possibility that the three degrees of freedom associate with the three POST KM spatial dimensions. The oscillator that associates with $s_{-2} = 0$ associates with one generator and one degree of freedom. The one degree of freedom might associate with the notion of one POST KM temporal dimension.

For a field that POST QM associates with the word boson, the change - from before an interaction that associates with the field to after the interaction - associates with the POST notion of excitation or de-excitation of the field. For an excitation, the excitation number (of the field) changes from a nonnegative integer n to $n+1$. The POST boson raising operator $a^{+}|n>= (1+n)^{(1/2)}|n+1>$ pertains. For a de-excitation, the excitation number changes from a positive integer n to $n-1$. The POST lowering operator $a^{-}|n>= n^{(1/2)}|n-1>$ pertains.

POST does not use harmonic oscillator mathematics to underlie modeling for excitations and de-excitations of fermion fields.

SOMA posits that equation (46) might show a fermion raising operator and that equation (47) might show a fermion lowering operator. The raising operator and the lowering operator confine the integer n to the domain $0 \leq n \leq 1$.

$$f^{+}|n>= (1-n)^{(1/2)}|n+1> \tag{46}$$

$$f^{-}|n>= n^{(1/2)}|n-1> \tag{47}$$

5.8. Some possibilities for detecting non-isomer-zero dark matter

Table 4 points to electromagnetic phenomena that associate with reaches of two and, thereby, suggests that OM equipment might be able to catalyze or detect transitions within isomer-three atoms. Discussion related to table 14 suggests that INFDA points to detection, by OM equipment, of light emitted by transition events that associate with isomer-three atoms. Presumably, some isomer-three atoms pass (essentially unimpeded by isomer-zero stuff) through isomer-zero stuff that is near to and includes the Earth. This paper suggests possibilities for doing experiments - based on OM-produced light and OM-detected light - to detect (via transition events that associate with isomer-three atoms) isomer-three atomic stuff. This paper does not discuss notions regarding whether techniques are now or when techniques might become sufficiently sensitive that such experiments would be feasible.

6. Discussion - From POST, via SOMA, toward NEST

6.1. Anomalous magnetic moments

In SOMA, one-some use of the 1g1'2 solution-pair associates with the one-some property of magnetic moment. Three-some use of the 1g1'2 solution-pair associates with contributions - to the magnetic field - based on moving charge.

POST QF associates with two complementary aspects of magnetic moment - nominal magnetic moment and anomalous magnetic moment. POST QF calculates anomalous magnetic moments that match INFDA regarding the electron and the muon. The calculations feature notions of virtual photons.

SOMA associates two solution-pairs - 1g1'2 and 3g1'2 - with the Γ that equals 1'2.

Regarding NEST, SOMA posits that one-some use of the solution-pair 1g1'2 associates with the one-some property of nominal magnetic moment and that one-some use of the solution-pair 3g1'2 associates with the one-some property of anomalous magnetic moment.

Two three-some solution-pairs associate with one-some use of the 3g1'2 solution-pair. The 3g1'2'6 three-some solution-pair associates with $6 \in \Gamma$. SOMA posits that the strength of 3g1'2'6 can vary based on elementary fermion flavour. The 3g1'2'4 three-some solution-pair associates with $6 \notin \Gamma$. SOMA posits that the strength of 3g1'2'4 does not vary based on elementary fermion flavour.

SOMA posits that equation (48) approximates a_{cl}, the anomalous magnetic moment for the cl charged lepton. Here, each one of a_4 and a_6 is a constant with respect to a choice between $cl = e$ (for the electron), $cl = \mu$ (for the muon), and $cl = \tau$ (for the tau). a_4 associates with 3g1'2'4. a_6 associates with 3g1'2'6.

$$a_{cl} \approx a_4 + a_6 t_{cl} \tag{48}$$

Aspects of equation (29) feature squares of properties. (Regarding equation (28), equation (29), and equation (30), $\beta^2 = (G_N(m_\tau)^2)/(G_N(m_e)^2)$.) Squares of properties might associate with notions of self-interactions.

SOMA might assume that t_{cl} is $(\log(m_{\mathrm{cl}}/m_e))^2$.

Based on data that reference [54] provides regarding the electron and the muon, SOMA calculates a_4 and a_6. Then, SOMA calculates a value, $a_{\tau,\mathrm{EQ}}$, for a_τ. Reference [117] provides, based on POST SM, a first-order result - which SOMA calls $a_{\tau,\mathrm{SM}}$ - for a_τ. Here, SM denotes the two-word term Standard Model. The value of $a_{\tau,\mathrm{EQ}}$ results in a value of $(a_{\tau,\mathrm{EQ}} - a_{\tau,\mathrm{SM}})/a_{\tau,\mathrm{SM}}$ of approximately -0.00228. Each one of $a_{\tau,\mathrm{EQ}}$ and $a_{\tau,\mathrm{SM}}$ comports with INFDA that reference [54] provides.

Regarding anomalous magnetic moments, NEST EQ might provide an alternative to POST QF. The NEST EQ method might require more input - from INFDA or POST QF - than POST QF requires. Here, NEST EQ features mathematics that associates with the word algebra. Here, NEST EQ avoids directly using POST QF mathematics that associates with the two-word term conditionally convergent.

6.2. Properties that associate with elementary fermions

Per table 3 and table 9, one-some use of the solution pair 2g1'2'3 associates with gravitational attraction.

For each charged lepton, a gravitational energy might associate approximately with the expression that equation (49) shows. (Here, one might associate the symbol a_{2g2} with the symbol m, which associates with mass.) SOMA posits that $a_{2g1'2'3}$ is the same for each one of the three charged leptons.

$$(m_{\mathrm{cl}} + a_{2g1'2'3})c^2 \tag{49}$$

SOMA posits that equation (50) pertains, assuming that $n_e = 0$, $n_\mu = 17$, and $n_\tau = 26$.

$$m_{\mathrm{cl}} + a_{2g1'2'3} = (m_e + a_{2g1'2'3}) \cdot ((m_\mu + a_{2g1'2'3})/(m_e + a_{2g1'2'3}))^{n_{\mathrm{cl}}/17} \tag{50}$$

Based on results from equation (30), equation (51) pertains.

$$a_{2g1'2'3} \approx 0.0002256 \text{ MeV}/c^2 \tag{51}$$

SOMA posits that equation (28), equation (29), and equation (30) define the ratio m_τ/m_e in terms of constants that POST might consider to be independent of m_τ. SOMA posits that equation (49), equation (50), equation (51), and the notions that $n_e = 0$, $n_\mu = 17$, and $n_\tau = 26$ define the ratio m_μ/m_e in terms of constants that POST might consider to be independent of m_μ and m_τ.

SOMA posits that relevant effects related to $a_{2g1'2'3}$ scale linearly with the magnitude of the charge of fermion elementary particles.

Regarding replacing the left side of equation (32), the expression that equation (52) shows pertains.

$$m(l_m, l_q) + (l_q/3) \cdot a_{2g1`2`3} \tag{52}$$

Regarding replacing the right side of equation (32), the expression that equation (53) shows pertains.

$$(m_e + a_{2g1`2`3}) \cdot ((m_\mu + a_{2g1`2`3})/(m_e + a_{2g1`2`3}))^{n_{m(l_m,l_q)}/17} \cdot (\alpha^{-1/4})^{g(l_q)\cdot(1+l_m)+j'_{l_q} d'(l_m)} \tag{53}$$

Assuming that $n_{m(0,l_q)} = 0$, $n_{m(1,l_q)} = 9$, $n_{m(2,l_q)} = 17$, $n_{m(3,l_q)} = 26$, $d'(0) = +0.29$, $d'(1) = -1.06$, and $d'(2) = -1.51$, the equality between equation (52) and equation (53) produces results - for the masses of the nine charged elementary fermions - that are essentially as INFDA appropriate as are results that equation (32) produces. For each one of the five quarks other than the top quark, $m(l_m, l_q)$ is within 0.3 standard deviations of the INFDA value. For the top quark and each of the three INFDA-posited values, $m(l_m, l_q)$ is within 4.5 standard deviations of the INFDA-posited value. (Each one of the first calculated value and the third calculated value is greater than the respective INFDA-posited value. The second calculated value is less than the respective INFDA-posited value.) For the three arc elementary fermions, the values of mass would be $m(0,3) \approx 9.4$ MeV/c^2, $m(1,0) \approx 6.9$ MeV/c^2, and $m(2,0) \approx 102$ MeV/c^2 The rest energies for at least two of the three neutrinos would be $m(-4,0)c^2 \approx 0.03451$ eV. Regarding the heavy neutrinos, the results are essentially the same as the results that associate with equation (32).

6.3. Elementary bosons for which the spins exceed one

Regarding table 6, for each one of $\Phi = \mathrm{J}$, $\Phi = \mathrm{G}$, and $\Phi = \mathrm{L}$, equation (54) provides a symbol that denotes a possible maximum positive integer σ (as in $\sigma\Phi$) that a choice of modeling admits.

$$\sigma_{max,\Phi} \tag{54}$$

POST does not include elementary particles for which $\Phi = \mathrm{J}$.

For POST QF, $\sigma_{max,\mathrm{G}}$ is one and $\sigma_{max,\mathrm{L}}$ is one.

Per (for example) discussion related to equation (42), NEST posits that positive values of $\sigma_{max,\mathrm{J}}$ associate with possibilities for explaining some INFDA. NEST posits that positive values of $\sigma_{max,\mathrm{J}}$ associate with possibilities for developing useful NEST EQ that would not (regarding fermions) rely on antisymmetric wave functions. NEST posits that positive values of $\sigma_{max,\mathrm{J}}$ associate with possibilities for developing useful NEST CM (which would not rely on notions of wave functions). NEST posits that $\sigma_{max,\mathrm{J}}$ is at least one. NEST does not yet suggest an upper limit regarding $\sigma_{max,\mathrm{J}}$.

NEST posits that $\sigma_{max,\mathrm{G}} \geq 2$ might associate with possibilities for NEST ND, SR, and EQ to explain hadron physics without as much (as pertains regarding POST QF) reliance on conditionally convergent mathematics. This notion might parallel aspects regarding equation (48) and anomalous magnetic moments. (Regarding POST statistical mechanics, reference [118] and reference [119] provide an example - regarding calculating the free energy of metallic hydrogen - of working around otherwise seeming needs to deploy mathematics similar to conditionally convergent mathematics that POST QF uses and that POST QF associates with the two-word term Feynman diagrams.)

NEST posits that $\sigma_{max,\mathrm{L}} \geq 2$ might associate with possibilities for NEST ND, SR, and EQ to predict and explain aspects of gravity and other non-electromagnetic LRI. Based on conservation laws that associate with 3g' and 4g', NEST posits that NEST might associate with $\sigma_{max,\mathrm{L}} \geq 4$.

Table 6 does not necessarily couple sufficiently with table 2 and table 3 to suggest - based on the following two notions - that NEST might associate with $\sigma_{max,\mathrm{L}} \leq 4$. The first notion associates with the following sentence. Discussion related to equation (11) suggests the limit $n_0 \leq 3$ and hence a limit of $\Sigma \leq 4$ regarding the relevance of $\Sigma\mathrm{g}\Gamma$ solution-pairs for which Σ is the only element in the list Γ. The second notion associates with the following sentences. Equation (29) suggests that the solution-pair 1g1 associates with an interaction strength that includes a factor of four and that the solution-pair 2g2 associates with an interaction strength that includes a factor of three. Extrapolation suggests that the solution-pair 3g3 associates with an interaction strength that includes a factor of two, that the solution-pair 4g4 associates with an interaction strength that includes a factor of one, and that the solution-pair 5g5 would associate with an interaction strength that includes a factor of zero.

Reference [22] notes that POST QF suggests that zero-mass elementary particles do not have spins that exceed two.

SOMA is not necessarily incompatible with the notion that NEST EQ might not necessarily need to include zero-mass elementary bosons that have spins that exceed two. However, if NEST EQ includes 3L and 4L fields (and, in essence, 3L and 4L boson elementary particles), NEST EQ might provide useful alternatives to POST QF.

6.4. Angular momentum states and two-body modeling

Per equation (23) and equation (55), the solution-pair 0g1‘3‘5‘7 associates with $S = 0$.

$$0 = | + 1 - 3 - 5 + 7| \tag{55}$$

The following eight sets of solution-pairs associate with $\Sigma = 0$. The symbol 8ˆ associates with the series $+8 = +8$, $+8 = -8 + 16$, $+8 = -8 - 16 + 32$, and so forth. The symbol 16ˆ associates with the series $+16 = +16$, $+16 = -16 + 32$, $+16 = -16 - 32 + 64$, and so forth. Regarding the notions of 0g” and ΣgΓ, for each solution-pair, the integers shown below (or alluded to by the series just above) appear in Γ and no other integers appear in Γ. For each set, for other than the first solution-pair, each solution-pair cascades from the first solution-pair.

1. $|+1-2-4+5|$, $|-1+2-4-5+8\hat{}|$, $|+1+2-4-5+6|$, and $|+1-2+4-5-6+8\hat{}|$.
2. $|+2-3-4+5|$, $|-2+3-4-5+8\hat{}|$, $|-2-3+4-5+6|$, and $|+2-3+4-5-6+8\hat{}|$.
3. $|-1-2-5+8|$, $|-1-2-5-8+16\hat{}|$,$|+1+2-5-6+8|$, and $|+1+2-5-6-8+16\hat{}|$
4. $|-1-2-4+7|$, $|+1+2-4-7+8\hat{}|$, $|+1+2-4-6+7|$, and $|-1+2+4-6-7+8\hat{}|$.
5. $|+3-4-7+8|$, $|+3-4-7-8+16\hat{}|$, $|-3-4+6-7+8|$, and $|-3-4+6-7-8+16\hat{}|$.
6. $|+2-3-7+8|$, $|+2-3-7-8+16\hat{}|$, $|+2+3-6-7+8|$, and $|+2+3-6-7-8+16\hat{}|$.
7. $|+2-4-5+7|$, $|-2-4+5-7+8\hat{}|$, $|-2-4+5-6+7|$, and $|+2-4-5+6-7+8\hat{}|$.
8. $|+1-3-5+7|$, $|-1-3+5-7+8\hat{}|$, $|-1-3+5-6+7|$, and $|-1-3-5+6-7+8\hat{}|$.

For each set, SOMA posits that equation (23) pertains regarding the first two of the four expressions. Thus, each set includes exactly one expression for each nonnegative S for which $2S$ is an even integer. For each set, SOMA posits that equation (24) pertains regarding the second two of the four expressions. Thus, each set includes exactly one expression for each nonnegative S for which $2S$ is an odd integer.

SOMA posits that each set might have uses regarding modeling regarding the spins of objects. Regarding objects, SOMA posits that equation (20) pertains regarding spin and equation (21) pertains regarding charge.

SOMA posits that the following notions can pertain.

For the first three sets, $5 \in \Gamma$ and $7 \notin \Gamma$ and there are two sets that associate with $Q = 0$ and one set that associates with $Q \neq 0$. These sets can associate with one object.

For the next three sets, $5 \notin \Gamma$ and $7 \in \Gamma$ and there are two sets that associate with $Q = 0$ and one set that associates with $Q \neq 0$. These sets can associate with another object.

For the last two sets, $5 \in \Gamma$ and $7 \in \Gamma$ and there is one set that associates with $Q = 0$ and one set that associates with $Q \neq 0$. These sets can associate with a system that consists of the two objects.

SOMA posits that the above notions - which associate with 0g” - are compatible with notions - that associate with Sg” for which S is a positive integer - that associate with table 4. SOMA suggests that possibilities exist for modeling (based on the eight sets) that would associate with interactions between the first object and the second object and that would parallel modeling that table 4 suggests.

This paper does not explore the notion that adding (to the above collection of eight sets) sets - such as the set that would associate with $|-2-3-4+9|$, $|-2-3+4-8\hat{}+9|$, $|-2+3-4-6+9|$, and $|-2+3+4-6-8\hat{}+9|$ - might point toward useful three-body modeling.

6.5. Aspects that associate with general relativity

Discussion related to equation (13) points to situations for which POST GR would not apply adequately successfully. Regarding the stress energy tensor, (per table 3) the reach of a major contributor (2g2) to the energy component is six, the reach of an instance of a major contributor (2g2‘4) to the three pressure components is two, and the reach of an instance of a major contributor (2g1‘2‘3) to the twelve off-diagonal components (of which six components match the other six components) is one.

For situations that involve significant effects of stuff that associates with more than one isomer, problems can arise regarding POST GR. Adjustments regarding equations of state might adequately compensate for difficulties that would associate with the three pressure components of the stress-energy tensor. Yet, problems - related to the off-diagonal components of the stress-energy tensor - could remain.

SOMA suggests that phenomena that POST CC associates with the three-word term dark energy effects associate with SOMA-dipole components of 2L and SOMA-octupole components of 2L.

Such notions suggest the notion that some POST uses of the POST GR concept of a cosmological constant might not comport with INFDA.

POST has difficulties harmonizing POST QF and POST GR.

NEST posits the following notions. INFDA and SOMA are compatible regarding the extent to which POST GR comports with INFDA. NEST and SOMA are adequately compatible with POST ND and POST SR. NEST EQ, ND, and SR might be adequately compatible with INFDA. NEST does not necessarily (regarding explaining and predicting INFDA) need to be compatible with GR.

6.6. Information that gravitational waves convey

Reference [120] discusses opportunities for research regarding gravitational waves.

Table 4 suggests one-some uses of 2g" solution-pairs that might associate with the producing of gravitational waves. SOMA posits that one-some use of the 2g1'3'4 solution-pair is relevant. One-some use of the 2g1'3'4 solution-pair associates with a reach of six.

To the extent that one-some use of at least one of the other 2g" solution-pairs that table 4 suggests is relevant regarding the producing of gravitational waves, SOMA suggests that adequately detailed analyses of the gravitational signatures that associate with collisions of objects - such as black holes - might enable the development of INFDA that associate with the extents to which the colliding objects include stuff that associates with more than one isomer. For example, each one of 2g1'4'5 transitions and 2g3'4'5 transitions associates with a reach of two isomers.

6.7. Possibilities for explaining relationships among properties of boson elementary particles

Equation (56) shows the radial aspects of the Laplacian operator that associates with D dimensions.

$$\nabla_r{}^2 = r^{-(D-1)}(\partial/\partial r)(r^{D-1})(\partial/\partial r) - \Omega r^{-2} \tag{56}$$

Per reference [121], for partial differential equations that include the radial expression for the Laplacian operator that associates with D dimensions, solutions can feature the result $\Omega = P(P+D-2)$, in which $2P$ is an integer. (For $D=3$ and $P=S$ and $2S$ being a nonnegative integer, Ω can associate with the POST notion $S(S+1)\hbar^2$ that POST associates with angular momentum.)

For boson elementary particles, equation (27) posits links between mass, spin, and charge. The following notions might provide useful insight.

Each one of m' and S is always non-negative. Perhaps, regarding each one of m' and S, some modeling associates with two degrees of freedom - positive quantity and zero quantity. Regarding mathematics associated with Laplacian operators, for $D=2$ and for each of $P=m'$ and $P=S$, the factor $P^2 = P(P+D-2)$ pertains regarding aspects of the mathematics. Equation (27) includes a term $(m')^2$. Equation (27) includes a term S^2.

Charge - which is a basis for Q - can be positive, zero, or negative. Perhaps some modeling associates with three degrees of freedom - positive quantity, zero quantity, and negative quantity. Regarding mathematics associated with Laplacian operators, for $D=3$ and for $P=Q$, the factor $P(P+1) = P(P+D-2)$ pertains regarding aspects of the mathematics. Equation (27) includes a term $Q(Q+1)$.

Whether or not l_{ms} is nonzero associates with the POST notion of whether longitudinal polarization can pertain. Regarding mathematics associated with Laplacian operators, for $D=2$ and for $P=(l_{ms})^{1/2}$, the factor $P^2 = P(P+D-2)$ pertains regarding aspects of the mathematics. Equation (27) includes a term $l_{ms} = ((l_{ms})^{1/2})^2$,

This insight might provide a basis for modeling that would underlie SOMA or a basis for other results. This paper does not further discuss uses for the insight.

6.8. Possibilities regarding NEST, SOMA, and EQ

Discussion above associates with the notion that INFDA, NEST, SOMA, and various KM might co-evolve to explain and predict more (compared to INFDA, POST, and POST KM) INFDA. Relevant branches of physics include EP, CA, and branches (other than EP) that address tiny objects such as hadrons, atomic nuclei, and atoms.

Regarding KM for hadrons, modeling based on σG for which $\sigma \geq 2$ might reduce dependence on conditionally convergent mathematics that associates with QF.

Regarding KM for nuclear physics, modeling based on σG for which $\sigma \geq 1$ might pertain regarding attractive aspects of the residual strong force and modeling based on σJ for which $\sigma \geq 1$ might pertain regarding repulsive aspects of the residual strong force.

Opportunities to develop - based in part on INFDA or on outputs from QF - non-QF KM might lead to non-QF KM that expands the scope of successful KM and simplifies KM methods. Some such non-QF KM might, for example, provide alternatives to some POST GR or harmonize aspects of CM and QM.

7. Concluding remarks

Each of the following sentences describes a physics challenge that has persisted for the most recent eighty or more years. Interrelate physics models. Interrelate physics properties, properties of objects, and physics constants. Provide, for elementary particles, an analog to the periodic table for chemical elements. Describe bases for phenomena that POST (or, modeling that has bases in popular modeling notions of space-time coordinates) associates with the two-word term dark matter. Describe bases for phenomena that POST associates with the two-word term dark energy. Explain the overall evolution of the universe.

Physics amasses data that people can use as bases for developing and evaluating modeling aimed at addressing the challenges.

SOMA (or, modeling that - for a single object - points to multiple attributes) addresses those physics challenges and has bases in the following mathematics - integer arithmetic, multipole expansions, Diophantine equations, and multidimensional harmonic oscillators.

SOMA unites and decomposes aspects of electromagnetism and gravity. For each of those two long-range interactions, the decomposition associates with properties - of objects - that people can measure and that POST features. For electromagnetism, the properties include charge and magnetic moment. For gravity, the properties include mass and components of stress-energy.

SOMA points to all known elementary particles and to some would-be elementary particles. SOMA includes a notion of isomers of elementary particles that do not mediate long-range interactions. SOMA features a notion of instances of components of long-range interactions.

SOMA explains data regarding dark matter. SOMA points to possible resolutions for tensions - between data and POST - regarding effects of dark energy. SOMA provides insight regarding galaxy formation and evolution.

SOMA matches data that POST matches, suggests explanations for data that POST seems not to explain, suggests results regarding data that people have yet to gather, and points to possible opportunities to develop models that unite aspects of physics and physics modeling.

In summary, SOMA suggests augmentations - to POST - that might achieve the following results. Extend the list of elementary particles. Predict masses for at least two neutrinos. Predict masses - that would be more accurate than known masses - for some other elementary particles. Describe dark matter. Explain ratios of dark matter effects to ordinary matter effects. Provide insight regarding galaxy formation. Describe bases for phenomena that associate with the two-word term dark energy. Explain eras in the history of the universe. Link properties of objects. Suggest attributes for NEST (or, new modeling that has bases in space-time coordinates) and a NEST-related aspect that would be similar to POST quantum field theory. Interrelate physics models. Point to opportunities for developing modeling that would underlie NEST and SOMA. Provide bases for further integrating and extending physics modeling.

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