The setbacks of theoretical physics

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Abstract

The search for a reliable foundation of physical reality has had many setbacks and is slow. Side roads were taken that did not lead to the desired goal. This document shows that there is an alternative path that leads to a better result. This result can be reproduced in a single sentence. This very short summary does need the necessary explanation. The paper provides this explanation. The paper also shows the relation between the foundation and several aspects of physics, such as quantum physics, classical physics, optics, and cosmology.

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1 Introduction

With some arrogance, I dare to say that the most important part of the foundations of physical reality is now exposed. Some mysteries remain, but these can be clearly described. For me, these mysteries exist because my knowledge of mathematics does not allow me to explain the origin of these mysteries. It is also possible that this mathematics does not yet exist. The foundation of physics can be represented in a single sentence that reflects the structure and behavior of the observable universe. "The universe that manifests itself to researchers is one continuous film of the possible coverages of space with versions of number systems belonging to the associative division rings."

2 Explanation

This short description can be explained with the observation that humans cannot think and communicate about things without providing these things with identification in the form of a name or pointer and a short compact description. The curious thing is that physical reality can function without these limitations. Yet physical reality also appears to have to adhere to strict rules and existing structures. The researchers have come to know these rules and structures to a large extent, and they formulate them in what they call mathematics and physics. Several researchers doubt whether people can discover the calculation rules that physical reality uses. Your writer does not belong to this group.

My arrogance is based on my conviction that those with education at the level of a bachelor in exact sciences mathematics or physics should easily be able to follow the argument given here and check it as desired. With considerably less prior knowledge, a large part of the argument is easy to follow. I have done my best to make as many details as possible freely accessible. Many treated subjects that are accessible on the internet are pointed to by in brackets enumerated URLs. Because formulas scare off many readers, they are housed in separate places. This applies to the calculation rules, the bra-ket procedure of Paul Dirac, and important equations. The formulas are placed in a separate chapter. The formulas have already been published elsewhere. [1]

3 Clarification

When people focus their research on space, they quickly realize that an empty space represents the ultimate nothingness. There is nothing in this space to which one could orient oneself. There is no center and there are no boundaries. It's not hard to imagine that the space could contain many anonymous locations. However, for humans, it is not possible to track the behavior of these locations without giving them identification and a precise description. Locations are point-shaped objects that can occupy a position in space. That position differs per location. Identification can be achieved by using number systems. The values of the elements of the number systems can indicate the position of the locations.

This paper introduces a structure that harbors a system of Hilbert spaces that all share the same underlying vector space. That system puts number systems in a well-defined interrelationship.

4 Vector space

There is still no possibility to point to the position. The pointer can consist of a base location and a pointing location connected by a direction-line. The vector has a length that can be characterized by a simple scalar number. Scientists call this pointer a vector and a space in which vectors occur a vector space. The vector is fully characterized by its direction-line and its length. The integrity of the vector does not change when it is shifted parallel. The parallel shift can take place on the direction-line but may also take place in another direction. Direction-lines can therefore be shifted parallel in the vector space. They have no beginning and no end. This immediately provides the operation with which two vectors can be added. If the base point is shifted from one vector to the pointer of the other vector, then the non-overlapping points form a new vector called the sum vector. If the direction-lines are different, the sum vector can use a new direction-line.

The two possibilities form a parallelogram in which the sum vectors are parallel and have an equal length.

By multiplying the vector by a scalar, the length is multiplied by that scalar. This creates a new vector. When the scalar is negative, the base point and the pointer point change function, and the vector gets the opposite direction. At the same time, its length may change. These simple calculation rules allow vectors to pinpoint all locations in the vector space. The section Vector arithmetic in the chapter Formulas contains the formulas.

4.1 Independent directions

Vector arithmetic enables a scalar product of two vectors. The scalar product can demonstrate the independence of the direction-lines of vectors. The scalar product of independent vectors is equal to zero. In this way, in the vector space, several mutually independent basic direction-lines can be detected. Since direction-lines can be shifted in parallel, the vector space can be covered by a raster of direction lines. The raster can form a primitive coordinate system.

5 Number systems

5.1 Real numbers

With their calculation rules, vectors can help to construct number systems. As an example, an ongoing addition of a starting vector and vectors equal to the starting vector and located on the same direction-line yields an ordered series of designated locations that collectively represent the natural numbers. By using the natural numbers as a label, we can count collections of locations. By removing locations from the collection, the subtraction procedure is created, and the countdown procedure is shown. We meet the number zero on the basis point of the original starting vector and then follow the negative integers. By adding groups of vectors several times, the procedure for multiplying numbers is created. That doesn't provide new integers. The reversal of multiplication is called division and delivers fractions. These can be new numbers. The integer numbers together with the fractions form the rational numbers.

5.2 Phase transitions

Scholars have shown that there are as many rational numbers as there are natural numbers. This means that all rational numbers can be labeled with a natural number. This only works if both number sets contain an infinite number of elements. The transition from finitely many elements to infinitely many elements implies a change in state for the set. In the new phase, the collection exhibits different behavior. It is not possible to achieve this phase transition step-by-step. Also, the way back does not go in a step-by-step way. The terms phase and phase transition are not often used in relation to number systems. This paper uses these terms to indicate the change of the status of the number system.

Adding or removing elements does not change the state of the infinite set. The infinite set of well-ordered rational numbers fills a large part of the same direction-line. Any location on this line can be approached arbitrarily close by a rational number. Nevertheless, there are still many locations on this line that cannot be designated by rational numbers. We call the numbers that these places indicate irrational numbers. Together, the set of rational numbers and the set of irrational numbers again form a set that can be seen as another phase. The phase transition happens again in one go and cannot be achieved step-by-step. The new phase of the collection can no longer be counted. In this set, all series of converging members end in a limit that is a member of the set. The phase transition adds several new calculation rules that manage the change of cohesive parts of the collection. Mathematicians call these extra calculation rules differential calculus. The differential calculus is closely related to the calculation rules of the rational numbers. The calculation rules can even mix. Without disturbing actuators, nothing will change in the new phase. If something is disturbed, then this phase of the collection tends to remove the disturbance as quickly as possible by sending away the consequences of the disturbance in all possible directions until the consequences eventually disappear into infinity. As mentioned, that vanishing area is never reached step by step. The result is that the number-covered area expands. The differential calculus tells exactly how that happens. On the so far considered direction-line the response acts in a single dimension.

The rational numbers treated so far, when multiplied by themselves, yield a positive number that is on the direction-line of the natural numbers. We call the numbers that behave in this way real numbers. We use this name for all numbers on this direction-line and therefore for all phases of the numbers that are on this direction-line. Multiplying by oneself is called squaring. The section Arithmetic of the real numbers contains the formulas.

5.3 Spatial numbers

There also appear to be systems of numbers that, when multiplied by themselves, yield a negative number which is located on the direction-line of the real numbers. We call these numbers spatial numbers. Often these numbers are called imaginary numbers. This name is not used here because the qualification imaginary also has completely different meanings. The spatial numbers no longer fit on the direction-line of the real numbers. They occupy one or three dimensions. This is because if spatial numbers fall outside the first spatial dimension, the calculation rules of the spatial numbers ensure that in addition to the second spatial dimension, a third spatial dimension is also filled with spatial numbers. The result of the product of two spatial numbers consists of an internal product that provides a real number and an external product that is zero or produces a result in a direction that is independent of the direction-lines of both factors. The internal product is the reason for the negative square. The calculation rules of the spatial numbers, therefore, differ from the calculation rules of the real numbers. The reaction to a disturbance of the third phase of spatial numbers is more spectacular in the three-dimensional spatial number system than in the one-dimensional spatial number system. The section Arithmetic of spatial numbers contains the formulas.

5.4 Division rings

Nevertheless, real numbers can be added to spatial numbers, and spatial numbers can be multiplied by real numbers. This creates new number systems. The real numbers, together with the one-dimensional spatial numbers, form the two-dimensional set of what are called complex numbers. The real numbers, together with the three-dimensional spatial numbers, form the four-dimensional set of what are called quaternions. The Mixed Arithmetic section of the chapter Formulas contains the corresponding formulas.

5.5 Confusing calculation rules

Two vectors can together deliver a scalar product. That scalar product is zero or positive and provides for two equal vectors the square of the length of the vector. This length is the norm of the vector. The almost identical effect of the inner product of spatial numbers has led to confusion among many mathematicians and physicists so that spatial numbers were sometimes mistaken for vectors. This happened, among other things, with the discoverer of the quaternions. This confusion led to a public scandal that caused the quaternions to fall into oblivion after the sixties of the last century. As we will see, this had major consequences for both mathematics and physics. [2]

6 History

Simple fractions were already discovered by the Egyptians before Christ. Cantor discovered the second and third phases of real numbers around 1870. Cantor did not use the designations phase and phase transition. He and others then turn their attention to various kinds of infinities of sets. This document deals with only two forms of infinity. These are the countable infinity of the second phase of numbers and the uncountable infinity of the phase of numbers.

The complex numbers were discovered as early as 1545 by Gerolamo Cardano. The quaternions were discovered in 1854 by Sir William Rowan Hamilton. He formulated his discovery using the four basic numbers. One real base number and three spatial base numbers. The external product appears in the outcome of the product of the first two spatial base numbers. Hamilton discovered this formula during a walk with his wife over a sandstone bridge in Dublin and out of joy he scratched the formula into the wall of the bridge. The rain quickly erased the inscription. Hamilton's students immortalized the formula on the bridge through a bronze commemorative plaque. [3]

7 Set theory

7.1 Collections in space

Around the turn of the nineteenth to the twentieth century, a group of mathematicians and mathematical physicists led by David Hilbert had an intense discussion about set theory. [4] [5]

This discussion focused mainly on the various forms of infinity and countability. A lot of attention was also paid to the phases and phase transitions of the collection. For example, there was a lot of attention to the continuum hypothesis. [6]

The discussion ignored the container of the set and also paid no attention to the type of objects that formed the set. For physical reality, these choices play a major role. By choosing space as a container and locations as elements of the set, the number systems used to identify the locations acquire additional properties that both human researchers and physical reality must consider. These additional properties are the symmetries that represent the freedom of choice that is not defined by the calculation rules. As a result, the number systems exist in many versions that are distinguished by their symmetry. For example, the location of the geometric center of the number system can in principle be anywhere in the vector space. Also, the arrangement of the numbers along the direction-lines in one direction or the opposite direction can proceed. Physical reality must adhere to the rules of calculation and will use as many symmetry choices as possible. A different choice of symmetry yields a different version of the number system. The word symmetry has several different meanings. This also occurs in this paper.

8 Coordinates

There are three associative division rings. [7]

These are the real numbers, the complex numbers, and the quaternions. Each of these number systems exists in many versions that differ in their symmetry. Recording the symmetry is possible with coordinate markers. These markers use the location that indicates the value of the number. A Cartesian coordinate system records all the selection freedoms of a version of a number system. The record removes the selection freedom and helps establish the version of the number system.[8]

The geometric symmetry is created by the limitations imposed by the vector space. If number systems are designed without these limitations, then the geometric symmetries are not encountered.

9 Hilbert spaces

David Hilbert discovered an extension of the concept of vector space called Hilbert space by his assistant John von Neumann. The Hilbert spaces have the surprising property that they can archive elements of the version of the number system used by the Hilbert space and then retrieve them in an orderly manner. The Hilbert space is often described as a vector space that is provided with an internal product. As previously argued, each vector space has a scalar product and not an internal product. Moreover, it is difficult to imagine that a vector that depicts itself via the scalar product yields a complex number or quaternion as an eigenvalue. Instead, Paul Dirac has found a significantly better procedure for converting a vector space into a Hilbert space. This procedure combines covariant ket vectors and contravariant bra vectors. These are not real vectors but are closely related to them. One problem is that Dirac only demonstrated this for the real numbers and the complex numbers. There was too little interest in quaternionic Hilbert spaces in that period. With a small effort, the procedure can be adapted so that it can also be used for quaternions. Hilbert spaces can thus work with any of the associative division rings. Each Hilbert space chooses a private version of one of these number systems. As mentioned, the Hilbert space can archive collections of elements of this version and retrieve them in an orderly manner. This also applies to the entire chosen version of this number system. There is a dedicated operator who manages this collection. I call this operator the ***reference***

operator. This means that each Hilbert space has a private parameter space. This is a countable parameter space. It also means that Hilbert space is characterized by the symmetry of the version of the number system. The first version of the bra-ket process works with countable number systems and yields Hilbert spaces that use a countable number of independent base vectors and are therefore called separable. In section Dirac's bra-ket procedure the formulas are treated.

9.1 Function space

The private parameter space turns every Hilbert space into a function space. Through the functions, Dirac's bra-ket procedure defines new operators who manage the target space of the sampled function as eigenspace.

9.2 Quantum logic

To the surprise of many mathematicians, the set of the closed subspaces of Hilbert space appears to have a lattice structure that is slightly different from the lattice structure of classical logic. Some suggested that this deviation could be the cause of the quantum structure of the energy exchange observed in small particles and atoms. Therefore, the name quantum logic has been assigned to this new lattice. [9] A more obvious explanation is given by differential calculus. Differential calculus only comes into effect in the third phase of number systems. Function theory and differential calculus describe the third phase of number systems. The Arithmetic of changes section describes the formulas that govern the third phase of number systems.

The countable parameter space of the separable Hilbert space concerns the first two phases of the number systems, or it is uncountable and concerns the untouched third phase. In that case, the Hilbert space is no longer separable. The non-separable Hilbert space provides operators with eigenspaces that are uncountable or can manage multiple phases of the chosen number system. The non-separable Hilbert space uses a modified version of Paul Dirac's bra-ket procedure in which integrals of functions are used instead of sums of series. This modified version provides insight into the workings of uncertainties and the expectation value of a stochastically spread series of numbers.

9.3 Other features of Hilbert spaces

By playing with subspaces of the Hilbert space several special features will be revealed. Subdividing into subspaces does not prohibit the content of the subspace from functionally relating to the content of other subspaces.

9.3.1 Subdividing into Hilbert spaces

The version of the number system that defines the private parameter space can be subdivided into other number systems with a lower number of dimensions. For example, for every direction-line in the spatial part of a quaternionic number system that crosses the number 0, the quaternionic number system contains a complex number system. The complex number system contains a real number system. Thus, the quaternionic Hilbert space contains complex-number-based Hilbert spaces as subspaces. These complex-number-based Hilbert spaces contain real-number-based Hilbert spaces as a subspace. These Hilbert spaces support their own function space.

9.3.2 Subdividing into parameter space and targe space

When visualizing functions, humans intuitively put the parameter space and the target space into separated independent space parts. We will share that habit.

The parameters relate to the target values. In non-separable Hilbert spaces, functions usually act in the third phase of the number system. However, sampled functions can be represented both in separable and non-separable Hilbert spaces.

The subdivisions require extra dimensions. The vector space possesses ample space to harbor these extra dimensions. We call the subspace space that contains the target spaces of all functions the common target space. In a separable Hilbert space, the common target space can be spanned by an orthonormal set of base vectors that each represent a target value of one or more functions.

9.3.2.1 Keeping the relation between parameter value and target value

In the target space, the original arrangement of locations in parameter space can be destroyed. This would occur when oscillations or rotations are involved. This endangers the relation between parameter value and target value. In the model, this is resolved by embedding other Hilbert spaces or clusters of Hilbert spaces into the target space. The embedding plots the image of the Hilbert space or the cluster of Hilbert spaces into the target space. The embedded Hilbert spaces or Hilbert space clusters will implement the oscillations and rotations. The system of interacting Hilbert spaces is treated in A system of Hilbert spaces.

9.3.2.2 The Hilbert Book model

The Hilbert Book Model applies the real part of the parameter space to implement the indicator for the progression of change. It applies the common target space to harbor a collection of target spaces of static functions that each belong to the values of the corresponding progression indicator. We will call the value of the progression indicator a timestamp. This introduces the notion of time into the model. This subdivision acts as the functionality of a book in which each page represents an instant of the history of the common target space.

9.3.2.3 Separating the target space into a mirror-symmetric and an anti-mirror-symmetric part

Along direction-lines on each page of the common target space, the mirror-symmetric functions can be represented by superpositions of cosine functions. The anti-mirror-symmetric functions can be represented by superpositions of sine functions.

At the geometrical center of the parameter space, the cosine functions have a maximum. At the geometrical center of the parameter space, the sine functions switch from negative to positive. The anti-mirror-symmetric target spaces will be placed in a separate subspace. In the formulas, this will be indicated by the imaginary factor i. In the Hilbert space, this imaginary factor represents a split into another subspace.

A cosine function can be combined with a sine function that owns the same frequency into a complex-number valued exponential function. The imaginary factor i belongs to the direction of that same direction-line. The resulting complex exponential function has the remarkable property that it relates to the partial differential change operator that belongs to the selected direction-line. The details are presented in section Fourier transform in the formula chapter.

The sine and cosine functions use spatial frequencies as their parameters. This introduces a frequency parameter space parallel to the spatial position parameter space. In the realm of the quaternionic Hilbert space, the frequency parameter space covers three dimensions. The frequency parameter space serves spectral functions that populate the common target space. We also call this representation the ***change space***.

9.3.2.4 Separating the target space into scalar function targets and spatial function targets

The split into mirror symmetric target space and anti-mirror symmetric target space can be done separately for the scalar function targets and the spatial function targets.

9.3.3 Adding change with time

If also change with time is included in the split into mirror-symmetric and anti-mirror-symmetric dependency, then the frequency parameter space will cover four dimensions. Fourier series show that the base vectors that span the location parameter space all are superpositions of the base vectors of the frequency parameter space with all Hansvl00##123456n coefficients having the same amplitude. This also holds vice-versa.

10 Potentials and forces

In physics, potential energy is the energy held by an object because of its position relative to other objects.

The potential at a location is equal to the work (energy transferred) per unit of actuator influence that would be needed to move an object to that location from a reference location where the value of the potential equals zero.

We consider the potential to be zero at infinity. Thus, if infinity is selected as the reference location, then the potential at a considered location is equal to the work (energy transferred) per unit of actuator influence that would be needed to move an object from infinity to that location. The potential at a location represents the reverse action of the combined actuator influences that act at that location.

10.1 Center of influence of actuators

The influence of similar actuators can superpose. Thus. a geometrical center of these influences defines the location of the virtual location of a representant of the considered group of actuators.

This virtual representant has a potential that has the potential of a point-like actuator of the same influence type. In the Hilbert Book Model static actual point-like actuators other than charges do not exist because the embedding field tends to remove them as quickly as possible. However static virtual point-like actuators can be defined.

10.2 Forces

The first-order change contains five terms, two scalar terms, and three spatial terms. In each of these subgroups, the terms can compensate each other. In the group of spatial terms, the gradient of the scalar part of the quaternionic field can compensate for the time variation of the spatial part of the quaternionic field. If the curl of the part of the quaternionic field can be neglected, then the gradient of a local potential can cause a time variation of a spatial field that describes the movement of influenced objects. If these are massive objects, then these objects will be accelerated. So, the spatial field will represent a force field.

10.3 Actuators

The actuators of spherical responses discussed in this paper are listed in the table.

Actuator	Description	Influenced objects	Symbol Θ	Symbol θ
Actual electric charge	Electric charges that are the sources or sinks of electrical fields and cause potentials in both the electrical field and the dynamic universe field. The influenced objects are other electric charges. These charges locate at the geometrical centers of floating Hilbert spaces.	Other electric charges	Q	q
Virtual electric Charge	Virtual charges that represent a collection of electric charges	Other electric charges	Q	q
Isotropic pulse	Isotropic pulses that are embeddings of hop landings of the state vector of floating Hilbert spaces into the dynamic universe field. These pulse responses cause spherical pulse responses in the form of spherical shock fronts.	Other massive objects	M	m

Floating Hilbert space	Virtual mass that represents a collection of isotropic pulses that are generated by a floating Hilbert space.	Other massive objects	M	m
Virtual mass	Virtual masses that represent a collection of masses of floating Hilbert spaces.	Other massive objects	M	m

Electric fields and gravitational fields differ fundamentally in their start and boundary conditions.

Electric charges can attract or repel each other.

Masses will always attract each other.

Spherical pulse responses in the form of spherical shock fronts are dark matter objects.

11 Stochastic processes

The characteristic function of a stochastic process that resides in the change space can control the spread of the location density distribution of the produced location swarm that resides in position space.

The stochastic process consists of a Poisson process that regulates the distribution in the real-number-based progression space which is a subspace of the quaternionic Hilbert space and a binomial process that regulates the distribution in position space. This distribution is described by a location density distribution.

The production of the stochastic process is archived in the eigenspace of a dedicated footprint operator that after reordering the timestamps stores its eigenvalues in quaternionic storage bins that consists of a real number valued timestamp and a three-dimensional spatial number value that represents a hop landing location. After sequencing the timestamps in equidistant steps, the hop landing locations represent a hopping path of a point-like object. The hopping path regularly regenerates a coherent hop landing location swarm. The location density distribution describes this swarm.

If this location density distribution is a Gaussian distribution, then its Fourier transform determines exactly the location density distribution of the swarm. The Fourier transform is again a Gaussian distribution, but it has different characteristics. The Fourier transform of the convolution of two functions equals the product of the Fourier transforms of the functions.

The described stochastic process can deliver the actuators that generate the pulse responses that may deform the dynamic universe field. In some way, an ongoing embedding process must map the eigenspace of the footprint operator onto the embedding field. As previously argued, the footprint operator's eigenspace corresponds to a dynamic footprint vector that defines a location density function and a probability amplitude. The footprint vector resides in the underlying vector space and has a representation in Hilbert space via the footprint operator. The footprint vector acts as the state vector of the separable Hilbert space and the probability amplitude corresponds to what physicists call the wavefunction of the represented moving particle.

11.1 Optical Transfer Function and Modulation Transfer function

The stochastic processes that own a characteristic function which is described here, are in common use in the qualification of imaging quality via the Optical Transfer Function of an imaging process or imaging equipment. The Optical Transfer Function is the Fourier transform of the Point Spread Function. For spatial locations, the PSF acts as a location density distribution. The modulus of the Optical

Transfer Function is a symmetric function and is called the Modulation Transfer Function. The vertical axis of the MTF shows the energy distribution of the spatial spectrum. In the case of light, it is the chromatic distribution of the PSF. A central peak in the form of a quick decrease of the MTF at low spatial frequencies indicates the existence of a veiling glare or halo. It is energy that is less correlated to location.

The Line Spread Function equals the integral over the Point Spread Function in the direction of the line. The Fourier transform of the Line Spread Function equals the cut through the center of the Optical Transfer Function. The cut is taken perpendicular to the direction of the line. The LSF can be a function of the direction of the line. In that case, the PSF has a non-isotropic angular distribution. The result of the Fourier transform conforms to the convolution of the OTF with the Fourier transform of the blade sharp pulse that corresponds to the Fourier transform of the integral along the line.

If the PSF is generated in a dynamic ongoing process, then also a phase distribution will occur. The Optical Transfer Function combines the Modulation Transfer Function and the Phase Transfer Function. The Phase Transfer Function is the argument of the Optical Transfer Function.

A system of Hilbert spaces that share the same underlying vector space can perform the job of the imaging platform. In this system, the imaging process will be called the embedding process. This explanation still says nothing about the essence of the necessary underlying stochastic selection process. That remains a mystery.

The concept of the Optical Transfer Function also makes sense for dependence on time. For time dependence the tool is called Fourier analysis. Together the two tools perform a four-dimensional spectral analysis.

11.2 Photons

Photons are not electromagnetic waves. Instead, photons consists of chains of equidistant one-dimensional shock fronts that travel along a geodesic. The one-dimensional shock fronts are ***dark energy objects***. See the section on differentiation.

11.3 Light

Light is a distribution of photons. A beam of light can have an angular distribution, a chromatic distribution, and a phase distribution. A homogeneous beam of light contains a single frequency and usually a narrow angular distribution.

11.4 Refraction

Refraction occurs at the borders of transparent media in which information transfer occurs with constant speed. The information transfer can take place by chains of absorption and reemission cycles. In free space nothing exist that absorbs or emits photons, but photons can travel through free space [10].

Refraction enables the construction of lenses, fiber plates, optical fibers, prisms, and mirrors.

Refraction is covered by a separate part of optics. [11]

11.5 Holographic imaging

Transparent optical lenses and tiny apertures can act as Fourier transformers. They map distributions of photons in position space into distributions in frequency space. These distributions are called holograms. [12] Holograms can be captured in photographs. Also, metal mirrors imprinted with phase patterns can generate holograms when a coherent beam of light is reflected by the imprinted mirror.

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11.6 Electron optics

Electron optics concerns the imaging of charged particles by artificially constructed electric or magnetic fields, or by electromagnetic fields [13][14]. Construction elements are metallic electrodes that are put at a given voltage or coils that carry electric currents.

Radio transmission is a special kind of electron optics.

12 Social influences

The promising discussion about set theory and number systems was disrupted by the rise of National Socialism in Hitler's Nazi Germany. Key participants in the discussion were threatened or had to flee to safer places. Many of them fled to the United States of America and were morally obliged to cooperate in the fight against Nazism by participating in the development of new weapon systems, such as the atomic bomb. Their attention was no longer focused on sets and number systems. This effect was exacerbated by the success of the complex functional analysis with which singularities can be treated. [15]

Joshua Willard Gibbs and Oliver Heaviside led the physicists in the direction of vector analysis. [16]

As a result, attention to geometric differential theory grew. [17]

In this way, it was thought that the spatial functions would be sufficient to explain physical phenomena. However, this choice is at the expense of the relationship with the real functions, which is more clearly regulated in quaternionic function theory. Many physicists no longer understood the reason why Hilbert spaces were brought to their attention. The complex Hilbert spaces became a toy of the mathematicians who developed all kinds of fancy complex Hilbert spaces.

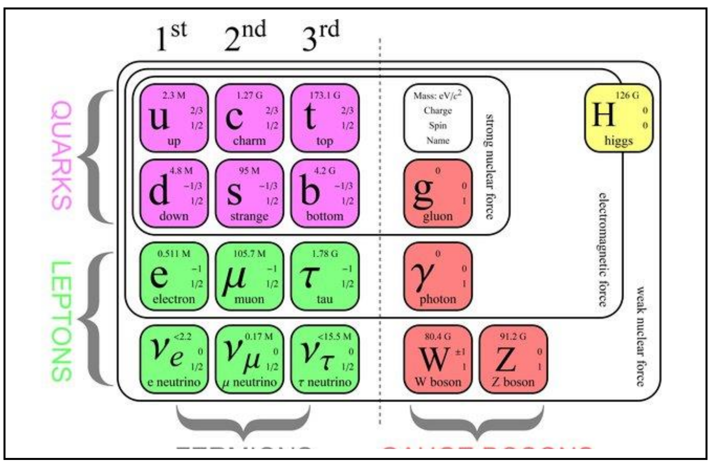
13 Ongoing investigation

At CERN in Geneva, sufficiently far from the Nazi sphere of influence, a small group continued with quantum logic and Hilbert spaces. My attention to this group was guided to quaternionic Hilbert spaces by reading the book "Foundations of quantum mechanics", written by Josef M. Yauch. [18]

Due to too few results, this research languished and died out in the sixties.

14 New insight

Now we are taking a giant step. This step concerns a significant difference in understanding between me and mainstream theoretical physics. This difference is prompted by the curious short list of properties of the electric charges of the first generation of elementary fermions. This list covers charges with values -1, -2/3, -1/3, 0, +1/3, +2/3, and +1. This list is included as part of the Standard Model of the experimental particle physicists who have summarized their main observations in that Standard Model. [19]



Multiplying with 3 turns the list into a list of integers -3, -2, -1, 0, +1, +2, and +3. This is the list of differences between a reference symmetry and other symmetries of versions of quaternionic number systems, when the coordinate axes are confined to be parallel.

We limit our use of the Standard Model to a subset and exclude the bosons and the gluons. We also exclude theoretical theories like Quantum Field Theory, Quantum Electro Dynamics, and Quantum Chromo Dynamics that were inserted into the

Standard Model by opportunistic theoretical physicists that spoiled the experimental results with their not-so-well-founded theoretical ideas. These theories base om the minimal action principle from which a Lagrangian is derived. These concepts play in the third phase of number systems. The calculation rules and restrictions of the third phase are set in the first and second phases. Therefore, these theories cannot explain the existence of electric charges and the existence of different types of fermions. These theories have no good explanation for the existence of the wavefunction and their explanation for the existence of conglomerates is questionable.

The similarity with the symmetries of versions of number systems stimulated me. It is not the similarity with the symmetries themselves that provides the reason, but instead, the differences between the symmetries of the versions of the number systems that float with their separable Hilbert space and the symmetry of a version of the number system that acts as a background platform control the situation. This is what happens in a system of separable Hilbert spaces that all apply the same underlying vector space.

15 A System of Hilbert spaces

The author calls the system of Hilbert spaces the Hilbert repository because it stores all data of a multiverse. Two types of systems of Hilbert spaces exist. Both systems contain a member that acts as a background platform.

The first type is a system of separable Hilbert spaces. The background platform owns a companion non-separable Hilbert space that embeds its separable companion. This companion archives a dynamic universe field. The floating separable members can harbor an electric charge that locate at their geometric center. A dark hole harbors the countable parameter space of the separable Hilbert space that acts as the background platform.

The second type is a system of non-separable Hilbert spaces. The background platform is a non-separable Hilbert space, that archives a dynamic multiverse field. The parameter space of the background platform is a continuum and therefore it is not contained in a black hole. The floating members of the system are the background platforms of systems of separable Hilbert spaces that own a companion non-separable Hilbert space that embeds its separable partner. The parameter space of the separable part of the background platform is contained in a dark hole.

This configuration represents a dynamic multiverse that divides part of the underlying vector space into a set of compartments, that each supports a dynamic universe.

15.1 A System of separable Hilbert spaces

We limit ourselves to Hilbert spaces that are all derived from the same vector space. We choose four mutually independent directions in the underlying vector space. The axes of the Cartesian coordinate system of the number system shall be parallel to one of the chosen direction-lines. This choice, therefore, leaves only a small number of different symmetry types. The exact reason why this restriction is enforced is not clear. However, it is obvious that the limitation makes it easier to compare symmetries and compute symmetry differences.

To understand the consequences of the limitation, we put the symmetries of the remaining versions of the quaternionic number system in a table whose lines we arrange with binary written hexadecimal rank numbers. We choose one of the

No	R	G	B	real	Difference	charge	type	Rgb
0	0	0	0	0	0	0	background	
1	1	0	0	0	1	-1/3	down	R
2	0	1	0	0	1	-1/3	down	G
3	1	1	0	0	2	-2/3	anti-up	B
4	0	0	1	0	1	-1/3	down	B
5	1	0	1	0	2	-2/3	anti-up	G
6	0	1	1	0	2	-2/3	anti-up	R
7	1	1	1	0	3	-3/3	electron	
8	0	0	0	1	0	0	neutrino	
9	1	0	0	1	-1	1/3	anti-down	B
A	0	1	0	1	-1	1/3	anti-down	G
B	1	1	0	1	-2	2/3	up	R
C	0	0	1	1	-1	1/3	anti-down	R
D	1	0	1	1	-2	2/3	up	G
E	0	1	1	1	-2	2/3	up	B
F	1	1	1	1	-3	3/3	positron	
	B	G	R					

sixteen remaining versions as a frame of reference platform and place this version at the front of the queue. In the table, the fitting fermions are mentioned by name.

You will notice that the anti-attribute raises a conflict between symmetries and the electric charges of the Standard Model. The reason might be that the anti-attribute is not measurable.

All these Hilbert spaces are separable and use number systems that belong to the first or second phase.

The remaining system of Hilbert spaces contains a Hilbert space that can serve as a background platform. We assume that the reference version acts as background platform.

The background platform must have an infinite number of subspaces. This means that the version of the number system chosen by this Hilbert space has an infinite number of elements.

15.2 A modelling platform

A system of Hilbert spaces that all share the same underlying vector space can act as a modeling platform that not only supports dynamic fields that obey quaternionic differential equations. The model can in principle capture all phenomena that exists in a dynamic universe.

The system of separable Hilbert spaces applies the structured storage capacity of the Hilbert spaces that are members of the system. The requirement that all member Hilbert spaces must share the same underlying vector space restricts the types of Hilbert spaces that can be a member of the system of separable

Hilbert spaces. In the chapter about change, we already restricted the definition of partial change along the directions of the Cartesian coordinate system. It appears that the coordinate systems that determine the symmetry type of the members of the system of separable Hilbert spaces must have the Cartesian coordinate axes in parallel. Possibly this is due to the existence of the primitive coordinate system in the underlying parameter space. The restriction enables the determination of differences in symmetry. Only the sequence along the axis can be freely selected up or down. It also means that partial change has a systemwide significance. This also means that only a small set of symmetry types will be tolerated. One of the Hilbert spaces will act as the background platform and its symmetry will act as background symmetry. Its natural parameter space will act as background parameter space of the system. All other members of the system will float with the geometric center of their parameter space over the background parameter space. This already generates a dynamic system. The symmetry differences generate symmetry-related sources or sinks that will be located at the geometric center of the natural parameter space of the corresponding floating Hilbert space. The sources and sinks correspond to symmetry-related charges that generate symmetry-related fields. In physics these symmetry-related charges are electric charges

Not the symmetries of the floating Hilbert spaces are important. Instead, the differences between the symmetry of the floating member and the background symmetry are important for establishing the type of the member Hilbert space. The counts of the differences in symmetry restrict to the shortlist -3, -2, -1, 0, +1, +2, +3.

The existence of symmetries and symmetry differences can be comprehended. The existence of corresponding symmetry-related charges is counterintuitive. The realization of these charges as sources or sinks of symmetry-related fields is not yet explained.

All floating Hilbert spaces are separable. The background Hilbert space is an infinite-dimensional separable Hilbert space. It owns a non-separable companion Hilbert space that embeds its separable partner.

The system of separable Hilbert spaces supports the containers of footprints that can map into the quaternionic fields. The vectors that represent the footprint vectors originate in the underlying spatial field. They act as state vectors for the Hilbert spaces that act as containers for the footprints. The state vector represents the vector from the underlying vector space that aims at the geometric center of the floating Hilbert space. This enables the maps of these state vectors and the corresponding footprint in the dynamic universe field. The state vector represents a vector from the underlying vector space that tries to locate the position of the geometric center of the floating platform in the parameter space of the background platform. State vectors are special footprint vectors. Together this entwined locator installs an ongoing embedding process that acts as an imaging process of the geometric center of the floating platform onto the background parameter space. The eigenspace of a dedicated operator maps this image into the dynamic field that represents the universe.

In this way, a huge amount of ongoing hopping paths are mapped onto the embedding field. Physicists call this dynamic field the universe. On the floating platforms, the hopping paths are closed. The movement of the floating platforms breaks the closure of the images of the hopping paths.

15.2.1 Conglomerates

Elementary fermions appear to behave as elementary modules. The conglomerates of these elementary modules populate the dynamic field that we call our universe. All massive objects, except black holes, are conglomerates of elementary fermions. All conglomerates of elementary fermions own mass. This means that the universe is covered by massive modular systems.

A private stochastic process determines the complete local life story of each elementary fermion. That stochastic process is controlled in the change space of its private Hilbert space. The private stochastic process produces an ongoing hopping path and corresponds to a footprint vector that consists of a dynamically changing superposition of the reference operator's eigenvectors. This is explained in the section of the formula chapter that treats the arithmetic of change. Each floating platform of the system of separable Hilbert spaces owns a single private footprint vector. The footprint vector acts as the state vector of the elementary fermion and the probability amplitude corresponds to what physicists call the wavefunction of the particle.

This invites the idea that conglomerates of elementary fermions are defined by stochastic processes whose characteristic functions are defined in the change space of the background platform. In this change space, the characteristic function of a stochastic process that defines a conglomerate is a superposition of the characteristic functions of the components of the conglomerate. The dynamic superposition coefficients act as displacement generators. This means that these displacement generators define the internal oscillations of the components within the conglomerates. It might not hold for higher order conglomerates, but it holds for the lower order conglomerates.

Since in change space, the position is not defined, the fact that a component belongs to a conglomerate does not restrict the distance between the components. This way of defining the membership of a conglomerate introduces entanglement. Independent of their mutual distance, components of a conglomerate must still obey the Pauli exclusion principle.

15.2.2 Interaction with black holes

Field excitations cannot enter or leave black holes, but the Hilbert spaces that represent elementary fermions may hover over the enclosed region of the black hole. So, part of the footprint of the elementary particle may be mapped into the region of the black hole. The mass of the black hole attracts nearby elementary fermions. Together with the effect of hovering this may enable the growth of black holes and the merge of approaching black holes. It may also explain the merge of a black hole and a dense star.

15.2.3 Hadrons

Hadrons can be mesons or baryons. They are conglomerates of quarks. Quarks can only bind via oscillations and via the attraction that is induced by their electric charges. Since the symmetry of quarks does not differ from the background symmetry in an isotropic way, the footprint of quarks does not deform the embedding field. So, mass does not help to bind the quarks until they reach an isotropic symmetry difference. This phenomenon is called color confinement. Hadrons feature mass. Thus, these conglomerates are sufficiently isotropic to deform the embedding field. Once configured, the mutual binding of baryons is very strong. The nuclei of atoms are constituted by baryons.

15.2.4 Atoms

Compound modules are composite modules for which the images of the geometric centers of the platforms of the components coincide in the background platform. The charges of the platforms of the elementary modules establish the primary binding of the corresponding platforms. Physicists and chemists call these compound modules atoms or atomic ions.

In free compound modules, the geometric symmetry-related charges do not take part in the internal oscillations. The targets of the private stochastic processes of the elementary modules oscillate. This means that the hopping path of the elementary module folds around the oscillation path and the hop landing location swarm gets smeared along the oscillation path. The oscillation path is a solution to the Helmholtz equation. Each fermion must use a different oscillation mode. A change of the oscillation mode goes together with the emission or absorption of a photon. As suggested earlier the emission or absorption of a photon involves a switch from the quaternionic Hilbert space to a subspace that is represented by a complex-number-based Hilbert space. The duration of the switch lasts a full particle regeneration cycle. During that cycle, the stochastic mechanism does not produce a swarm of hop landing locations that produce pulses that generate spherical shock fronts, but instead, it produces a one-dimensional string of equidistant pulse responses that cause one-dimensional shock fronts. The center of emission coincides with the geometrical center of the compound module. This ensures that the emitted photon does not lose its integrity. All photons will share the same emission duration, and that duration will coincide with the regeneration cycle of the hop landing location swarm. This is the reason that photons obey the Planck-Einstein relation $E = h\nu$. Absorption cannot be interpreted so easily. It can only be comprehended as a time-reversed emission act. Otherwise, the absorption would require an incredible aiming precision for the photon. The number of one-dimensional pulses in the string corresponds to the step in the energy of the Helmholtz oscillation.

The type of stochastic process that controls the binding of components appears to be responsible for the absorption and emission of photons and the change of oscillation modes. If photons arrive with too low energy, then the energy is spent on the kinetic energy of the common platform. If photons arrive with too high energy, then the energy is distributed over the available oscillation modes, and the rest is spent on the kinetic energy of the common platform, or it escapes into free space. The process must somehow archive the modes of the components. It can apply the private platform of the components for that purpose. Most probably, the current value of the dynamic superposition coefficient is stored in the eigenspace of a special superposition operator.

15.2.5 Molecules

Molecules are conglomerates of compound modules that each keep their private geometrical center. However, electron oscillations are shared among the compound modules. Together with the geometric symmetry-related charges, this binds the compound modules into the molecule.

15.2.6 Earth

On Earth, conglomerates of molecules can form living species. Living species archive essential properties in RNA and DNA molecules.

15.2.7 Particles and fields

The floating elements of the system can be interpreted as particles. In contrast, the background platform cannot be interpreted as a particle. Still, all elements of the system of Hilbert spaces are platforms that show similar capabilities and properties. All floating platforms act like symmetry-related fields and these fields correspond to symmetry-related charges. The background platform does not show a symmetry-related field and a symmetry-related charge. Instead, it acts as a universe-wide embedding field that can be deformed by the presence of floating members. Mainstream physics considers the Higgs particle responsible for the capabilities that this paper assigns to the background platform. In this paper the background platform including its non-separable companion implements the origin of the gravitational potential via the action of spherical shock fronts that are generated by actuators that cause isotropic pulses.

15.3 A System of non-separable Hilbert spaces

This system shows some similarities with the holographic principle that is promoted by some theoretical physicists [20]. However, this resemblance is reached without the tools of string theory or quantum gravity because in this paper the black hole is supposed to contain a countable parameter space that relates to a continuous surrounding common target space. The system does not show the recycling universe of Sir Roger Penrose.

The floating members of the system are universes that are relationally connected to a private black hole. The corresponding compartment of space represents the influence range of this black hole. The countable parameter space contained in the black hole relates to the content of the compartment. The borders of the compartments do not act as barriers for photons, fermions, atoms, planets, or stars. The background member of the system contains the continuum parameter space of the whole multiverse. It relates to all the contained universes

Astronomers observe that black holes can merge and that neutron stars can collapse into new black holes. These events redistribute the compartments. These events cause graphical shock fronts that are constituted of a huge number of superposed spherical shock fronts which are generated in a small region and in a small period. Gravitational wave is a misnomer for these phenomena.

The parameter space of the multiverse adapts to the changes of the covered compartments.

16 Conclusions

The Hilbert Book Model applies the system of Hilbert spaces that all share the same underlying vector space. The author calls this system the Hilbert repository. This approach differs on several essential points from the approach that mainstream physics follows. Still, an astonishing agreement exists between the Standard Model of the elementary fermions that is contained in the Stand Model of the experimental particle physicists and the system of separable Hilbert spaces.

In the system of separable Hilbert spaces, spatial coordinate axes play an important role. These axes must be systemwide in parallel. In spatial continuums, first-order change usually occurs along the spatial coordinate axes. In locally spherical symmetric conditions change covers all directions. The freedom of choice left by spatial arithmetic always occurs along the Cartesian coordinate axes. Possibly this is due to the adaptation to the primitive coordinate system that exists in the underlying parameter space.

In the Hilbert Book Model (HBM), the footprints of all massive objects are recurrently regenerated with a high repetition rate that corresponds with the duration of the emission of photons.

Mainstream physics still has not found a suitable explanation for dark matter objects and dark energy objects. The HBM explains these objects as field excitations that behave as shock fronts and are described in detail by solutions of second-order quaternionic partial differential equations. The spherical shock fronts are the only field excitations that deform the field that embeds them. Photons are strings of equidistant one-dimensional shock-fronts. Black holes are slowly varying objects that contain a countable content. Black holes deform their continuous surround.

Elementary fermions are complicated objects that are represented by a private quaternionic separable Hilbert space that manages the properties of the fermion. These Hilbert spaces own a private parameter space and a private symmetry. The separable Hilbert spaces float with the geometric center of their parameter space over a background parameter space that is managed by a background separable Hilbert space. This background Hilbert space owns a non-separable Hilbert space The non-separable Hilbert space embeds its separable companion. The non-separable Hilbert space manages several continuums in the eigenspace of a corresponding dedicated operator. One of the continuums is a dynamic field, which physicists call universe. The universe field embeds the images of the geometric centers of the floating separable Hilbert spaces. This map is blurred by stochastic disturbances of the locator vector that resides in the underlying vector space and points to the geometric center of the floating Hilbert space. Depending on the difference in symmetry, the embedding of the image may cause a spherical shock response that will temporarily deform the universe field. The corresponding shock front moves away in all directions until it vanishes at infinity. The content of the shock front expands the covered volume of the field. An isotropic symmetry difference with the background platform is required for the generation of the spherical shock front. Only a few fermions feature an isotropic symmetry difference. Isolated quarks do not possess the required isotropic symmetry difference and will not produce a deformation of the universe. However, combined in a hadron such that the combination features an isotropic symmetry difference, the hadron can cause deformation. This phenomenon is known as color confinement.

The non-separable Hilbert space embeds its separable partner. Consequently, the parameter space of the non-separable Hilbert space is the parameter space of the separable companion Hilbert space where the irrational numbers are added to the rational numbers. The result is a continuum. The parameter spaces are not affected by deforming actuators. However, the continuum eigenspaces of other operators than the reference operator of the non-separable can be vibrated, deformed, and expanded.

Symmetry-related charges are located at the geometric centers of the floating Hilbert spaces. The charges depend on the difference in symmetry between the floating platform and the background platform. The charges act as sources or sinks of corresponding symmetry-related fields. These fields differ fundamentally from the universe field. However, both types of fields obey the same quaternionic field equations. They differ in their start and boundary conditions.

The archival of the footprint in the floating separable Hilbert space enables the independent retrieval of that footprint at a later instance. Thus, the footprint can have been generated in an episode before the beginning of the flow of time. The retrieval can occur as a function of the flow of time and uses the archived timestamps for synchronizing the retrieval. This means that at the instant of time zero, none of the archived footprint data was retrieved. Without deforming actuators, the embedding field stays flat. Thus, at the beginning of the flow of time, the embedding field was in its maiden state. The function that described the universe field was equal to its parameter space. Immediately after that instant, the locator landings started, distributed randomly over that parameter space, to mark the locations of the geometrical centers of the floating Hilbert spaces. Depending on the symmetry of the floating Hilbert space this resulted in a corresponding spherical shock front. This certainly does not look like the Big Bang that mainstream physics promotes. Instead, already at its start, the ongoing embedding was a quiet imaging process.

The background non-separable Hilbert space defines in change space the conglomerates of elementary fermions as superpositions. For that reason, it applies the characteristic functions of the stochastic mechanisms that generate the footprints of the elementary fermions. In change space, the position is not defined. This is the reason for the existence of entanglement. The Pauli exclusion principle works independently of the distance between the elements of the conglomerate.

Elementary fermions act like elementary modules. Together they constitute all massive objects that occur in the universe. The notorious exception is formed by black holes. For the rest, the content of the universe is one large modular system that produces a huge number and enormous diversity of modular subsystems. Atoms, molecules, rocks, planets, stars, galaxies, and living species are all examples of modular systems. Every human is a modular system. On planet earth, before the arrival of humans, modularization happened in a stochastic way. Since the arrival of humans, modularization can happen intelligently. Computers and robots are excellent examples of this development.

Once the elementary fermions were formed, the rest of the content of the universe followed automatically. Modular systems that care for their own community and that take care of the modular systems on which they depend have the greatest chance to survive. See “A law of nature” in [21].

Mainstream physics usually bases on the steady action principle. The steady action principle does not request a recurrent regeneration of the objects that occur in the universe. It does not request that conglomerates be generated in a modular way. It also does not oppress the strange reaction of continuums to disruptions by actuators.

Forces require a point of engagement. Fields do not own a point of engagement. For quaternionic functions, the first-order change already connects the gradient of a scalar field to the time variation of the corresponding spatial part of the field. It suffices that the universe field shows a gradient in its scalar part and that the spatial part of the field moves uniformly. Thus, a gravitational potential raises an acceleration of the moving spatial field. Intuition cannot tell you this. But mathematics does.

Finally, the paper introduces the system of non-separable Hilbert spaces. This system concerns a multiverse consisting of universes that all apply a black hole to archive the private parameter space of the background platform of the system of separate Hilbert spaces that represents the considered dynamic universe. The system of non-separable Hilbert spaces corresponds to a coverage of space by compartments that each contain a dynamic universe and a private black hole.

Astronomers observe that black holes can merge and that neutron stars can collapse into new black holes. These events redistribute the compartments.

17 Formulas

This chapter applies MathType to formulate equations

17.1 Physical units

This chapter applies mathematical formulas that do not contain physical units. Physical units represent the adaptation of the considered subject to units that experimental physicists use to measure that subject. In fact, lightspeed c is such physical unit because it represents a physical unit measured in meters per second. Physicists use the permittivity $\varepsilon = \varepsilon_0 \varepsilon_1$ for the electrical field. In free space $\varepsilon_1 = 1$. Physicists use the permeability $\mu = \mu_0 \mu_1$ for the electrical field. In free space $\mu_1 = 1$.

The two physical units are related via light speed c [22] [23].

$$c^2 = \frac{1}{\varepsilon_0 \mu_0} \qquad (17.1.1)$$

17.2 Vector arithmetic

In this section vectors that reside in a vector space will be indicated with boldface and scalars will be indicated with italics.

The addition of vectors is commutative. It can be done by shifting one of the vectors in parallel until it coincides with the alternative point of the other vector. Now the two resulting points represent the vector sum. The arithmetic of scalars resembles the arithmetic of rational members of the real number systems. Vector addition is commutative. The addition creates new vectors.

$$\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v} \qquad (17.2.1)$$

Vector addition is also associative.

$$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w}) \qquad (17.2.2)$$

Multiplication with a scalar is commutative. This multiplication may change the length and thus the integrity of the vector. It may create a new vector.

$$\mathbf{w} = a\mathbf{v} = \mathbf{v}a \tag{17.2.3}$$

Multiplication with scalars is distributive for scalars and vectors.

$$\begin{gathered}(a+b)\mathbf{v} = a\mathbf{v} + b\mathbf{v} \\ a(\mathbf{v}+\mathbf{w}) = a\mathbf{v} + a\mathbf{w}\end{gathered} \tag{17.2.4}$$

Multiplication with negative scalars reverses the direction of the vector. In particular

$$(-1)\mathbf{v} = -\mathbf{v} \tag{17.2.5}$$

Vectors obey a scalar product. However, they do not obey an outer product. Otherwise, their arithmetic would be equal to the arithmetic of the spatial numbers, and the dimension of the vector space would be limited by three.

17.2.1 Base vectors

A selected base $\{\mathbf{u}_i\}$ is a subset of the vectors that is used to define another vector as a superposition of the members of the base.

$$\mathbf{v} = \sum_{i=0}^{i=N} v_i \mathbf{u}_i \tag{17.2.6}$$

A scalar product $\langle \mathbf{v}, \mathbf{w} \rangle$ of two vectors $\mathbf{v}$ and $\mathbf{w}$ would be defined in terms of the orthonormal base $\{\mathbf{u}_i\}$ as

$$\langle \mathbf{v}, \mathbf{w} \rangle = \sum_{i=0}^{i=N} v_i w_j \langle \mathbf{u}_i, \mathbf{u}_j \rangle \tag{17.2.7}$$

while

$$\langle \mathbf{u}_i, \mathbf{u}_j \rangle = \delta_{ij} \tag{17.2.8}$$

If the orthonormal base spans the full vector space, then the vector space contains N dimensions. N can be infinite.

The scalar product that is taken over all dimensions generates a metric. That metric can establish the length $\ell_{\mathbf{a}}$ of the vector $\mathbf{a}$ as a scalar. The scalar product can indicate the length of a vector

$$\begin{gathered} \ell_{\mathbf{a}} = \|\mathbf{a}\| \\ \langle \mathbf{a}, \mathbf{a} \rangle = \|\mathbf{a}\|^2 \end{gathered} \tag{17.2.9}$$

If the scalar product equals zero, then either one of the vectors has zero length, or the two vectors live in different dimensions. In that case, the vectors are independent. In a N dimensional vector space precisely N vectors can be mutually independent.

The scalar product can be applied to construct a set of coordinate markers that together form a coordinate system.

17.3 Arithmetic of real numbers

We will indicate the real numbers with the suffix $_{\mathrm{r}}$.

For real numbers, addition and multiplication are commutative, associative, and distributive.

$$\begin{gathered} b_r + a_r = a_r + b_r \\ (a_r + b_r) + c_r = a_r + (b_r + c_r) \end{gathered} \tag{17.3.1}$$

$$\begin{gathered} b_r a_r = a_r b_r \\ (a_r b_r) c_r = a_r (b_r c_r) \end{gathered} \tag{17.3.2}$$

$$a_r (b_r + c_r) = a_r b_r + a_r c_r \tag{17.3.3}$$

For real numbers, the square is zero or it is positive

$$a_r a_r \geq 0 \qquad (17.3.4)$$

17.4 Arithmetic of spatial numbers

For spatial numbers, addition and multiplication are commutative and associative.

$$\begin{aligned} \vec{b}+\vec{a} &= \vec{a}+\vec{b} \\ \left(\vec{a}+\vec{b}\right)+\vec{c} &= \vec{a}+\left(\vec{b}+\vec{c}\right) \end{aligned} \qquad (17.4.1)$$

The product d of two spatial numbers $\vec{a}$ and $\vec{b}$ results in a real scalar part d_r and a new spatial part $\vec{d}$

$$d = d_r + \vec{d} = \vec{a}\vec{b} \qquad (17.4.2)$$

$d_r = -\left\langle \vec{a},\vec{b} \right\rangle$ is the inner product of $\vec{a}$ and $\vec{b}$

For the inner product and the norm $\|\vec{a}\|$ holds $\langle \vec{a},\vec{a} \rangle = \|\vec{a}\|^2$

$$\left\langle \vec{a},\vec{b} \right\rangle = \|\vec{a}\|\left\|\vec{b}\right\|\cos(\alpha) \qquad (17.4.3)$$

The angle α between the spatial numbers $\vec{a}$ and $\vec{b}$ is measured in radians.

The square of a spatial number equals zero or it is a negative real number.

$$\vec{a}\vec{a} = -\langle \vec{a},\vec{a} \rangle \leq 0 \qquad (17.4.4)$$

$\vec{d} = \vec{a}\times\vec{b}$ is the outer product of $\vec{a}$ and $\vec{b}$

The spatial part $\vec{d}$ is independent of $\vec{a}$ and independent of $\vec{b}$. This means that $\langle \vec{a}, \vec{d} \rangle = 0$ and $\langle \vec{b}, \vec{d} \rangle = 0$

$$\begin{aligned} \left\| \vec{a} \times \vec{b} \right\| &= \|\vec{a}\| \left\| \vec{b} \right\| \left| \sin(\alpha) \right| \\ \vec{a} \times \vec{b} &= -\vec{b} \times \vec{a} \end{aligned} \tag{17.4.5}$$

It is possible to write spatial numbers as superpositions of base numbers. For the three-dimensional spatial numbers, this means.

$$\begin{aligned} \vec{a} &= a_i \vec{i} + a_j \vec{j} + a_k \vec{k} \\ \vec{i} \ \vec{j} &= \pm \vec{k} \end{aligned} \tag{17.4.6}$$

The $\pm$ sign indicates the chiral choice of the handedness of the outer product.

17.5 Mixed arithmetic

The addition and multiplication of real numbers with spatial numbers are commutative.

$$\begin{aligned} a_r + \vec{b} &= \vec{b} + a_r \\ a_r \vec{b} &= \vec{b} a_r \end{aligned} \tag{17.5.1}$$

Mixed numbers are indicated without suffixes and caps. In the next formula c is a mixed number.

$$c = c_r + \vec{c} \tag{17.5.2}$$

Quaternionic multiplication obeys the equation

$$c = c_r + \vec{c} = ab = (a_r + \vec{a})(b_r + \vec{b})$$
$$= a_r b_r - \langle \vec{a}, \vec{b} \rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \quad (17.5.3)$$

The $\pm$ sign indicates the freedom of choice of the handedness of the product rule that exists when selecting a version of the quaternionic number system. In this way, the handedness of the product rule is treated as a special kind of symmetry. The version must be selected before it can be used in calculations.

Two quaternions that are each other's inverse can rotate the spatial part of another quaternion.

$$c = ab / a \quad (17.5.4)$$

The construct rotates the spatial part of b that is perpendicular to $\vec{a}$ over an angle that is twice the angular phase θ of $a = \|a\| e^{\vec{i}\theta}$ where $\vec{i} = \vec{a} / \|\vec{a}\|$.

Cartesian quaternionic functions apply a quaternionic parameter space that is sequenced by a Cartesian coordinate system. In the parameter space, the real parts of quaternions are often interpreted as instances of (proper) time, and the spatial parts are often interpreted as spatial locations. With these interpretations, the real parts of quaternionic functions represent dynamic scalar fields. The spatial parts of quaternionic functions represent dynamic spatial fields. These fields are often called vector fields. This is a misleading name. Vectors obey different arithmetic.

17.6 Arithmetic of change

In continuums, all convergent series of numbers end in a limit that is a member of that continuum. This fact enables the differentiation of the continuum. Differential calculus shows that a continuum can change.

The continuum shows astonishing behavior. It has the habit to remove deformations. Without disturbing actuators, the continuum stays flat.

17.6.1 Differentiation

Along a direction-line, change can be described by a partial differential. If in a region of the space coverage inside this direction-line all converging series of coordinate markers result in a limit that is a coordinate marker, then the partial change of the space coverage along the direction of r is defined as the limit

$$\frac{\partial\psi}{\partial r} = \lim_{\delta r \to 0} \frac{\psi\left(r+\delta r\right)-\psi(r)}{\delta r} \tag{17.6.1}$$

If the region is covered by all its irrational numbers, then this limit exists. The existence of the limit is not ensured. If the limit does not exist, then the location represents a singular point. It is also possible that the surrounding region is covered by a discrete set of point-like objects.

If the spatial part of the neighborhood is isotropic and the limit also exists in the real number space, then the total differential change df of field f equals

$$df = \frac{\partial f}{\partial \tau} d\tau + \frac{\partial f}{\partial x}\vec{i}dx + \frac{\partial f}{\partial y}\vec{j}dy + \frac{\partial f}{\partial z}\vec{k}dz \tag{17.6.2}$$

In this equation, the partial differentials $\frac{\partial f}{\partial \tau}, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial y}$ behave like quaternionic differential operators.

The quaternionic nabla ∇ assumes the ***special condition*** that partial differentials direct along the axes of the Cartesian coordinate system in a natural parameter space of a non-separable Hilbert space. Thus,

$$\nabla = \sum_{i=0}^{4} \vec{e}_i \frac{\partial}{\partial x_i} = \frac{\partial}{\partial \tau} + \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} + \vec{k}\,\frac{\partial}{\partial z} \tag{17.6.3}$$

This will be applied in the next section by splitting both the quaternionic nabla and the function in a scalar part and a spatial part.

The first-order partial differential equations divide the first-order change of a quaternionic field into five different parts that each represent a new field. We will represent the quaternionic field change operator by a quaternionic nabla operator. This operator behaves like a quaternionic multiplier.

The first-order partial differential follows from

$$\nabla = \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} = \nabla_r + \vec{\nabla} \tag{17.6.4}$$

The spatial nabla $\vec{\nabla}$ is well-known as the del operator and is treated in detail in Wikipedia. The partial derivatives in the change operator only use parameters that are taken from the natural parameter space.

$$\begin{aligned} \phi = \nabla \psi &= \left(\frac{\partial}{\partial \tau} + \vec{\nabla} \right) \left(\psi_r + \vec{\psi} \right) \\ &= \nabla_r \psi_r - \langle \vec{\nabla}, \vec{\psi} \rangle + \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \end{aligned} \tag{17.6.5}$$

In a selected version of the quaternionic number system, only the corresponding version of the quaternionic nabla is active. In a selected Hilbert space, this version is always and everywhere the same.

The differential $\nabla \psi$ describes the change of field ψ. The five separate terms in the first-order partial differential have separate physical meanings. All basic fields feature this decomposition. The terms may represent new fields.

$$\phi_r = \nabla_r \psi_r - \left\langle \vec{\nabla}, \vec{\psi} \right\rangle \tag{17.6.6}$$

ϕ_r is a scalar field.

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \tag{17.6.7}$$

$\vec{\phi}$ is a spatial field.

$\vec{\nabla} f$ is the gradient of f .

$\left\langle \vec{\nabla}, \vec{f} \right\rangle$ is the divergence of $\vec{f}$.

$\vec{\nabla} \times \vec{f}$ is the curl of $\vec{f}$.

Important properties of the del operator are

$$\left(\vec{\nabla}, \vec{\nabla}\right) \psi = \Delta \psi = \nabla^2 \psi \tag{17.6.8}$$

$$\left(\vec{\nabla}, \vec{\nabla} \times \vec{\psi}\right) = 0 \tag{17.6.9}$$

$$\vec{\nabla} \times \left(\vec{\nabla} \psi_r\right) = 0 \tag{17.6.10}$$

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi}\right) = \vec{\nabla}\left(\vec{\nabla}, \vec{\psi}\right) - \left(\vec{\nabla}, \vec{\nabla}\right) \vec{\psi} \tag{17.6.11}$$

Sometimes parts of the change get new symbols

$$\vec{E} = -\nabla_r \vec{\psi} - \vec{\nabla} \psi_r \tag{17.6.12}$$

$$\vec{B} = \vec{\nabla} \times \vec{\psi} \tag{17.6.13}$$

The formula (17.6.5) does not leave room for gauges. In Maxwell equations, the equation (17.6.6) is treated as a gauge.

$$\left(\vec{\nabla}, \vec{B}\right) = 0 \tag{17.6.14}$$

$$\vec{\nabla} \times \vec{E} = -\nabla_r \vec{\nabla} \times \vec{\psi} - \vec{\nabla} \times \vec{\nabla} \psi_r = -\nabla_r \vec{B} \tag{17.6.15}$$

$$\left(\vec{\nabla},\vec{E}\right)=-\nabla_r\left(\vec{\nabla},\vec{\psi}\right)-\left(\vec{\nabla},\vec{\nabla}\right)\psi_r \tag{17.6.16}$$

The conjugate of the quaternionic nabla operator defines another type of field change.

$$\nabla^*=\nabla_r-\vec{\nabla} \tag{17.6.17}$$

$$\begin{aligned}\zeta=\nabla^*\phi&=\left(\frac{\partial}{\partial\tau}-\vec{\nabla}\right)\left(\phi_r+\vec{\phi}\right)\\&=\nabla_r\phi_r+\left\langle\vec{\nabla},\vec{\phi}\right\rangle+\nabla_r\vec{\phi}-\vec{\nabla}\phi_r\mp\vec{\nabla}\times\vec{\phi}\end{aligned} \tag{17.6.18}$$

All dynamic quaternionic fields obey the same first-order partial differential equations (17.6.5) and (17.6.18).

$$\nabla^\dagger=\nabla^*=\nabla_r-\vec{\nabla}=\nabla_r+\vec{\nabla}^\dagger=\nabla_r+\vec{\nabla}^* \tag{17.6.19}$$

In the Hilbert space, the quaternionic nabla is a normal operator. The operators

$$\nabla^\dagger\nabla=\nabla\nabla^\dagger=\nabla^*\nabla=\nabla\nabla^*=\nabla_r\nabla_r+\left\langle\vec{\nabla},\vec{\nabla}\right\rangle \tag{17.6.20}$$

are normal operators who are also Hermitian operators.

The separate operators $\nabla_r\nabla_r$ and $\left\langle\vec{\nabla},\vec{\nabla}\right\rangle$ are also Hermitian operators.

$\left\langle\vec{\nabla},\vec{\nabla}\right\rangle$ is known as the Laplace operator.

The two operators can also be combined as $\square=\nabla_r\nabla_r-\left\langle\vec{\nabla},\vec{\nabla}\right\rangle$. This is the d'Alembert operator.

The solutions to $\nabla_r \nabla_r + \langle \vec{\nabla}, \vec{\nabla} \rangle = 0$ and $\nabla_r \nabla_r - \langle \vec{\nabla}, \vec{\nabla} \rangle = 0$ differ. These two equations offer different solutions and for that reason, they deliver different dynamic behavior of the field. The equations control the behavior of the embedding field that physicists call their universe. This dynamic field exists everywhere in the reach of the parameter space of the function. Both equations also control the behavior of the symmetry-related fields. The homogeneous d'Alembert equation is known as the wave equation and offers waves and wave packages as its solutions. Both equations offer shock fronts as solutions but only the operators in (17.6.20) deliver shock fronts that feature a spin or polarization vector. Integration over the time domain turns both equations in the Poisson equation and removes the spin or polarization vector. Shock fronts require a corresponding actuator and occur only in odd numbers of participating dimensions. Spherical shock fronts require an isotropic actuator. Otherwise, the shock front does not appear.

17.6.1.1 Continuity equations

Continuity equations are partial quaternionic differential equations.

The dynamic changes in the field are interpreted as field excitations as field deformations or field expansions.

The field excitations that will be discussed here are solutions to mentioned second-order partial differential equations. Without a corresponding actuator, the field will not react. It appears that spherical pulses are the only actuators that deform the field. The field reacts to these pulses by quickly removing the deformation by sending the deformation away in all directions in the form of shock fronts until these fronts vanish at infinity. This follows from the solutions presented in (17.6.29) and (17.6.31).

One of the second-order partial differential equations results from combining the two first-order partial differential equations $\phi = \nabla\psi$ and $\zeta = \nabla^*\phi$.

$$\zeta = \nabla^*\varphi = \nabla^*\nabla\psi = \nabla\nabla^*\psi = \left(\nabla_r + \vec{\nabla}\right)\left(\nabla_r - \vec{\nabla}\right)\left(\psi_r + \vec{\psi}\right)$$
$$= \left(\nabla_r\nabla_r + \left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)\psi$$

(17.6.21)

All other terms vanish. $\left\langle\vec{\nabla},\vec{\nabla}\right\rangle$ is known as the Laplace operator.

Integration over the time domain results in the Poisson equation

$$\rho = \left\langle\vec{\nabla},\vec{\nabla}\right\rangle\psi \qquad (17.6.22)$$

Under isotropic conditions, a very special solution of the Poisson equation is the Green's function $\frac{1}{4\pi\left|\vec{q}-\overrightarrow{q'}\right|}$ of the affected field. This solution is the spatial Dirac $\delta(\vec{q})$ pulse response of the field under strict isotropic conditions.

$$\nabla\frac{1}{\left|\vec{q}-\overrightarrow{q'}\right|} = -\frac{\left(\vec{q}-\overrightarrow{q'}\right)}{\left|\vec{q}-\overrightarrow{q'}\right|^3} \qquad (17.6.23)$$

$$\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\frac{1}{\left|\vec{q}-\overrightarrow{q'}\right|} \equiv \left\langle\vec{\nabla},\vec{\nabla}\frac{1}{\left|\vec{q}-\overrightarrow{q'}\right|}\right\rangle$$
$$= -\left\langle\vec{\nabla},\frac{\left(\vec{q}-\overrightarrow{q'}\right)}{\left|\vec{q}-\overrightarrow{q'}\right|^3}\right\rangle = 4\pi\delta\left(\vec{q}-\overrightarrow{q'}\right) \qquad (17.6.24)$$

This solution corresponds with an ongoing source or sink that exists in the field. A point-like stationary spatial pulse cannot start a shock front. The stationary spatial point-like object must be a sink or a source. In physics, this means that stationary point-like masses do not exist in physical reality.

Change can take place in one spatial dimension or combined in two or three spatial dimensions.

Under the proper conditions, the dynamic pulse response of the field is a solution of a special form of the equation (17.6.21).

$$\left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi = 4\pi\delta\left(\vec{q} - \overrightarrow{q'}\right)\theta\left(\tau \pm \tau'\right) \qquad (17.6.25)$$

Here $\theta(\tau)$ is a temporal step function and $\delta(\vec{q})$ is a spatial Dirac pulse response. For the spherical pulse response, the pulse must be isotropic.

After the instant τ', the equation turns into a homogeneous equation.

A remarkably simple solution is the shock front in one dimension along the line $\vec{q} - \overrightarrow{q'}$.

$$\psi = f\left(\left|\vec{q} - \overrightarrow{q'}\right| \pm c\left(\tau - \tau'\right)\vec{n}\right) \qquad (17.6.26)$$

Here $\vec{n}$ is a normed spatial quaternion. This spatial quaternion has an arbitrary direction that does not vary in time. Here, the normalized spatial number $\vec{n}$ can be interpreted as the polarization of the solution. We intentionally placed the normalized spatial number $\vec{n}$ close to speed c. The function f can be a primitive shock front, but it can also be a superposition of primitive shock fronts. The single primitive shock-front solution represents a ***dark energy object***. It represents a quantum of energy.

In isotropic conditions, we better switch to spherical coordinates. Then the equation gets the form

$$\left(\frac{\partial^2}{\partial\tau^2}+\frac{\partial^2}{\partial r^2}+2\frac{\partial}{r\partial r}\right)\psi$$
$$=\left(\frac{\partial^2}{\partial\tau^2}+\frac{\partial^2}{\partial r^2}\right)(\psi r)=0 \tag{17.6.27}$$

The second line describes the second-order change of ψr in one dimension along the radius r. That solution is described above. A solution to this equation is

$$\psi r = f\left(r \pm c\tau\vec{n}\right) \tag{17.6.28}$$

The solution of (17.6.27) is described by

$$\psi = \frac{f\left(\left|\vec{q}-\overrightarrow{q'}\right| \pm c\left(\tau-\tau'\right)\vec{n}\right)}{\left|\vec{q}-\overrightarrow{q'}\right|} \tag{17.6.29}$$

The normalized spatial number $\vec{n}$ can be interpreted as the spin of the solution. It might be related to the direction that is selected when the quaternion-based Hilbert space is temporarily reduced to a subspace that contains a complex-number-based Hilbert space. The spherical pulse response acts either as an expanding or as a contracting spherical shock front. Over time this pulse response integrates into the green's function. This means that the isotropic pulse injects the volume of the green's function into the field. Subsequently, the front spreads this volume over the field. The contracting shock front collects the volume of the green's function and sucks it out of the field. The $\pm$ sign in the equation (17.6.25) selects between injection and subtraction. The shock front moves away from the pulse that caused the front. Finally, it vanishes at infinity. The inserted volume expands the field.

Spherical shock fronts are ***dark matter objects***.

Shock fronts only occur in one and three dimensions. A pulse response can also occur in two dimensions, but in that case, the pulse response is a complicated vibration that looks like the result of a throw of a stone in the middle of a pond.

Equations (17.6.21) and (17.6.22) show that the operators $\frac{\partial^2}{\partial\tau^2}$ and $\langle\vec{\nabla},\vec{\nabla}\rangle$ are valid second-order partial differential operators. These operators combine in the quaternionic equivalent of the wave equation.

$$\varphi = \left(\frac{\partial^2}{\partial\tau^2} - \langle\vec{\nabla},\vec{\nabla}\rangle\right)\psi = \Box\psi \qquad (17.6.30)$$

This equation also offers one-dimensional and three-dimensional shock fronts as its solutions.

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q'}\right| \pm c(\tau-\tau')\right)}{\left|\vec{q}-\vec{q'}\right|} \qquad (17.6.31)$$

$$\psi = f\left(\left|\vec{q}-\vec{q'}\right| \pm c(\tau-\tau')\right) \qquad (17.6.32)$$

These pulse responses do not contain the normed spatial number $\vec{n}$. Apart from pulse responses, the wave equation offers waves as its solutions.

If locally the field can be split into a time-dependent part $T(\tau)$ and a location-dependent part $A(\vec{q})$, then the homogeneous version of the wave equation can be transformed into the Helmholtz equation.

$$\frac{\partial^2\psi}{\partial\tau^2} = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi = -\omega^2\psi \tag{17.6.33}$$

$$\psi(\vec{q},\tau) = A(\vec{q})T(\tau) \tag{17.6.34}$$

$$\frac{1}{T}\frac{\partial^2 T}{\partial\tau^2} = \frac{1}{A}\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A = -\omega^2 \tag{17.6.35}$$

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A + \omega^2 A = 0 \tag{17.6.36}$$

$$\frac{\partial^2 T}{\partial\tau^2} + \omega^2 T = 0 \tag{17.6.37}$$

ω acts as quantum coupling between(17.6.36) and (17.6.37).

The time-dependent part $T(\tau)$ depends on initial conditions, or it indicates the switch of the oscillation mode.

During the switch, the quaternionic Hilbert space temporarily switches to a complex-number-based Hilbert space that is a subspace of the Hilbert space. The switch takes a corresponding interval and during that interval, the subspace emits or absorbs a sequence of equidistant one-dimensional shock fronts. Together, these shock fronts constitute a photon. The one-dimensional shock fronts are discussed above. The switch of the oscillation mode means that temporarily the oscillation is stopped and instead an object is emitted or absorbed that compensates for the difference in potential energy. The location-dependent part of the field $A(\vec{q})$ describes the possible oscillation modes of the field and depends on boundary conditions. The oscillations have a binding effect. They keep moving objects within a bounded region.

For three-dimensional isotropic spherical conditions, the solutions have the form

$$A(r,\theta,\varphi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left\{\left(a_{lm}j_l(kr)\right)+b_{lm}Y_l^m(\theta,\varphi)\right\} \qquad (17.6.38)$$

Here j_l and y_l are the spherical Bessel functions, and Y_l^m are the spherical harmonics. These solutions play a role in the spectra of atomic modules.

Planar and spherical waves are the simpler wave solutions to the equation (17.6.33)

$$\psi(\vec{q},\tau)=\exp\left\{\vec{n}\left(\left\langle\vec{k},\vec{q}-\vec{q}_0\right\rangle-\omega\tau+\varphi\right)\right\} \qquad (17.6.39)$$

$$\psi(\vec{q},\tau)=\frac{\exp\left\{\vec{n}\left(\left\langle\vec{k},\vec{q}-\vec{q}_0\right\rangle-\omega\tau+\varphi\right)\right\}}{\left|\vec{q}-\vec{q}_0\right|} \qquad (17.6.40)$$

A more general solution is a superposition of these basic types.

Two quite similar homogeneous second-order partial differential equations exist. They are the homogeneous versions of equations (17.6.25) and (17.6.30). The equation (17.6.25) has spherical shock-front solutions with a spin vector that behaves like the spin of elementary particles. Obviously, the field only reacts dynamically when it gets triggered by corresponding actuators. Pulses may cause shock fronts that after the trigger keep traveling. Oscillations of type (17.6.39) and (17.6.40) must be triggered by periodic actuators.

The inhomogeneous pulse-activated equations are

$$\left(\nabla_r\nabla_r\pm\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)\psi=4\pi\delta\left(\vec{q}-\vec{q}'\right)\theta(\tau\pm\tau') \qquad (17.6.41)$$

Without the interaction with actuators, all vibrations and deformations of the field keep busy vanishing until the affected field resembles a flat field. Only an ongoing stream of actuators can generate a more persistently deformed field. This is provided by an ongoing embedding of the actuators into the eigenspaces of operators that archive the dynamic fields.

17.6.1.2 Isotropic conditions

The two shock-front solutions show an interesting property of the Laplace operator. In isotropic conditions, the Poisson equation can be rewritten as

$$\varphi = \langle \vec{\nabla}, \vec{\nabla} \rangle \psi = \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right) \psi = \frac{1}{r} \frac{\partial^2}{\partial r^2} (r\psi) \qquad (17.6.42)$$

The product $\phi = (r\psi)$ is a solution of a one-dimensional equation in which r plays the variable.

The same thing holds for all differential equations that contain the Laplace operator $\langle \vec{\nabla}, \vec{\nabla} \rangle$

So, spherical solutions of the second-order differential equations ξ / r can be obtained from the solutions ξ of one-dimensional second-order differential equations by dividing ξ by the distance r to the center.

It looks as if in isotropic conditions the quaternionic differential calculus can be scaled down to complex-number-based differential calculus. This already works at local scales. If on larger scales the isotropic condition is violated, then the coordinates of the complex-number-based abstraction must be adapted to the possibly deformed Cartesian coordinates of the quaternionic platform. This makes sense in the presence of moderate deformations of the quaternionic field. After adaptation, the map of each complex-number-based coordinate line becomes a geodesic.

These tricks are possible because complex-number-based Hilbert spaces can be considered subspaces of quaternionic Hilbert spaces.

If the dimension of the quaternionic Hilbert space is reduced to the dimension of a subspace that contains a complex-number-based Hilbert space, then it might become important whether the selected direction involves a polar angle or an azimuth angle. In mathematics, the range of the polar angle is twice the range of the azimuth angle. In physics, the two ranges are exchanged.

17.6.2 Enclosure balance equations

Enclosure balance equations are quaternionic integral equations that describe the balance between the inside, the border, and the outside of an enclosure.

These integral balance equations base on replacing the del operator $\vec{\nabla}$ with a normed vector $\vec{n}$. The vector $\vec{n}$ is oriented outward and perpendicular to a local part of the closed boundary of the enclosed region.

$$\vec{\nabla}\psi \Leftrightarrow \vec{n}\psi \tag{17.6.43}$$

This approach turns part of the differential continuity equation into a corresponding integral balance equation.

$$\iiint \vec{\nabla}\psi dV = \oiint \vec{n}\psi dS \tag{17.6.44}$$

$\vec{n}\ dS$ plays the role of a differential surface. $\vec{n}$ is perpendicular to that surface.

This result separates into three parts

$$\begin{aligned}\vec{\nabla}\psi &= -\langle \vec{\nabla},\vec{\psi}\rangle + \vec{\nabla}\psi_r \pm \vec{\nabla}\times\vec{\psi} \Leftrightarrow \vec{n}\psi \\ &= -\langle \vec{n},\vec{\psi}\rangle + \vec{n}\psi_r \pm \vec{n}\times\vec{\psi}\end{aligned} \tag{17.6.45}$$

The first part concerns the gradient of the scalar part of the field

$$\vec{\nabla}\psi_r \Leftrightarrow \vec{n}\psi_r \tag{17.6.46}$$

$$\iiint \vec{\nabla}\psi_r dV = \oiint \vec{n}\psi_r dS \tag{17.6.47}$$

The divergence is treated in an integral balance equation that is known as the Gauss theorem. It is also known as the divergence theorem [24].

$$\left\langle \vec{\nabla}, \vec{\psi} \right\rangle \Leftrightarrow \left\langle \vec{n}, \vec{\psi} \right\rangle \tag{17.6.48}$$

$$\iiint \left\langle \vec{\nabla}, \vec{\psi} \right\rangle dV = \oiint \left\langle \vec{n}, \vec{\psi} \right\rangle dS \tag{17.6.49}$$

The curl is treated in a corresponding integrated balance equation

$$\vec{\nabla} \times \vec{\psi} \Leftrightarrow \vec{n} \times \vec{\psi} \tag{17.6.50}$$

$$\iiint \vec{\nabla} \times \vec{\psi} dV = \oiint \vec{n} \times \vec{\psi} dS \tag{17.6.51}$$

Equation (17.6.49) and equation (17.6.51) can be combined in the extended theorem

$$\iiint \vec{\nabla} \, \vec{\psi} dV = \oiint \vec{n} \; \vec{\psi} dS \tag{17.6.52}$$

The method also applies to other partial differential equations. For example

$$\begin{aligned} \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) = \vec{\nabla} \left\langle \vec{\nabla}, \vec{\psi} \right\rangle - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \vec{\psi} \Leftrightarrow \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) \\ = \vec{n} \left\langle \vec{n}, \vec{\psi} \right\rangle - \left\langle \vec{n}, \vec{n} \right\rangle \vec{\psi} \end{aligned} \tag{17.6.53}$$

$$\iiint_V \left\{ \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) \right\} dV = \oiint_S \left\{ \vec{\nabla} \left\langle \vec{\nabla}, \vec{\psi} \right\rangle \right\} dS - \oiint_S \left\{ \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \vec{\psi} \right\} dS$$

(17.6.54)

One dimension less, a similar relation exists.

$$\iint_S \left(\left\langle \vec{\nabla} \times \vec{a}, \vec{n} \right\rangle \right) dS = \oint_C \left\langle \vec{a}, d\vec{l} \right\rangle \tag{17.6.55}$$

This is known as the Stokes theorem[25]

The curl can be presented as a line integral

$$\left\langle \vec{\nabla} \times \vec{\psi}, \vec{n} \right\rangle \equiv \lim_{A \to 0} \left(\frac{1}{A} \oint_C \left\langle \vec{\psi}, d\vec{r} \right\rangle \right) \tag{17.6.56}$$

17.6.2.1 Derivation of physical laws

The quaternionic equivalents of Ampère's law are [26]

$$\vec{J} \equiv \vec{\nabla} \times \vec{B} = \nabla_r \vec{E} \Leftrightarrow \vec{J} \equiv \vec{n} \times \vec{B} = \nabla_r \vec{E} \qquad (17.6.57)$$

$$\iint_S \left\langle \vec{\nabla} \times \vec{B}, \vec{n} \right\rangle dS = \oint_C \left\langle \vec{B}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{J} + \nabla_r \vec{E}, \vec{n} \right\rangle dS \qquad (17.6.58)$$

The quaternionic equivalents of Faraday's law are [27]:

$$\nabla_r \vec{B} = \vec{\nabla} \times (\nabla_r \vec{\psi}) = -\vec{\nabla} \times \vec{E} \Leftrightarrow \nabla_r \vec{B} = \vec{n} \times (\nabla_r \vec{\psi}) = -\vec{\nabla} \times \vec{E}$$

(17.6.59)

$$\oint_c \left\langle \vec{E}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{\nabla} \times \vec{E}, \vec{n} \right\rangle dS = -\iint_S \left\langle \nabla_r \vec{B}, \vec{n} \right\rangle dS \qquad (17.6.60)$$

$$\vec{J} = \vec{\nabla} \times \left(\vec{B} - \vec{E} \right) = \vec{\nabla} \times \vec{\varphi} - \nabla_r \vec{\varphi} = \vec{v} \rho \qquad (17.6.61)$$

$$\iint_S \left\langle \vec{\nabla} \times \vec{\varphi}, \vec{n} \right\rangle dS = \oint_C \left(\left\langle \vec{\varphi}, d\vec{l} \right\rangle \right) = \iint_S \left\langle \vec{v} \rho + \nabla_r \vec{\varphi}, \vec{n} \right\rangle dS \qquad (17.6.62)$$

The equations (17.6.60) and (17.6.62) enable the derivation of the ***Lorentz force*** [28].

$$\vec{\nabla} \times \vec{E} = -\nabla_r \vec{B} \qquad (17.6.63)$$

$$\frac{d}{d\tau} \iint_S \left\langle \vec{B}, \vec{n} \right\rangle dS = \iint_{S(\tau_0)} \left\langle \dot{\vec{B}}(\tau_0), \vec{n} \right\rangle ds + \frac{d}{d\tau} \iint_{S(\tau)} \left\langle \vec{B}(\tau_0), \vec{n} \right\rangle ds$$

(17.6.64)

The Leibniz integral equation states [29]

$$\frac{d}{dt} \iint_{S(\tau)} \left\langle \vec{X}(\tau_0), \vec{n} \right\rangle dS$$

$$= \iint_{S(\tau_0)} \left\langle \dot{\vec{X}}(\tau_0) + \left\langle \vec{\nabla}, \vec{X}(\tau_0) \right\rangle \vec{v}(\tau_0), \vec{n} \right\rangle dS - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0) \times \vec{X}(\tau_0), d\vec{l} \right\rangle$$

(17.6.65)

With $\vec{X} = \vec{B}$ and $\left\langle \vec{\nabla}, \vec{B} \right\rangle = 0$ follows

$$\frac{d\Phi_B}{d\tau} =$$

$$\frac{d}{d\tau}\iint_{S(\tau)} \left\langle \dot{\vec{B}}(\tau),\vec{n}\right\rangle dS = \iint_{S(\tau_0)} \left\langle \vec{B}(\tau_0),\vec{n}\right\rangle dS - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle$$

$$= -\oint_{C(\tau_0)} \left\langle E(\tau_0),d\vec{l}\right\rangle - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle$$

(17.6.66)

The electromotive force (EMF) ε equals [30]

$$\varepsilon = \oint_{C(\tau_0)} \left\langle \frac{\vec{F}(\tau_0)}{q},d\vec{l}\right\rangle = -\frac{d\Phi_B}{d\tau}\bigg|_{\tau=\tau_0} \qquad (17.6.67)$$

$$= \oint_{C(\tau_0)} \left\langle \vec{E}(\tau_0),d\vec{l}\right\rangle + \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle$$

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B} \qquad (17.6.68)$$

17.7 Dirac’bra-ket procedure

Paul Dirac introduced a handy notation for the relationship that exists between the ingredients of a Hilbert space. The bra-ket combination provides the opportunity to use complex numbers and quaternions as superposition coefficients. The bra-ket combination restricts the applied numbers to members of an associative division ring. This reduces the choice to real numbers, complex numbers, and quaternions. The bra-ket combination selects a private version of that associative division ring. First, we focus on separable Hilbert spaces. Inside separable Hilbert spaces, the applied sets of numbers are countable. With that restriction, the bra-ket combination turns the underlying vector space into a separable Hilbert space.

17.7.1 Countable number systems

Paul Dirac introduced a handy notation for the relationship that exists between the ingredients of a Hilbert space. The bra-ket combination provides the opportunity to use complex numbers and quaternions as superposition coefficients. The bra-ket combination restricts the applied numbers to members of an associative division ring. This reduces the choice to real numbers, complex numbers, and quaternions. The bra-ket combination selects a private version of that associative division ring. First, we focus on separable Hilbert spaces. In separable Hilbert spaces, the applied sets of numbers are countable. With that restriction, the bra-ket combination turns the underlying vector space into a separable Hilbert space.

By selecting a version of the number system, the symmetry of the number system is fixed. This section treats the case that the Hilbert space applies quaternions to specify the values of bra-ket combinations. The format of the formulas that are shown also holds for complex numbers and real numbers. The values of bra-ket combinations will be used in linear combinations of vectors and as eigenvalues of operators.

To make this possible, the bra-ket method distinguishes the vectors from the underlying vector space into two types of vectors with different arithmetic. The two types represent different views of the underlying simple vector space. The ket $\langle \mathbf{f} |$ is a covariant vector, and the bra $| \mathbf{g} \rangle$ is a contravariant vector. The vectors $\mathbf{f}$ and $\mathbf{g}$ reside in the underlying vector space. The arithmetic of the ket vectors differs from the arithmetic of the bra vectors. The bra-ket combination $\langle \mathbf{f} | \mathbf{g} \rangle$ has a quaternionic value. If the underlying vectors $\mathbf{f}$ and $\mathbf{g}$ are equal, then the bra-ket combination can act as a metric. Since the product of quaternions is not commutative, care must be taken with the format of the formulas when quaternions are applied.

17.7.1.1 Ket vectors

The addition of ket vectors is commutative and associative.

$$|\mathbf{f}\rangle + |\mathbf{g}\rangle = |\mathbf{g}\rangle + |\mathbf{f}\rangle = |\mathbf{f} + \mathbf{g}\rangle \tag{17.7.1}$$

$$\left(|\mathbf{f} + \mathbf{g}\rangle\right) + |\mathbf{h}\rangle = |\mathbf{f}\rangle + \left(|\mathbf{g} + \mathbf{h}\rangle\right) = |\mathbf{f} + \mathbf{g} + \mathbf{h}\rangle \tag{17.7.2}$$

Together with quaternions, a set of ket vectors forms a ket vector space. Ket vectors are covariant vectors.

A quaternion α can be used to construct a covariant linear combination with the ket vector $|\mathbf{f}\rangle$

$$|\alpha\mathbf{f}\rangle = |\mathbf{f}\rangle\alpha \tag{17.7.3}$$

17.7.1.2 Bra vectors

For bra vectors hold

$$\langle\mathbf{f}| + \langle\mathbf{g}| = \langle\mathbf{g}| + \langle\mathbf{f}| = \langle\mathbf{f} + \mathbf{g}| \tag{17.7.4}$$

$$\left(\langle\mathbf{f} + \mathbf{g}|\right) + \langle\mathbf{h}| = \langle\mathbf{f}| + \left(\langle\mathbf{g} + \mathbf{h}|\right) = \langle\mathbf{f} + \mathbf{g} + \mathbf{h}| \tag{17.7.5}$$

Bra vectors are contravariant vectors.

$$\langle\alpha\mathbf{f}| = \alpha^*\langle\mathbf{f}| \tag{17.7.6}$$

Quaternions can constitute linear combinations with bra vectors.

A set of bra vectors form the vector space that is adjunct to the vector space of ket vectors that are the origins of these maps. If the map images the adjunct space onto the original vector space, then the bra vectors may be mapped onto the corresponding ket vector.

17.7.1.3 Bra-ket combination

For the bra-ket combination holds

$$\langle\mathbf{f}|\mathbf{g}\rangle = \langle\mathbf{g}|\mathbf{f}\rangle^* \tag{17.7.7}$$

For quaternionic numbers α and β hold

$$\langle \alpha\mathbf{f} \mid \mathbf{g}\rangle = \langle \mathbf{g} \mid \alpha\mathbf{f}\rangle^* = \left(\langle \mathbf{g} \mid \mathbf{f}\rangle \alpha\right)^* = \alpha^* \langle \mathbf{f} \mid \mathbf{g}\rangle \tag{17.7.8}$$

$$\langle \mathbf{f} \mid \beta\mathbf{g}\rangle = \langle \mathbf{f} \mid \mathbf{g}\rangle \beta \tag{17.7.9}$$

$$\begin{aligned} \langle (\alpha+\beta)\mathbf{f} \mid \mathbf{g}\rangle &= \alpha^* \langle \mathbf{f} \mid \mathbf{g}\rangle + \beta^* \langle \mathbf{f} \mid \mathbf{g}\rangle \\ &= (\alpha+\beta)^* \langle \mathbf{f} \mid \mathbf{g}\rangle \end{aligned} \tag{17.7.10}$$

This corresponds with (17.7.3) and (17.7.6)

$$\langle \alpha\mathbf{f}| = \alpha^* \langle \mathbf{f}| \tag{17.7.11}$$

$$|\alpha\mathbf{g}\rangle = |\mathbf{g}\rangle \alpha \tag{17.7.12}$$

We made a choice. Another possibility would be $\langle \alpha\mathbf{f}| = \alpha\langle \mathbf{f}|$ and $|\alpha\mathbf{g}\rangle = |\mathbf{g}\rangle \alpha^*$

17.7.1.4 Operator construction

$|\mathbf{f}\rangle\langle \mathbf{g}|$ is a constructed operator.

$$|\mathbf{g}\rangle\langle \mathbf{f}| = \left(|\mathbf{f}\rangle\langle \mathbf{g}|\right)^\dagger \tag{17.7.13}$$

The superfix † indicates the adjoint version of the operator.

For the orthonormal base $\{|q_i\rangle\}$ consisting of eigenvectors of the reference operator, holds

$$\langle q_n \mid q_m\rangle = \delta_{nm} \tag{17.7.14}$$

Eigenvectors belong to the underlying vector space. Eigenvalues belong to the natural parameter space which represents a selected version of the applied number system. The ***bra-ket method*** enables the definition of new operators that are defined by quaternionic functions.

$$\langle \mathbf{g} | F | \mathbf{h} \rangle = \sum_{i=1}^{N} \{ \langle \mathbf{g} | q_i \rangle F(q_i) \langle q_i | \mathbf{h} \rangle \} \quad (17.7.15)$$

The symbol F is used both for the operator F and the sampled quaternionic function $F(q)$. This enables the shorthand

$$F \equiv |q_i\rangle F(q_i) \langle q_i| \quad (17.7.16)$$

for operator F. It is evident that for the adjoint operator

$$F^{\dagger} \equiv |q_i\rangle F^{*}(q_i) \langle q_i| \quad (17.7.17)$$

For ***reference operator*** $\mathfrak{R}$ holds

$$\mathfrak{R} = |q_i\rangle q_i \langle q_i| \quad (17.7.18)$$

If $\{q_i\}$ consists of all rational values of the version of the quaternionic number system that Hilbert space $\mathfrak{H}$ applies then the eigenspace of $\mathfrak{R}$ represents the natural parameter space of the separable Hilbert space $\mathfrak{H}$. It is also the parameter space of the function $F(q)$ that defines the natural operator F in the formula (17.7.16). This formula turns the separable Hilbert space into a sampled function space.

17.7.1.5 Expected value

Any bra vector $\langle \mathbf{g}|$ can be written as a linear combination of the bra base vectors $\{\langle q_i|\}$.

$$\langle \mathbf{g}| = \sum_{i=1}^{N} \{ \langle \mathbf{g} | q_i \rangle \langle q_i | \} \quad (17.7.19)$$

Any ket vector $|\mathbf{g}\rangle$ can be written as a linear combination of the ket base vectors $\{|q_i\rangle\}$.

$$|\mathbf{g}\rangle = \sum_{i=1}^{N} \{ |q_i\rangle \langle q_i|\mathbf{g}\rangle \} \quad (17.7.20)$$

The eigenvalues are archived as a combination of a real value and a spatial value. These parts take independent dimensions. If the real parts are sequenced, then the sequence of eigenvalues represents an ongoing hopping path. If this ongoing hopping path recurrently regenerates the same hop landing location swarm, then the hop landing locations can be summed over the regeneration period in the cells of a dense spatial grid. The total sum results in a spatial center location. The sums in the cells describe a location density distribution. The center location acts as the expected spatial value of the hop landing locations. A hop landing location distribution will describe the hop landing location swarm. If the swarm covers a larger number of locations, then the description by the location density distribution will be more accurate. If the results for the grid cells are sampled over a larger part of the real numbers, then the describing location density distribution approaches a continuous function.

This means that $\left|\langle \mathbf{g}|\vec{q}_i\rangle\right|^2 = \langle \mathbf{g}|\vec{q}_i\rangle\langle \vec{q}_i|\mathbf{g}\rangle$ can take the role of a hop landing location distribution. Here, we only used the spatial parts of the eigenvalues.

The expected spatial value for operator $\mathfrak{R}$ and vector $\mathbf{g}$ is

$$\langle \mathfrak{R} \rangle_{\mathbf{g}} = \langle \mathbf{g}|\mathfrak{R}|\mathbf{g}\rangle = \sum_{i=1}^{N} \{ \langle \mathbf{g}|\vec{q}_i\rangle \vec{q}_i \langle \vec{q}_i|\mathbf{g}\rangle \} \quad (17.7.21)$$

The expected value plays its role in a series of subsequent observations or events. After sequencing the timestamps of the samples, the string of samples represents an ongoing hopping path. If the vector $\mathbf{g}$ aims at a special location inside the parameter space of the Hilbert space, then the mechanism that generates the ongoing hopping path recurrently regenerates a hop landing location swarm that is described by a stable location density distribution. For large values of N the location density distribution approaches a continuous function $\langle \mathbf{g}|\vec{q}\rangle\langle\vec{q}|\mathbf{g}\rangle$, and the distribution $\langle \mathbf{g}|\vec{q}\rangle$ can be interpreted as a probability amplitude. The square of the modulus of this probability amplitude is a probability density distribution. What these continuous functions approximately describe are discrete sets. The approach fits better if the number of elements in the set is larger and there exists a requirement that the coherence of the set is large. If at instant zero the vector $\mathbf{g}$ equals the eigenvector that belongs to eigenvalue zero, and the expectation value $\mathbf{g}$ also equals zero, then the hop landing locations $\{q_i\}$ will tend to stay awhile about the geometrical center of the Hilbert space. If the tendency lasts, then the vector $\mathbf{g}$ will act as a ***unique state vector*** of the Hilbert space.

To give the location density distribution a statistical sense, a stochastic selection process must be or have been active. That selection process is then represented by a footprint vector $|\mathbf{g}\rangle$ that varies over time. How $|\mathbf{g}\rangle$ varies over time is checked by the characteristic function of the selection process. The footprint vector is represented by a vector $\mathbf{g}$ in the underlying vector space. The Hilbert space can archive the life history of the footprint vector in the form of a cord of quaternionic eigenvalues from a dedicated footprint operator.

The state vector of the Hilbert space is a special footprint vector of the Hilbert space. It is the footprint vector that at every instant of time has the expectation value of zero. At instant zero the state vector equals the eigenvector that belongs to location zero. This still does not say everything about the essence of the required underlying stochastic selection mechanism. For example, this description does not explain the value and stability of the recurrence rate of the hop landing location swarm. It is not clear why the characteristic function of the stochastic mechanism is stable.

17.7.1.6 Operator types

I is used to indicate the identity operator.

For normal operator N holds $NN^{\dagger} = NN^{\dagger}$.

The normed eigenvectors of a normal operator form an orthonormal base of the Hilbert space.

For unitary operator U holds $UU^{\dagger} = U^{\dagger}U = I$

For Hermitian operator H holds $H = H^{\dagger}$

A normal operator N has a Hermitian part $\frac{N + N^{\dagger}}{2}$ and an anti-Hermitian part $\frac{N - N^{\dagger}}{2}$

For anti-Hermitian operator A holds $A = -A^{\dagger}$

A Hermitian operator has real eigenvalues. An anti-Hermitian operator has spatial eigenvalues.

The reference operator $\mathfrak{R}$ is a normal operator.

17.7.2 Uncountable number systems

Every infinite-dimensional separable Hilbert space owns a unique non-separable companion Hilbert space that embeds its separable partner. The non-separable Hilbert space allows operators that maintain eigenspaces that in every dimension and every spatial direction contain closed sets of rational and irrational eigenvalues. These eigenspaces are uncountable and behave as dynamic sticky continuums. These continuums can vibrate, deform, and expand.

Gelfand triple and ***Rigged Hilbert space*** are other names for the general non-separable Hilbert spaces.

In the non-separable Hilbert space, for operators with continuum eigenspaces, the bra-ket method turns from a summation into an integration.

$$\langle \mathbf{g} | F | \mathbf{h} \rangle \equiv \int\iiint \left\{ \langle \mathbf{g} | q \rangle F(q) \langle q | \mathbf{h} \rangle \right\} dV d\tau \qquad (17.7.22)$$

Here we omitted the enumerating subscripts that were used in the countable base of the separable Hilbert space. Instead, the integration applies the infinitesimal $dVd\tau$ that is taken from the continuum in the private parameter space.

The shorthand for the operator F is now

$$F \equiv |q\rangle F(q) \langle q| \qquad (17.7.23)$$

For eigenvectors $|q\rangle$, the function $F(q)$ defines as

$$F(q) = \langle q | Fq \rangle = \int\iiint \left\{ \langle q | q' \rangle F(q') \langle q' | q \rangle \right\} dV' d\tau' \qquad (17.7.24)$$

The function $F(q)$ is no longer sampled.

The reference operator $\mathcal{R}$ that provides the continuum natural parameter space as its eigenspace follows from

$$\langle \mathbf{g} | \mathcal{R} \mathbf{h} \rangle \equiv \int\iiint \left\{ \langle \mathbf{g} | q \rangle q \langle q | \mathbf{h} \rangle \right\} dV d\tau \qquad (17.7.25)$$

The corresponding shorthand is

$$\mathcal{R} \equiv |q\rangle q \langle q| \qquad (17.7.26)$$

The reference operator is a special kind of defined operator. Via the quaternionic functions that specify defined operators, the claim becomes clear that every infinite-dimensional separable Hilbert space owns a unique non-separable companion Hilbert space that can be considered to embed its separable companion.

The reverse bra-ket method combines Hilbert space operator technology with quaternionic function theory and indirectly with quaternionic differential and integral technology.

17.7.2.1 Expected spatial value

Like the situation in the separable Hilbert space, a grid overlay of the spatial part of the parameter space is applied to be able to integrate over the grid cells. The expected spatial value is averaged over a part of the real part of the parameter space.

In the non-separable Hilbert space, the expected spatial value is defined as an average over the spatial part of the parameter space.

$$\langle \mathfrak{R} \rangle_{\mathbf{g}} = \langle \mathbf{g} | \mathfrak{R} | \mathbf{g} \rangle = \iiint_{0} \{ \langle \mathbf{g} | q \rangle \vec{q} \langle q | \mathbf{g} \rangle \} dV \tag{17.7.27}$$

The real part of the parameter space is usually held fixed, and the integration is done over the spatial part of the parameter space.

The location density distribution is a continuous function with values corresponding to locations in the spatial part of the parameter space.

$$|\langle \mathbf{g} | q \rangle|^2 = \langle \mathbf{g} | q \rangle \langle q | \mathbf{g} \rangle \tag{17.7.28}$$

Thus, the variable $\vec{q}$ can be any value in the spatial part of the parameter space.

17.8 Fourier transform

A cosine function can be combined with a sine function that owns the same frequency into a complex-number valued exponential function.

The imaginary factor i belongs to the direction of that same direction-line.

$$\varphi(2\pi xp) = \cos(2\pi xp) + i\bullet\sin\left(2\pi xp\right) = \exp(i2\pi xp) \qquad (17.8.1)$$

This sum has the remarkable property that p resembles the partial differential change operator for the direction i of x

$$i\frac{\delta}{\delta x}\varphi = -2\pi p\varphi \qquad (17.8.2)$$

$$i\frac{\delta}{\delta p}\varphi = -2\pi x\varphi \qquad (17.8.3)$$

x and p are related via a Fourier transform [31].

In this section, we do not indicate in the exponentials the spatial direction number i with a vector cap. Instead, we use the convention that is applied in complex number versions of the exponential function.

The Fourier transform in a separable complex-number-based Hilbert space is given by the relation between $\psi(x)$ and $\tilde{\psi}\left(p_{xn}\right)$ in the sum

$$\psi(x) = \sum_{n=-\infty}^{\infty}\left\{\tilde{\psi}\left(p_{x,n}\right)e^{2\pi i x p_{x,n}}\left(p_{x,n+1} - p_{x,n}\right)\right\} \qquad (17.8.4)$$

In the limit where $\Delta p_x = \left(p_{x,n+1} - p_{x,n}\right) \to 0$ the sum becomes an integral

$$\psi(x) = \int_{-\infty}^{\infty}\left\{\tilde{\psi}\left(p_x\right)e^{2\pi i x p_x}\right\}dp_x \qquad (17.8.5)$$

The reverse Fourier transform runs as

$$\tilde{\psi}(p_x) = \int_{-\infty}^{\infty}\left\{\psi\left(x\right)e^{-2\pi i x p_x}\right\}dx \qquad (17.8.6)$$

In these formulas, the symbol i represents a normalized spatial number part of a complex number. i corresponds to the spatial direction that was selected for constructing the complex-number-based Hilbert space.

The function $e^{2\pi i x p_x}$ is an eigenfunction of the operator $\vec{p}_x = \vec{i}\,\frac{\partial}{\partial x}$ which is recognizable as part of the change operator (17.6.3).

$$\vec{i}\,\frac{\partial}{\partial x} e^{2\pi i x p_x} = 2\pi\,\vec{p}_x e^{2\pi i x p_x} \tag{17.8.7}$$

The eigenvalue p_x represents the eigenfunction and the eigenvector $\vec{p}_x$ in the change space. In the same sense, the function $e^{-2\pi i x p_x}$ is an eigenfunction of the position operator $-\vec{i}\,\frac{\partial}{\partial p_x}$ and corresponds with the eigenvalue x of that operator.

$$-\vec{i}\,\frac{\partial}{\partial p_x} e^{-2\pi i x p_x} = 2\pi x e^{-2\pi i x p_x} \tag{17.8.8}$$

The eigenvalue x represents the eigenfunction and the eigenvector x in the position space.

The Fourier transform of a Dirac delta function is

$$\tilde{\delta}(p_x) = \int_{-\infty}^{\infty} \left\{ \delta(x) e^{-2\pi i x p_x} \right\} dx = 1 \tag{17.8.9}$$

The inverse transform tells

$$\delta(x) = \int_{-\infty}^{\infty} \left\{ 1 \cdot e^{2\pi i x p_x} \right\} dp_x \tag{17.8.10}$$

$$\delta(x-a) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i(x-a)p_x} dp_x \tag{17.8.11}$$

$$e^{2\pi i p_x a} = \int_{-\infty}^{\infty} \delta(x-a) e^{2\pi i x p_x} dx \qquad (17.8.12)$$

The operator $\vec{p}_x = \vec{i}\,\frac{\partial}{\partial x}$ is often called the momentum operator for the spatial direction $\vec{i}$ of the coordinate x. $\vec{p}$ differs from the classical momentum which is defined as the product of velocity $\vec{v}$ and mass m. It is important to notice that every orthonormal base vector of the position space is a superposition of ALL orthonormal base vectors of the change space. Further, the norms of the superposition coefficients are all equal. Similarly, every orthonormal base vector of the change space is a superposition of ALL orthonormal base vectors of the position space. Again, the norms of the superposition coefficients are all equal. Thus, jumping between different bases completely randomizes the landing base vector.

Fourier transforms convert convolutions of functions into products of the Fourier transforms of the functions.

17.9 Uncertainty principle

The uncertainty principle states

$$\left(\int_{-\infty}^{\infty} (x-x_0)^2 \left|\psi(x)\right|^2 dx\right)\left(\int_{-\infty}^{\infty} (p_x - p_{x,0})^2 \left|\tilde{\psi}(p_x)\right|^2 dp_x\right) \geq \frac{1}{16\pi^2}$$

(17.9.1)

For a Gaussian distribution, the equality sign holds. The Fourier transform of a Gaussian distribution is again a Gaussian distribution that has a different standard deviation.

If $\psi(x)$ spreads, then $\tilde{\psi}(p_x)$ shrinks and vice versa.

17.10 Center of influence of actuators

The potential $V(r)$ describes the effect of a local response to an actual or virtual isotropic point-like actuator and reflects the work that must be done by an agent to bring a unit amount of the actuator influence from infinity to the considered location.

$$V(r) = \theta_p \varepsilon / r \tag{17.10.1}$$

Here θ_p represents the actuator influence. ε takes care of adaptation to physical units. r is the distance to the location of the point-like actuator.

A swarm of point-like actual or virtual actuators that superpose their potentials in the potential of a single actuator or virtual actuator produces a potential that viewed from a sufficient distance r has shape

$$V(r) = \Theta \varepsilon / r \tag{17.10.2}$$

Here Θ represents the actuator influence of the resulting actual or virtual actuator. r is the distance to the center of the actuator influence. This formula is valid at sufficiently large values of r such that the a swarm of actuators can be considered as a point-like object.

In a coherent swarm of actuating objects $\theta_i, i = 1,2,3,\ldots n$, each with static influence θ_i at locations r_i, the center of actuation $\vec{R}$ follows from

$$\sum_{i=1}^{n} \theta_i \left(\vec{r}_i - \vec{R} \right) = \vec{0} \tag{17.10.3}$$

Thus

$$\vec{R} = \frac{1}{\Theta} \sum_{i=1}^{n} \theta_i \vec{r}_i \tag{17.10.4}$$

Where

$$\Theta = \sum_{i=1}^{n} \theta_i \tag{17.10.5}$$

In the following, we will consider an ensemble of actuating objects that own a center of actuation $\vec{R}$ and a fixed combined actuation influence Θ as a single virtual actuation object that is located at $\vec{R}$. The separate actuators θ_i may differ because, at the instant of summation, the corresponding influence might have partly faded away.

$\vec{R}$ can be a dynamic location. In that case, the ensemble must move as one unit.

17.11 Forces

The first-order change of the quaternionic field can be divided into five separate partial changes. Some of these parts can compensate for each other.

Mathematically, the statement that in the first approximation nothing in the field ξ changes indicates that locally, the first-order partial differential $\nabla\xi$ will be equal to zero.

$$\zeta = \nabla\xi = \nabla_r\xi_r - \langle\vec{\nabla},\vec{\xi}\rangle + \vec{\nabla}\xi_r + \nabla_r\vec{\xi} \pm \vec{\nabla}\times\vec{\xi} = 0 \tag{17.11.1}$$

Thus

$$\zeta_r = \nabla_r\xi_r - \langle\vec{\nabla},\vec{\xi}\rangle = 0 \tag{17.11.2}$$

$$\vec{\zeta} = \vec{\nabla}\xi_r + \nabla_r\vec{\xi} \pm \vec{\nabla}\times\vec{\xi} = 0 \tag{17.11.3}$$

These formulas can be interpreted independently. For example, according to the equation (17.11.2), the variation in time of ξ_r can compensate the divergence of $\vec{\xi}$. The terms that are still eligible for change must together be equal to zero. For our purpose, the curl $\vec{\nabla}\times\vec{\xi}$ of the spatial field $\vec{\xi}$ is expected to be zero. The resulting terms of the equation (17.11.3) are

$$\nabla_r\vec{\xi} + \vec{\nabla}\xi_r = 0 \tag{17.11.4}$$

In the following text plays the role of the spatial field and ξ_r plays the role of the scalar potential of the considered object. The spatial part $\vec{\xi}$ conforms to the uniform speed of movement of the floating group of influenced objects. The main characteristic of this field is that it tries to keep its overall change at zero. The author calls ξ the ***conservation field***.

At a large distance r, we approximate this potential by using the formula

$$\zeta_r(r) \approx \frac{\Theta\varepsilon}{r} \tag{17.11.5}$$

The new artificial field $\xi = \left\{\frac{\Theta\varepsilon}{r}, \vec{v}\right\}$ considers a single uniformly moving influenced object or a set of influenced objects that move uniformly as a normal situation. It is a combination of scalar potential $\frac{\Theta\varepsilon}{r}$ and speed $\vec{v}$. This speed of movement is the relative speed between the floating platform and the background platform. At equilibrium this speed is uniform.

If the gradient of $\frac{\Theta\varepsilon}{r}$ differs from zero, then the artificial field $\left\{\frac{\Theta\varepsilon}{r}, \vec{v}\right\}$ tries to counteract this by changing field $\vec{v}$ into a field of accelerated objects $\vec{a}$.

$$\vec{a} = \dot{\vec{v}} = -\vec{\nabla}\left(\frac{\Theta\varepsilon}{r}\right) = \frac{\Theta\varepsilon\,\vec{r}}{|\vec{r}|^3} \tag{17.11.6}$$

In reverse, the accelerated spatial field $\vec{a}$ acts on actuator influences $\frac{\Theta\varepsilon}{r}$ that appear in its realm by afflicting a gradient to this potential.

Thus, if two uniformly moving actuator influences Θ_1 and Θ_2 exist in each other's neighborhood, then any disturbance of the equilibrium will cause the force $\vec{F}$

$$\vec{F}(\vec{r}_1 - \vec{r}_2) = \Theta_1 \vec{a} = \frac{\varepsilon\Theta_1\Theta_2(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3} = \frac{\varepsilon\Theta_1\Theta(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3} \tag{17.11.7}$$

The influenced objects own mass and can own electric charge. Electric charges only influence electric charges. Massive actuators only influence massive objects.

17.12 Deformation potentials

We consider the deformation potential to be zero at infinity. The deformation potential at a considered location is equal to the work (energy transferred) per unit mass that would be needed to move an object from infinity to that location. Isotropic pulses that deform the embedding field introduce an extra complication because the pulse response is a shock front that quickly fades away. Therefore we reinvestigate this kind of potential.

17.12.1 Center of deformation

If the actuator is a response to an isotropic pulse, then the deformation potential $V(r)$ describes the effect of a local response to an isotropic point-like actuator and reflects the work that must be done by an agent to bring a unit amount of the injected stuff from infinity back to the considered location.

$$V(r) = m_p G / r \qquad (17.12.1)$$

Here m_p represents the mass that corresponds to the full pulse response. G takes care of adaptation to physical units. r is the distance to the location of the pulse. The pulse response is a spherical shock front.

A stream of these deforming actuators recurrently regenerates a coherent swarm of embedding locations in the dynamic universe field. Viewed from a sufficient distance r that swarm generates a potential

$$V(r) = MG / r \qquad (17.12.2)$$

Here M represents the mass that corresponds to the considered swarm of pulse responses. r is the distance to the center of the deformation. This formula is valid at sufficiently large values of r such that the whole swarm can be considered as a point-like object.

In a coherent swarm of massive objects $p_i, i=1,2,3,\ldots n$, each with static mass m_i at locations r_i, the center of mass $\vec{R}$ follows from

$$\sum_{i=1}^{n} m_i \left(\vec{r}_i - \vec{R}\right) = \vec{0} \qquad (17.12.3)$$

Thus

$$\vec{R} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i \qquad (17.12.4)$$

Where

$$M = \sum_{i=1}^{n} m_i \qquad (17.12.5)$$

In the following, we will consider an ensemble of massive objects that own a center of mass $\vec{R}$ and a fixed combined mass M as a single massive object that is located at $\vec{R}$. The separate masses m_i may differ because, at the instant of summation, the corresponding deformation might have partly faded away.

$\vec{R}$ can be a dynamic location. In that case, the ensemble must move as one unit. The problem with the treatise in this paragraph is that in physical reality, point-like objects that possess a static mass do not exist. Only pulse responses that temporarily deform the field exist. Except for black holes, these pulse responses constitute all massive objects that exist in the universe.

17.12.2 Pulse location density distribution

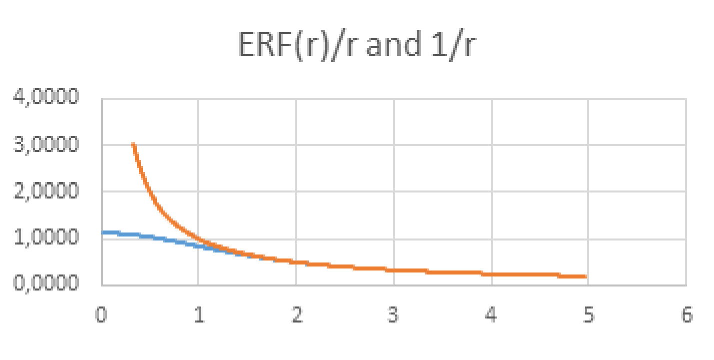
It is false to treat a pulse location density distribution as a set of point-like masses as is done in formulas (17.12.3) and (17.12.4). Instead, the deformation potential follows from the convolution of the location density distribution and the green's function. This calculation is still not correct, because the exact result depends on the fact that the deformation that is due to a pulse response quickly fades away and the result also depends on the density of the distribution. If these effects can be ignored, then the resulting deformation potential of a Gaussian density distribution would be given by

$$g(r) \approx GM \frac{ERF(r)}{r} \tag{17.13.1}$$

Where $ERF(r)$ is the well-known error function. Here the deformation potential is a perfectly smooth function that at some distance from the center equals the approximated deformation potential that was described above in the equation (17.12.2). As indicated above, the convolution only offers an approximation because this computation does not account for the influence of the density of the swarm, and it does not compensate for the fact that the deformation by the individual pulse responses quickly fades away. Thus, the exact result depends on the duration of the recurrence cycle of the swarm.

In the example, we apply a normalized location density distribution, but the actual location density distribution might have a higher amplitude.

This might explain why some elementary module types exist in multiple generations. These generations appear to have their mass. For example, elementary fermions exist in three generations. The two more massive generations usually get the name muon or tau generation.



This might also explain why different first-generation elementary particle types show different masses. Due to the convolution, and the coherence of the location density distribution, the blue curve does not show any sign of the singularity that is contained in the red curve, which shows the green's function.

In physical reality, no point-like static mass object exists. The most important lesson of this investigation is that far from the deformation center of the distribution the deformation of the field is characterized by the here shown simplified form of the deformation potential

$$\phi(r) \approx \frac{GM}{r} \tag{17.13.2}$$

Warning: This simplified form shares its shape with the green's function of the deformed field. This does not mean that the green's function owns a mass that equals $M_G = \frac{1}{G}$. The functions only share the form of their tail.

17.12.3 Rest mass

The weakness in the definition of the deformation potential is the definition of the unit of mass and the fact that shock fronts move with a fixed finite speed. Thus, the definition of the deformation potential only works properly if the geometric center location of the swarm of injected spherical pulses is at rest in the affected embedding field. The consequence is that the mass that follows from the definition of the deformation potential is the ***rest mass*** of the considered swarm. We will call the mass that is corrected for the motion of the observer relative to the observed scene the ***inertial mass***.

17.12.4 Observer

The inspected location is the location of a hypothetical test object that owns an amount of mass. It can represent an elementary particle or a conglomerate of such particles. This location is the target location in the embedding field. The embedding field is supposed to be deformed by the embedded objects.

Observers can access information that is retrieved from storage locations that for them have a historic timestamp. That information is transferred to them via the dynamic universe field. This dynamic field embeds both the observer and the observed event. The dynamic geometric data of point-like objects are archived in Euclidean format as a combination of a timestamp and a three-dimensional spatial location. The embedding field affects the format of the transferred information. The observers perceive in spacetime format. A hyperbolic Lorentz transform converts the Euclidean coordinates of the background parameter space into the spacetime coordinates that are perceived by the observer.

17.12.4.1 Lorentz transform

In dynamic fields, shock fronts move with speed c. In the quaternionic setting, this speed is unity.

$$x^2 + y^2 + z^2 = c^2\tau^2 \qquad (17.15.1)$$

In flat dynamic fields, swarms of triggers of spherical pulse responses move with lower speed v.

For the geometric centers of these swarms still holds:

$$x^2 + y^2 + z^2 - c^2\tau^2 = x'^2 + y'^2 + z'^2 - c^2\tau'^2 \qquad (17.15.2)$$

If the locations $\{x,y,z\}$ and $\{x',y',z'\}$ move with uniform relative speed v, then

$$ct' = ct\cosh(\omega) - x\sinh(\omega) \tag{17.15.3}$$

$$x' = x\cosh(\omega) - ct\sinh(\omega) \tag{17.15.4}$$

$$\cosh(\omega) = \frac{\exp(\omega)+\exp(-\omega)}{2} = \frac{c}{\sqrt{c^2 - v^2}} \tag{17.15.5}$$

$$\sinh(\omega) = \frac{\exp(\omega)-\exp(-\omega)}{2} = \frac{v}{\sqrt{c^2 - v^2}} \tag{17.15.6}$$

$$\cosh(\omega)^2 - \sinh(\omega)^2 = 1 \tag{17.15.7}$$

This is a hyperbolic transformation that relates two coordinate systems, which is known as a Lorentz boost.

This transformation can concern two platforms P and P' on which swarms reside and that move with uniform relative speed.

However, it can also concern the storage location P that contains a timestamp τ and spatial location $\{x,y,z\}$ and platform P' that has coordinate time t' and location $\{x',y',z'\}$.

In this way, the hyperbolic transform relates two platforms that move with uniform relative speed. One of them may be a floating Hilbert space on which the observer resides. Or it may be a cluster of such platforms that cling together and move as one unit. The other may be the background platform on which the embedding process produces the image of the footprint.

The Lorentz transform converts a Euclidean coordinate system consisting of a location $\{x,y,z\}$ and proper timestamps τ into the perceived coordinate system that consists of the spacetime coordinates $\{x',y',z',ct'\}$ in which t' plays the role of coordinate time. The uniform velocity v causes time dilation $\Delta t' = \frac{\Delta\tau}{\sqrt{1-\frac{v^2}{c^2}}}$ and length contraction $\Delta L' = \Delta L\sqrt{1-\frac{v^2}{c^2}}$

17.12.4.2 Minkowski metric

Spacetime is ruled by the Minkowski metric.

In flat field conditions, proper time τ is defined by

$$\tau = \pm \frac{\sqrt{c^2 t^2 - x^2 - y^2 - z^2}}{c} \tag{17.15.8}$$

And in deformed fields, still

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \tag{17.15.9}$$

Here ds is the spacetime interval and $d\tau$ is the proper time interval. dt is the coordinate time interval

17.12.4.3 Schwarzschild metric

Polar coordinates convert the Minkowski metric to the Schwarzschild metric. The proper time interval $d\tau$ obeys

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2 d\varphi^2\right) \tag{17.15.10}$$

Under pure isotropic conditions, the last term on the right side vanishes.

According to mainstream physics, in the environment of a black hole, the symbol r_s stands for the Schwarzschild radius.

$$r_s = \frac{2GM}{c^2} \tag{17.15.11}$$

The variable r equals the distance to the center of mass of the massive object with mass M .

The Hilbert Book model finds a different value for the boundary of a spherical black hole. That radius is a factor of two smaller.

17.12.4.4 Event horizon

The deformation potential energy $U(r)$

$$U(r) = \frac{mMG}{r} \tag{17.15.12}$$

at the event horizon $r = r_{eh}$ of a black hole is supposed to be equal to the mass-energy equivalent of an object that has unit mass $m = 1$ and is brought by an agent from infinity to that event horizon. Dark energy objects are energy packages in the form of one-dimensional shock fronts that are a candidate for this role. Photons are strings of equidistant samples of these energy packages. The energy equivalent of the unit mass objects is

$$E = mc^2 = \frac{mMG}{r_{eh}} \tag{17.15.13}$$

Or

$$r_{eh} = \frac{MG}{c^2} \tag{17.15.14}$$

At the event horizon, all energy of the dark energy object is consumed to compensate for the deformation potential energy at that location. No field excitation and in particular no shock front can pass the event horizon.

17.12.5 Inertial mass

The Lorentz transform also gives the transform of the rest mass to the mass that is relevant when the embedding field moves relative to the floating platform of the observed object with uniform speed $\vec{v}$.

In that case, the inertial mass M relates to the test mass M_0 as

$$M = \gamma M_0 = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{17.16.1}$$

This indicates that the formula (17.12.2) for the deformation potential at distance r must be changed to

$$V(r)=\frac{M_0 G}{r\sqrt{1-\frac{v^2}{c^2}}} \tag{17.16.2}$$

17.12.6 Inertia

The relation between inertia and mass is complicated. We apply an artificial field that resists its change. The condition that for each type of massive object, the deformation potential is a static function, and the condition that in free space, the massive object moves uniformly, establish that inertia rules the dynamics of the situation. These conditions define an artificial quaternionic field that resists change. The scalar part of the artificial field is represented by the deformation potential, and the uniform speed of the massive object represents the spatial part of the field.

The first-order change of the quaternionic field can be divided into five separate partial changes. Some of these parts can compensate for each other.

Mathematically, the statement that in the first approximation nothing in the field ξ changes indicates that locally, the first-order partial differential $\nabla\xi$ will be equal to zero.

$$\zeta=\nabla\xi=\nabla_r\xi_r-\left\langle\vec{\nabla},\vec{\xi}\right\rangle+\vec{\nabla}\xi_r+\nabla_r\vec{\xi}\pm\vec{\nabla}\times\vec{\xi}=0 \tag{17.17.1}$$

Thus

$$\zeta_r=\nabla_r\xi_r-\left\langle\vec{\nabla},\vec{\xi}\right\rangle=0 \tag{17.17.2}$$

$$\vec{\zeta}=\vec{\nabla}\xi_r+\nabla_r\vec{\xi}\pm\vec{\nabla}\times\vec{\xi}=0 \tag{17.17.3}$$

These formulas can be interpreted independently. For example, according to the equation (17.17.2), the variation in time of ξ_r can compensate the divergence of $\vec{\xi}$. The terms that are still eligible for change must together be equal to zero. For our purpose, the curl $\vec{\nabla} \times \vec{\xi}$ of the spatial field $\vec{\xi}$ is expected to be zero. The resulting terms of the equation (17.17.3) are

$$\nabla_r \vec{\xi} + \vec{\nabla} \xi_r = 0 \tag{17.17.4}$$

In the following text plays $\vec{\xi}$ the role of the spatial field and ξ_r plays the role of the scalar deformation potential of the considered object. For elementary modules, this special field concerns the effect of the hop landing location swarm that resides on the floating platform on its image in the embedding field. It reflects the activity of the stochastic process and the uniform movement of the geometric center of the floating platform over the embedding field in the background platform. It is characterized by a mass value and by the uniform velocity of the floating platform concerning the background platform. The real (scalar) part conforms to the deformation that the stochastic process causes. The spatial part conforms to the speed of movement of the floating platform. The main characteristic of this field is that it tries to keep its overall change at zero. The author calls ξ the ***conservation field***.

At a large distance r, we approximate this potential by using the formula

$$\zeta_r(r) \approx \frac{GM}{r} \tag{17.17.5}$$

Here M is the inertial mass of the object that causes the deformation. The new artificial field $\xi = \left\{ \frac{GM}{r}, \vec{v} \right\}$ considers a uniformly moving mass as a normal situation. It is a combination of scalar potential $\frac{GM}{r}$ and speed $\vec{v}$. This speed of movement is the relative speed between the floating platform and the background platform. At rest this speed is uniform.

If this object accelerates, then the new field $\left\{\frac{GM}{r},\vec{v}\right\}$ tries to counteract the change of the spatial field $\vec{v}$ by compensating this with an equivalent change of the scalar part $\frac{GM}{r}$ of the new field ξ. According to the equation (17.17.4), this equivalent change is the gradient of the real part of the field.

$$\vec{a}=\dot{\vec{v}}=-\vec{\nabla}\left(\frac{GM}{r}\right)=\frac{GM\,\vec{r}}{|\vec{r}|^{3}} \qquad (17.17.6)$$

This generated spatial field acts on masses that appear in its realm.

Thus, if two uniformly moving masses m and M exist in each other's neighborhood, then any disturbance of the situation will cause the deformation force

$$\vec{F}\left(\vec{r}_1-\vec{r}_2\right)=m_0\vec{a}=\frac{Gm_0M\left(\vec{r}_1-\vec{r}_2\right)}{\left|\vec{r}_1-\vec{r}_2\right|^{3}}=\gamma\frac{Gm_0M_0\left(\vec{r}_1-\vec{r}_2\right)}{\left|\vec{r}_1-\vec{r}_2\right|^{3}} \qquad (17.17.7)$$

Here $M=\gamma M_0$ is the inertial mass of the object that causes the deformation. m_0 is the rest mass of the observer.

The inertial mass M relates to its rest mass M_0 as

$$M=\gamma M_0=\frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (17.17.8)$$

This formula holds for all elementary particles except for quarks.

The problem with quarks is that these particles do not provide an isotropic symmetry difference. They must first combine into hadrons to be able to generate an isotropic symmetry difference. This phenomenon is known as ***color confinement***.

17.12.7 Momentum

In the formula (17.17.7) that relates mass to force the factor γ that corrects for the relative speed can be attached to m_0 or to M_0

$$\vec{F}\left(\vec{r}_1 - \vec{r}_2\right) = \gamma \frac{G m_0 M_0 \left(\vec{r}_1 - \vec{r}_2\right)}{\left|\vec{r}_1 - \vec{r}_2\right|^3} \quad (17.18.1)$$

The force relates to the temporal change of the momentum vector $\vec{P}$ of the observer

$$\vec{F} = \dot{\vec{P}} = \frac{d\vec{P}}{dt} \quad (17.18.2)$$

The momentum vector $\vec{P}$ is part of a quaternionic momentum P. The momentum depends on the relative speed of the moving object that causes the deformation which defines the mass. The speed is determined relative to the field that embeds the object and that gets deformed by the investigated object. For free elementary particles, the speed equals the floating speed of the platform on which the particle resides.

$$P = P_r + \vec{P} \quad (17.18.3)$$

$$\|P\|^2 = P_r^2 + \|\vec{P}\|^2 \quad (17.18.4)$$

$$\vec{P} = \gamma m_0 \vec{v} \quad (17.18.5)$$

$$\|\vec{P}\|^2 = \gamma^2 m_0^2 \|\vec{v}\|^2 \quad (17.18.6)$$

$$\|P\|^2 = \gamma^2 m_0^2 c^2 = P_r^2 + \gamma^2 m_0^2 \|\vec{v}\|^2 \quad (17.18.7)$$

$$\|P\| = \gamma m_0 c = E / c \quad (17.18.8)$$

$$E = \gamma m_0 c^2 \quad (17.18.9)$$

$$P_r^2 = \gamma^2 m_0^2 c^2 - \gamma^2 m_0^2 \|\vec{v}\|^2$$
$$= \gamma^2 m_0^2 \left(c^2 - \|\vec{v}\|^2\right) = \gamma^2 m_0^2 c^2 \left(1 - \left\|\frac{\vec{v}}{c}\right\|^2\right) = m_0^2 c^2 \tag{17.18.10}$$

$$P_r = m_0 c = \frac{E}{\gamma c} \tag{17.18.11}$$

$$\|\vec{P}\| = \gamma m_0 \|\vec{v}\| \tag{17.18.12}$$

$$P = P_r + \vec{P} = m_0 c + \gamma m_0 \vec{v} = \frac{E}{\gamma c} + \gamma m_0 \vec{v} \tag{17.18.13}$$

If $\vec{v} = \vec{0}$ then $\vec{P} = \vec{0}$ and $\|P\| = P = P_r = m_0 c$

Here Einstein’s famous mass-energy equivalence is involved.

$$E = \gamma m_0 c^2 = mc^2 \tag{17.18.14}$$

The disturbance by the ongoing expansion of the embedding field suffices to put the deformation force into action. The description also holds when the field ξ describes a conglomerate of platforms and M represents the mass of the conglomerate.

The artificial field ξ represents the habits of the underlying model that ensures the constancy of the deformation potential and the uniform floating of the considered massive objects in free space.

Inertia ensures that the third-order differential (the third-order change) of the deformed field is minimized. It does that by varying the speed of the platforms on which the massive objects reside.

Inertia bases mainly on the definition of mass that applies to the region outside the sphere where the deformation potential behaves like the green's function of the field. There, the formula $\xi_r = \frac{GM}{r}$ applies. Further, it bases on the intention of modules to keep the deformation potential inside the mentioned sphere constant. At least that holds when this potential is averaged over the regeneration period. In that case, the overall change $\nabla\xi$ in the conservation field ξ equals zero. Next, the definition of the conservation field supposes that the swarm which causes the deformation moves as one unit. Further, the fact is used that the solutions of the homogeneous second-order partial differential equation can superpose in new solutions of that same equation.

The popular sketch in which the deformation of our living space is presented by smooth dips is obviously false. The story that is represented in this paper shows the deformations as local extensions of the field, which represents the universe. In both sketches, the deformations elongate the information path, but none of the sketches explain why two masses attract each other. The above explanation founds on the habit of the stochastic process to recurrently regenerate the same time average of the deformation potential, even when that averaged potential moves uniformly. Without the described habit of the stochastic processes, inertia would not exist.

The applied artificial field also explains the deformation attraction by black holes.

The artificial field that implements mass inertia also plays a role in other fields. Similar tricks can be used to explain the electrical force from the fact that the electrical field is produced by sources and sinks that can be described with the green's function.

17.12.7.1 Forces

In the system of separable Hilbert spaces, all symmetry-related charges are located at the geometric center of an elementary particle and all these particles own a footprint that for isotropic symmetry differences can deform the embedding field. In that case, the particle features mass and forces might be coupled to acceleration via

$$F = m\vec{a} \tag{17.18.15}$$

Or to momentum via

$$F = \dot{\vec{P}} \tag{17.18.16}$$

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