Elementary proof of Fermat-Wiles' Theorem

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Fermat-Wiles' Theorem :

(1) « the equality $x^n+y^n=z^n$, with $n, x, y, z \in N^*$, is impossible for $n>2$. »

Abstract of proof :

In the division of $x^n = z^n - y^n$ by $x^{n-1} = az^{n-1} - by^{n-1}$, $(a,b) \in Z^2$, remainder must be zero implying the equality $b^2 y^{n-2} = a^2 z^{n-2}$ which is impossible for $n>2$ since $x^{n-1} = az^{n-1} - by^{n-1}$ and x, y, z are coprim numbers.

The application of the procedure scheme of Euclidian division until remainder equal to $z^n- y^n$, and the evaluation of remainders and partial quotients allow to obtain the unique remainder which can and must be equal to zero.

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We suppose x, y and z are coprim numbers.

Given gcd(y,z)=1 and the corollary of the Bachet's theorem (1624), it exists two relative integers a and b such that :

(2) $x^{n-1} = az^{n-1} - by^{n-1}$

In the division $(z^n-y^n) : (az^{n-1}-by^{n-1})$ $(x= x^n/x^{n-1})$ remainder must be zero.

Let us put the division and carry out the operations until obtain the remainder equal to dividend $z^n - y^n$ and then obtain the candidate remainders to be zero.

$x^n = z^n - y^n$ | $x^{n-1} = az^{n-1} - by^{n-1}$

$- z^n + (b/a)zy^{n-1}$ $\quad$ $z/a + y/b - z/a - y/b$

--------------------- Evaluation of remainders and partial quotients :

$R_0 = - y^n + (b/a)zy^{n-1}$ $\quad$ $R_0 = 0 \Rightarrow (q)=x= z/a \Rightarrow$ **$ax=z$** $\Rightarrow R_0 \neq 0$

$+ y^n - (a/b)yz^{n-1}$

$R_1 = (b/a)zy^{n-1} - (a/b)yz^{n-1}$ $\quad$ $R_1 =0 \Rightarrow b^2y^{n-2}-a^2 z^{n-2}=0 \Rightarrow (q)= x = z/a + y/b$

$(b/a)zy^{n-1} + z^n$

$R_2 = z^n - (a/b)yz^{n-1}$ $\quad$ $R_2=0 \Rightarrow (q)= x = z/a + y/b - z/a \Rightarrow$ **$bx=y$** $\Rightarrow R_2\neq 0$

$+ (a/b)yz^{n-1} - y^n$

$R_3 = z^n - y^n \neq 0$, $\quad$ end of the operations cycle.

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Evaluation of remainders and partial quotients :

If the remainder R_0 is zero then the quotient is $x = z/a$, so $ax = z$, which is impossible since gcd(x,z)=1.

If the remainder R_2 is zero then the quotient is $x = z/a + y/b - z/a = y/b$, so $bx = y$, which is impossible since gcd(x,y)=1.

$R_3 = z^n - y^n \neq 0$, $x, y, z \in N^*$ and gcd(y,z)=1.

The application of the procedure scheme of the Euclidean division allowed to obtain the remainders and the remainder which can and must be zero is unique and obtained by deduction : three remainders out of the four obtained cannot be equal to zero.

So the problem of the existence of unique remainder zero does not arise.

Therefore only the remainder R_1 can and must be equal to zero :

(3) $R_1 = (b/a)zy^{n-1} - (a/b)yz^{n-1} = ((b/a)y^{n-2} - (a/b)z^{n-2})yz = 0$

So $(b/a)y^{n-2} - (a/b)z^{n-2} = 0$ which implies the equality :

(4) $b^2 y^{n-2} = a^2 z^{n-2}$

where, for n>2, as gcd(y,z)=1, y divides a^2 and z divides b^2, so gcd(a,y) >1 and gcd(b,z) >1.

Then, according to the equality $x^{n-1} = az^{n-1} - by^{n-1}$ (2), gcd(a,y) >1 => gcd(x,y) >1 and gcd(b,z) >1 => gcd(x,z) >1, but gcd(x,y)= gcd(x,z)=1 (hypothesis).

Therefore, the equalities $b^2y^{n-2} - a^2z^{n-2} = 0$ (R), $x^{n-1} = az^{n-1} - by^{n-1}$ (d), $x^n = z^n - y^n$ (D) are impossible for n>2.

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Division with integer numbers :

$a \ * \ z^n - y^n \ (D_0) \qquad | \ az^{n-1} - by^{n-1} \quad (d)$

$=> az^n - ay^n \qquad z + ay - bz + bz$

$-az^n + bzy^{n-1} \qquad$ as we have multiplied D_0 by a, then D_1 by b,

we have z/a + ay/ab –bz/ab + bz/ab

------------------- Evaluation of remainders and partial quotients :

$b \ * \ bzy^{n-1} - ay^n \ (D_1) \qquad R_0 = 0 => (q)= x = z/a => \mathbf{ax = z} => R_0 \neq 0$

$D_1=R_0= \ => b^2zy^{n-1} - aby^n$

$-a^2yz^{n-1} + aby^n$

$>>>>R_1= \ b^2zy^{n-1} - a^2yz^{n-1} \ (D_2) \ R_1 =0 => b^2y^{n-2}-a^2 z^{n-2}=0 => (q)= x = z/a + y/b$

$-b^2zy^{n-1} + abz^n$

$D_3=R_2= \ abz^n - a^2yz^{n-1} \ (D_3) \qquad R_2=0 =>(q)= x= z/a+y/b - z/a => \mathbf{bx = y} => R_2\neq 0$

$-abz^n + b^2zy^{n-1}$

$R_1 <<<< \ b^2zy^{n-1} - a^2yz^{n-1}$ end of the operations cycle.

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Remark :

Let the system :

(5) $a^x + b^y= c^z$, $(a, b, c, x, y, z) \in N^{*6}$ et a, b, c coprim integers.

(6) $a^x = c^z - b^y$

(7) $a^{x-1} = uc^{z-1} - vb^{y-1}$, $(u, v) \in Z^2$

In application of the algorithm described above to the division c^z-b^y: $uc^{z-1}- vb^{y-1}$, the remainder which can and must be zero implies the equality :

(8) $v^2b^{y-2} = u^2c^{z-2}$,

which is impossible for y>2 or z>2 and, by symmetry, for x>2 and z>2.

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