

# Dimensionless Unification of the Fundamental Interactions

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## Abstract

In this paper in an elegant way will be present the simple unification of the fundamental interactions. We will calculate the unity formulas that connect the coupling constants of the fundamental forces. We will find the formulas for the Gravitational constant. It will be presented that the gravitational fine-structure constant is a simple analogy between atomic physics and cosmology. We will find the expression that connects the gravitational fine-structure constant with the four coupling constants. Perhaps the gravitational fine-structure constant is the coupling constant for the fifth force. Also will be presented the simple unification of atomic physics and cosmology. We will find the formulas for the cosmological constant and we will propose a possible solution for the cosmological parameters. Perhaps the shape of the universe is Poincaré dodecahedral space. Finally will be presented the law of the gravitational fine-structure constant followed by ratios of maximum and minimum theoretical values for natural quantities. This article will be followed by the energy wave theory and the fractal space-time theory.

## Keywords

Fine-structure constant , Proton to electron mass ratio , Dimensionless physical constants , Coupling constant , Gravitational constant , Avogadro's number , Fundamental Interactions , Gravitational fine-structure constant , Cosmological parameters , Cosmological constant

## 1. Introduction

In ancient Greek philosophy, the pre-Socratic philosophers speculated that the apparent diversity of observed phenomena was due to a single type of interaction, namely the motions and collisions of atoms. The concept of “atom” proposed by Democritus was an early philosophical attempt to unify phenomena observed in nature. Archimedes was possibly the first philosopher to have described nature with axioms and then deduce new results from them. In the late 17th century, Isaac Newton's description of the long-distance force of gravity implied that not all forces in nature result from things coming into contact. Newton's work in his *Mathematical Principles of Natural Philosophy* dealt with this in a further example of unification, in this case unifying Galileo's work on terrestrial gravity, Kepler's laws of planetary motion and the phenomenon of tides by explaining these apparent actions at a distance under one single law: the law of universal gravitation

In 1814, building on these results, Laplace famously suggested that a sufficiently powerful intellect could, if it knew the position and velocity of every particle at a given time, along with the laws of nature, calculate the position of any particle at any other time. Laplace thus envisaged a combination of gravitation and mechanics as a theory of everything. In 1820, Hans Christian Ørsted discovered a connection between electricity and magnetism, triggering decades of work that culminated in 1865, in James Clerk Maxwell's theory of electromagnetism. In his experiments of 1849–50, Michael Faraday was the first to search for a unification of gravity with electricity and magnetism. However, he found no connection. In 1900, David Hilbert published a famous list of mathematical problems. In Hilbert's sixth problem, he challenged researchers to find an axiomatic basis to all of physics. In this problem he thus asked for what today would be called a theory of everything. After 1915, when Albert Einstein published the theory of gravity (general relativity), the search for a unified field theory combining gravity with electromagnetism began with a renewed interest. In Einstein's day, the strong and the weak forces had not yet been discovered, yet he found the potential existence of two other distinct forces, gravity and electromagnetism, far more alluring. coupling), is a number that determines the strength of the force exerted in an interaction. In attributing a relative strength to the four fundamental forces, it has proved useful to quote the strength in terms of a coupling constant. The coupling constant

for each force is a dimensionless constant.

In physics, a coupling constant or gauge coupling parameter is a number that determines the strength of the force exerted in an interaction. In attributing a relative strength to the four fundamental forces, it has proved useful to quote the strength in terms of a coupling constant. The coupling constant for each force is a dimensionless constant. A coupling constant is a parameter in field theory, which determines the relative strength of interaction between particles or fields. In the quantum field theory the coupling constants are associated with the vertices of the corresponding Feynman diagrams. Dimensionless parameters are used as coupling constants, as well as the quantities associated with them that characterize the interaction and have dimensions. In the paper [16] we presented the unity formulas for the coupling constants and the dimensionless physical constants.

In physics, the fundamental interactions, also known as fundamental forces, are the interactions that do not appear to be reducible to more basic interactions. There are four fundamental interactions known to exist: the gravitational and electromagnetic interactions, which produce significant long-range forces whose effects can be seen directly in everyday life, and the strong and weak interactions, which produce forces at minuscule, subatomic distances and govern nuclear interactions. Some scientists hypothesize that a fifth force might exist, but these hypotheses remain speculative. Each of the known fundamental interactions can be described mathematically as a field. The gravitational force is attributed to the curvature of spacetime, described by Einstein's general theory of relativity. The other three are discrete quantum fields, and their interactions are mediated by elementary particles described by the Standard Model of particle physics.

Archimedes constant  $\pi$  is a mathematical constant that appears in many types in all fields of mathematics and physics. It is also found in various types from other disciplines, such as Cosmology, numbers, Statistics, fractals, thermodynamics, engineering, and electromagnetism. Archimedes constant  $\pi$  appears in the cosmological constant, Heisenberg's uncertainty principle, Einstein's field equation of general relativity, Coulomb's law for the electric force in vacuum, Magnetic permeability of free space, Period of a simple pendulum with small amplitude, Kepler's third law of planetary motion, the buckling formula, etc.

Golden ratio  $\phi$  is an omnipresent number in nature, found in the architecture of living creatures as well as human buildings, music, finance, medicine, philosophy, and of course in physics and mathematics including quantum computation. It is the most irrational number known and a number-theoretical chameleon with a self-similarity property. The golden ratio can be found in nearly all domains of Science, appearing when self-organization processes are at play and/or expressing minimum energy configurations. Several non-exhaustive examples are given in biology (natural and artificial phyllotaxis, genetic code and DNA), physics (hydrogen bonds, chaos, superconductivity), astrophysics (pulsating stars, black holes), chemistry (quasicrystals, protein AB models), and technology (tribology, resistors, quantum computing, quantum phase transitions, photonics). The fifth power of the golden mean appears in Phase transition of the hard hexagon lattice gas model, Phase transition of the hard square lattice gas model, One-dimensional hard-boson model, Baryonic matter relation according to the E-infinity theory, Maximum quantum probability of two particles, Maximum of matter energy density, Reciprocity relation between matter and dark matter, Superconductivity phase transition, etc.

Euler's number  $e$  is an important mathematical constant, which is the base of the natural logarithm. All five of these numbers play important and repetitive roles in mathematics and these five constants appear in a formulation of Euler's identity. Euler's number has many practical uses, especially in higher level mathematics such as calculus, differential equations, trigonometry, complex analysis, statistics, etc. Euler's number frequently appears in problems related to growth or decay, where the rate of change is determined by the present value of the number being measured. One example is in biology, where bacterial populations are expected to double at reliable intervals. Another case is radiometric dating, where the number of radioactive atoms is expected to decline over the fixed half-life of the element being measured. From Euler's identity the following relation of the mathematical constant  $e$  can emerge  $e = i^{-2i/\pi}$ .

Gelfond's constant, in mathematics, is the number  $e^\pi$ ,  $e$  raised to the power  $\pi$ . Like  $e$  and  $\pi$ , this constant is a transcendental number. It was named after the Soviet mathematician Aleksandr Gelfond. Gelfond's constant were singled out in Hilbert's 7th problem as an example of numbers whose excess was an open problem. This was first established by Gelfond and may now be considered as an application of the Gelfond-Schneider theorem, noting that  $e^\pi = (e^{i\pi})^{-i} = (-1)^{-i} = i^{-2i}$ .

Euler's constant is a mathematical constant usually denoted by the lowercase Greek letter gamma ( $\gamma$ ). The number  $\gamma$  has not been proved algebraic or transcendental. In fact, it is not even known whether  $\gamma$  is irrational. The numerical value of Euler's constant is  $\gamma = 0,57721566490153286\dots$

The imaginary unit  $i$  is a solution to the quadratic equation  $x^2 + 1 = 0$ . Although there is no real number with this property, it can be used to extend the real numbers to what are called complex numbers, using addition and multiplication. Despite their misleading name, imaginary numbers are not only real but also very useful, with application in electricity, signal processing and many other applications. They are widely used in electronics, for the

representation of alternating currents and in waves.

Euler's identity is considered to be an exemplar of mathematical beauty as it shows a profound connection between the most fundamental numbers in mathematics:

$$e^{i\pi} + 1 = 0$$

The expression who connects the six basic mathematical constants, the number 0, the number 1, the golden ratio  $\phi$ , the Archimedes constant  $\pi$ , the Euler's number  $e$  and the imaginary unit  $i$  is:

$$e^{\frac{i\pi}{1+\phi}} + e^{\frac{-i\pi}{1+\phi}} + e^{\frac{i\pi}{\phi}} + e^{\frac{-i\pi}{\phi}} = 0$$

In [9] we presented exact and approximate expressions between the Archimedes constant  $\pi$ , the golden ratio  $\phi$ , the Euler's number  $e$  and the imaginary number  $i$ .

## 2. Dimensionless physical constants

In physics, a dimensionless physical constant is a physical constant that is dimensionless, a pure number having no units attached and having a numerical value that is independent of whatever system of units may be used. The term fundamental physical constant is used to refer to some universal dimensionless constants. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values. In the 1920s and 1930s, Arthur Eddington embarked upon extensive mathematical investigation into the relations between the fundamental quantities in basic physical theories, later used as part of his effort to construct an overarching theory unifying quantum mechanics and cosmological physics. The mathematician Simon Plouffe has made an extensive search of computer databases of mathematical formulas, seeking formulas for the mass ratios of the fundamental particles. An empirical relation between the masses of the electron, muon and tau has been discovered by physicist Yoshio Koide, but this formula remains unexplained.

Dimensionless physical constants cannot be derived and have to be measured. Developments in physics may lead to either a reduction or an extension of their number: discovery of new particles, or new relationships between physical phenomena, would introduce new constants, while the development of a more fundamental theory might allow the derivation of several constants from a more fundamental constant. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values. The laws of physics have a set of fundamental constants, and it is generally admitted that only dimensionless combinations of constants have physical significance. These combinations include the electromagnetic and gravitational fine structure, along with the ratios of elementary particles masses. Cosmological measurements clearly depend on the values of these constants in the past and can therefore give information on their time dependence if the effects of time-varying constants can be separated from the effects of cosmological parameters.

The fine-structure constant is one of the most fundamental constants of physics. It describes the strength of the force of electromagnetism between elementary particles in what is known as the standard model of particle physics. In particular, the fine-structure constant sets the strength of electromagnetic interaction between light (photons) and charged elementary particles such as electrons and muons. The quantity  $\alpha$  was introduced into physics by A. Sommerfeld in 1916 and in the past has often been referred to as the Sommerfeld fine-structure constant. In order to explain the observed splitting or fine structure of the energy levels of the hydrogen atom, Sommerfeld extended the Bohr theory to include elliptical orbits and the relativistic dependence of mass on velocity.

One of the most important numbers in physics is the fine-structure constant  $\alpha$  which defines the strength of the electro-magnetic field. It is a dimensionless number independent of how we define our units of mass, length, time or electric charge. A change in these units of measurement leaves the dimensionless constant unchanged. The number can be seen as the chance that an electron emits or absorbs a photon. It's a pure number that shapes the universe to an astonishing degree. Paul Dirac considered the origin of the number «the most fundamental unsolved problem of physics». The constant is everywhere because it characterizes the strength of the electromagnetic force affecting charged particles such as electrons and protons. Many eminent physicists and philosophers of science have pondered why  $\alpha$  itself has the value that it does, because the value shows up in so many important scenarios and aspects of physics. Nobody has come up with any ideas that are even remotely convincing. A similar situation occurs with the proton-electron mass ratio  $\mu$ , not because of its ubiquity, but rather how chemistry can be based on two key electrically charged particles of opposite electric charge that are opposite but of seemingly identical magnitude while their masses have a ratio that is quite large yet finite. These two questions have a huge bearing on why physics and chemistry behave the way they do. The product of the two quantities appears, at least at first glance, not to be so important. The

fine-structure constant  $\alpha$  is defined as:

$$\alpha = \frac{q_e^2}{4\pi\epsilon_0\hbar c}$$

The 2018 CODATA recommended value of the fine-structure constant is  $\alpha=0,0072973525693(11)$  with standard uncertainty  $0,0000000011\times 10^{-3}$  and relative standard uncertainty  $1,5\times 10^{-10}$ . Also the fine-structure constant is universal scaling factor:

$$\alpha = \frac{2\pi r_e}{\lambda_e} = \frac{\lambda_e}{2\pi\alpha_0} = \frac{r_e}{l_{pl}} \frac{m_e}{m_{pl}} = \sqrt{\frac{r_e}{\alpha_0}}$$

There is a dream, which, albeit more often not confessed, occupies the most secret aspirations of theoreticians and is that of reducing the various constants of Physics to simple formula involving integers and transcendent numbers. The fine-structure constant plays an important role in modern physics. Yet it continues to be a mystery as to exactly what it represents and why it has the mystical value it has. The purpose of many sciences is to find the most accurate mathematical formula that can be found in the experimental value of fine-structure constant. Attempts to find a mathematical basis for this dimensionless constant have continued up to the present time. However, no numerological explanation has ever been accepted by the physics community. We propose in [10] the exact formula for the fine-structure constant  $\alpha$  with the golden angle, the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1} = 360 \cdot \varphi^{-2} - 2 \cdot \varphi^{-3} + (3 \cdot \varphi)^{-5} \quad (1)$$

with numerical value:

$$\alpha^{-1} = 137,035999164...$$

Another beautiful forms of the equations are:

$$\begin{aligned} \frac{1}{\alpha} &= \frac{360}{\varphi^2} - \frac{2}{\varphi^3} + \frac{1}{3^5\varphi^5} \\ \frac{1}{\alpha} &= \frac{360}{\varphi^2} - \frac{2}{\varphi^3} + \frac{3^{-5}}{\varphi^5} \end{aligned} \quad (2)$$

Other equivalent expressions for the fine-structure constant are:

$$\begin{aligned} \alpha^{-1} &= (362 - 3^{-4}) \cdot \varphi^{-2} - (1 - 3^{-5}) \cdot \varphi^{-1} \\ \alpha^{-1} &= (362 - 3^{-4}) + (3^{-4} + 2 \cdot 3^{-5} - 364) \cdot \varphi^{-1} \\ \alpha^{-1} &= 1 - 2 \cdot \varphi^{-1} + 360 \cdot \varphi^{-2} - \varphi^{-3} + (3 \cdot \varphi)^{-5} \end{aligned}$$

Also we propose in [11] a simple and accurate expression for the fine-structure constant  $\alpha$  in terms of the Archimedes constant  $\pi$ :

$$\alpha^{-1} = \frac{2.706}{43} \pi \ln 2 \quad (3)$$

with numerical value:

$$\alpha^{-1} = 137,035999078...$$

Other equivalent expression for the fine-structure constant is:

$$\alpha^{-1} = 2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1} \cdot \pi \cdot \ln 2 \quad (4)$$

The equivalent expressions for the fine-structure constant with the madelung constant  $b_2(2)$  are:

$$\alpha^{-1} = -\frac{2.706}{43} b_2(2) \quad (5)$$

$$\alpha^{-1} = -2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1} \cdot b_2(2) \quad (6)$$

In Physics, the ratio of the mass of a proton to an electron is simply the remainder of the mass of the proton divided by that of the electron, from the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious ratio of mass between a proton and an electron. The values of  $m_e$  and  $m_p$ , and the equilibrium between them, govern nuclear reactions such as the decay of protons and the nuclear synthesis of stars, leading to the formation of basic biochemical elements, including carbon. The space where stars and planets form and support life and molecular structures can appear. The mass ratio of protons to electrons, two constant particles that make up about 95% of the visible Universe, may be related to the total computational value of the Universe. Thus, as pure numbers they are supposed to be associated with prime numbers, entropy, binary and complexity. The proton to electron mass ratio  $\mu$  is a ratio of like-dimensioned physical quantities, it is a dimensionless quantity, a function of the dimensionless physical constants, and has numerical value independent of the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious mass ratio between the proton and electron. The numerical challenge of the mass ratio of proton to electron in the field of elementary particle physics began with the discovery of the electron by JJ Thomson in 1897, and with the identification of the point nature of the proton by E. Rutherford in 1911. These two particles have electric charges that are identical in size but opposite charges. The 2018 CODATA recommended value of the proton to electron mass ratio  $\mu$  is:

$$\mu = 1.836,15267343$$

with standard uncertainty 0,00000011 and relative standard uncertainty  $6,0 \times 10^{-11}$ . The value of  $\mu$  is a solution of the equation:

$$3 \cdot \mu^4 - 5.508 \cdot \mu^3 - 841 \cdot \mu^2 + 10 \cdot \mu - 2.111 = 0$$

We propose in [12] the exact mathematical expression for the proton to electron mass ratio using Fibonacci and Lucas numbers:

$$\mu^{32} = \phi^{-42} \cdot F_5^{160} \cdot L_5^{47} \cdot L_{19}^{40/19} \quad (7)$$

with numerical value:

$$\mu = 1.836,15267343...$$

Also we propose in [12] the exact mathematical expression for the proton to electron mass ratio:

$$\mu = 165 \sqrt[3]{\frac{\ln^{11} 10}{7}} \quad (8)$$

with numerical value:

$$\mu = 1836,15267392...$$

Other equivalent expressions for the proton to electron mass ratio are:

$$\mu^3 = 7^{-1} \cdot 165^3 \cdot \ln^{11} 10$$

$$7 \cdot \mu^3 = (3 \cdot 5 \cdot 11)^3 \cdot \ln^{11}(2 \cdot 5) \quad (9)$$

Also other exact mathematical expression in [12] for the proton to electron mass ratio is:

$$\mu = 6 \cdot \pi^5 + \pi^{-3} + 2 \cdot \pi^{-6} + 2 \cdot \pi^{-8} + 2 \cdot \pi^{-10} + 2 \cdot \pi^{-13} + \pi^{-15} \quad (10)$$

with numerical value:

$$\mu = 1.836,15267343...$$

In physics, the gravitational coupling constant  $\alpha_G$  is a constant that characterizes the gravitational pull between a given pair of elementary particles. For the electron pair this constant is denoted by  $\alpha_G$ . The choice of units of measurement, but only with the choice of particles. The gravitational coupling constant  $\alpha_G$  is a scaling ratio that can be used to compare similar unit values from different scaling systems (Planck scale, atomic scale, and cosmological scale). The gravitational coupling constant can be used for comparison of length, range and force values. The gravitational coupling constant  $\alpha_G$  is defined as:

$$\alpha_G = \frac{Gm_e^2}{\hbar c}$$

There is so far no known way to measure  $\alpha_G$  directly. The value of the constant gravitational coupling  $\alpha_G$  is only known in four significant digits. The approximate value of the constant gravitational coupling is  $\alpha_G = 1,7518099 \times 10^{-45}$ . Also the gravitational coupling constant is universal scaling factor:

$$\alpha_G = \frac{m_e^2}{m_{pl}^2} = \frac{\alpha_{G(p)}}{\mu^2} = \frac{\alpha}{\mu N_1} = \frac{\alpha^2}{N_1^2 \alpha_{G(p)}} = \left( \frac{2\pi l_{pl}}{\lambda_e} \right)^2 = \left( \alpha \frac{l_{pl}}{r_e} \right)^2 = \left( \frac{l_{pl}}{\alpha \alpha_0} \right)^2$$

The gravitational coupling constant  $\alpha_{G(p)}$  for the proton is produced similar to the electron, but replaces the mass of electrons with the mass of the protons. The gravitational coupling constant of the proton  $\alpha_{G(p)}$  is defined as:

$$\alpha_{G(p)} = \frac{Gm_p^2}{\hbar c}$$

The approximate value of the constant gravitational coupling of the proton is  $\alpha_{G(p)} = 5,9061512 \times 10^{-39}$ . Also other expression for the gravitational coupling constant is:

$$\alpha_{G(p)} = \frac{m_p^2}{m_{pl}^2} = \mu^2 \alpha_G = \frac{\alpha \mu}{N_1} = \frac{\alpha^2}{N_1^2 \alpha_G}$$

The enormous value of the ratio of electric force to gravitational force was first pointed out by Bergen Davis in 1904. But Weyl and Eddington suggested that the number was about  $10^{40}$  and was related to cosmological quantities. The ratio  $N_1$  of electric force to gravitational force between electron and proton is defined as:

$$N_1 = \frac{\alpha}{\mu \alpha_G} = \frac{\alpha \mu}{\alpha_{G(p)}} = \frac{\alpha}{\sqrt{\alpha_G \alpha_{G(p)}}} = \frac{k_e q_e^2}{G m_e m_p} = \frac{\alpha \hbar c}{G m_e m_p}$$

The approximate value of the ratio of electric force to gravitational force between electron and proton is  $N_1 = 2,26866072 \times 10^{39}$ . The ratio  $N_1$  of electric force to gravitational force between electron and proton can also be written in expression:

$$N_1 = \frac{5}{3} 2^{130} = 2,26854911 \times 10^{39}$$

According to current theories  $N_1$  should be constant. The ratio  $N_2$  of electric force to gravitational force between two electrons is defined as:

$$N_2 = \mu N_1 = \frac{\alpha}{\alpha_G} = \frac{N_1^2 \alpha_{G(p)}}{\alpha} = \frac{k_e q_e^2}{G m_e^2} = \frac{\alpha \hbar c}{G m_e^2}$$

The approximate value of  $N_2$  is  $N_2 = 4,16560745 \times 10^{42}$ . According to current theories  $N_2$  should grow with the expansion of the universe.

Avogadro's number  $N_A$  is defined as the number of carbon-12 atoms in twelve grams of elemental carbon-12 in its standard state. Avogadro's number  $N_A$  is the fundamental physical constant that links the macroscopic physical world of objects that we can see and feel with the submicroscopic, invisible world of atoms. The name honors the Italian mathematical physicist Amedeo Avogadro, who proposed that equal volumes of all gasses at the same temperature and pressure contain the same number of molecules. The most accurate definition of the Avogadro's number value involves the change in molecular quantities and, in particular, the change in the value of an elementary charge. The exact value of the Avogadro's number is  $N_A = 6,02214076 \times 10^{23}$ . The value of the Avogadro's number  $N_A$  can also be written in expressions:

$$N_A = 84.446.885^3 = 6,02214076 \times 10^{23}$$

$$N_A = 2^{79} = 6,04462909 \times 10^{23} \quad (11)$$

In [12] was presented the exact mathematical expressions that connects the proton to electron mass ratio  $\mu$  and the fine-structure constant  $\alpha$ :

$$9 \cdot \mu - 119 \cdot \alpha^{-1} = 5 \cdot (\varphi + 42) \quad (12)$$

$$\mu - 6 \cdot \alpha^{-1} = 360 \cdot \varphi - 165 \cdot \pi + 345 \cdot e + 12 \quad (13)$$

$$\mu - 182 \cdot \alpha = 141 \cdot \varphi + 495 \cdot \pi - 66 \cdot e + 231 \quad (14)$$

$$\mu - 807 \cdot \alpha = 1.205 \cdot \pi - 518 \cdot \varphi - 411 \cdot e \quad (15)$$

Also in [14] was presented the unity formula that connects the fine-structure constant and the proton to electron mass ratio. It was explained that  $\mu \cdot \alpha^{-1}$  is one of the roots of the following trigonometric equation:

$$2 \cdot 10^2 \cdot \cos(\mu \cdot \alpha^{-1}) + 13^2 = 0 \quad (16)$$

The exponential form of this equation is:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) + 13^2 = 0 \quad (17)$$

This exponential form can also be written with the beautiful form:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) = 13^2 \cdot e^{i\pi} \quad (18)$$

Also this unity formula can also be written in the form:

$$10 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha})^{1/2} = 13 \cdot i \quad (19)$$

So other beautiful formula that connects the fine-structure constant, the proton to electron mass ratio and the fifth power of the golden mean is:

$$5^2 \cdot (5 \cdot \varphi^{-2} + \varphi^{-5})^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) + (5 \cdot \varphi^2 - \varphi^{-5})^2 = 0 \quad (20)$$

The formula that connects the fine-structure constant, the proton to electron mass ratio and the mathematical constants  $\pi, \varphi, e$  and  $i$  is:

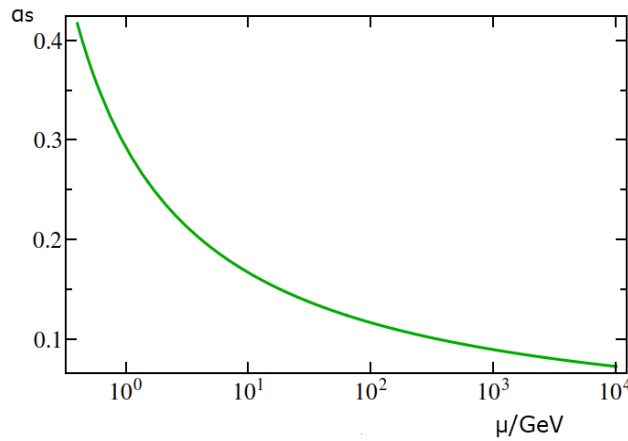
$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) = (5 \cdot \varphi^2 - \varphi^{-5})^2 \cdot e^{i\pi} \quad (21)$$

All these equations are simple, elegant and symmetrical in a great physical meaning.

### 3. Dimensionless unification of the strong nuclear and the weak nuclear interactions

In nuclear physics and particle physics, the strong interaction is one of the four known fundamental interactions, with the others being electromagnetism, the weak interaction, and gravitation. Strong force involves the exchange of huge particles and therefore has a very small range. It is clear that strong force is much stronger simply than the fact that the nuclear magnitude (dominant strong force) is about  $10^{-15}$  m while the atom (dominant electromagnetic force) has a size of about  $10^{-10}$  m. At the range of  $10^{-15}$  m, the strong force is approximately 137 times as strong as electromagnetism,  $10^6$  times as strong as the weak interaction, and  $10^{38}$  times as strong as gravitation. The strong coupling constant  $\alpha_s$  is one of the fundamental parameters of the typical model of particle physics.

The strong coupling constant  $\alpha_s$  is one of the fundamental parameters of the typical model of particle physics. The strong nuclear force confines quarks into hadron particles such as the proton and neutron. In addition, the strong force binds these neutrons and protons to create atomic nuclei, where it is called the nuclear force. Most of the mass of a common proton or neutron is the result of the strong force field energy; the individual quarks provide only about 1% of the mass of a proton. The electromagnetic force is infinite in range and obeys the inverse square law, while the strong force involves the exchange of massive particles and it therefore has a very short range. The value of the strong coupling constant, like other coupling constants, depends on the energy scale. As the energy increases, this constant decreases.



**Figure 1.** Strong coupling constant  $\alpha_s$  as a function of the energy.

The last measurement [24] in 23 November 2021 of European organization for nuclear research (CERN) is used in a comprehensive QCD analysis at next-to next-to-leading order, which results in significant improvement in the accuracy of the parton distributions in the proton. Simultaneously, the value of the strong coupling constant at the Z boson mass is extracted as:

$$\alpha_s(m_Z) = 0,1170 \pm 0,0019$$

A measurement of the inclusive jet production in proton-proton collisions at the LHC at  $\sqrt{s} = 13$  TeV is presented. The double-differential cross sections are measured as a function of the jet transverse momentum  $p_T$  and the absolute jet rapidity  $|y|$ . The anti-kT clustering algorithm is used with distance parameter of 0.4 (0.7) in a phase space region with jet  $p_T$  from 97 GeV up to 3.1 TeV and  $|y| < 2.0$ . Data collected with the CMS detector are used, corresponding to an integrated luminosity of 36.3 fb $^{-1}$  (33.5 fb $^{-1}$ ). The measurement is used in a comprehensive QCD analysis at next-to next-to-leading order, which results in significant improvement in the accuracy of the parton distributions in the proton. Simultaneously, the value of the strong coupling constant at the Z boson mass is extracted as  $\alpha_s(m_Z) = 0,1170 \pm 0,0019$ . For the first time, these data are used in a standard model effective field theory analysis at next to-leading order, where parton distributions and the QCD parameters are extracted simultaneously with imposed constraints on the Wilson coefficient  $c_1$  of 4-quark contact interactions.

Interaction phenomena in field theory are often defined using perturbation theory, in which the functions in the equations are extended to forces of constant interaction. Usually, for all interactions except the strong one, the coupling constant is much smaller than the unit. This makes the application of perturbation theory effective, as the contribution from the main terms of the extensions decreases rapidly and their calculation becomes redundant. In the case of strong interactions, perturbation theory becomes useless and other calculation methods are required. One of the predictions of quantum field theory is the so-called "floating constants" phenomenon, according to which interaction constants change slowly with the increase of energy transferred during the interaction of particles. Thus, the constant of the electromagnetic interaction increases, and the constant of the strong interaction decreases with increasing energy. For quarks in quantum chromodynamics, a strong interaction constant is introduced:

$$\alpha_s = \frac{q_{qg}^2}{4\pi\hbar c} = \frac{q_{qg}^2 \epsilon_0 \alpha}{q_e^2} = \frac{\epsilon_0 q_{qg}^2}{q_{pl}^2}$$



where  $g_{qq}$  is the active color charge of a quark that emits virtual gluons to interact with another quark. By reducing the distance between the quarks, which is achieved in high-energy particle collisions, a logarithmic reduction of  $\alpha_s$  and a weakening of the strong interaction (the effect of the asymptotic freedom of the quarks) is expected. In [15] we presented the recommended value for the strong coupling constant:

$$\alpha_s = \frac{\text{Eulers' number}}{\text{Gerford's constant}}$$

$$\alpha_s = \frac{e}{e^\pi}$$

$$\alpha_s = e^{1-\pi} \quad (22)$$

with numerical value:

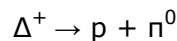
$$\alpha_s = 0,11746\dots$$

This value is the current world average value for the coupling evaluated at the Z-boson mass scale. It fits perfectly in the measurement of the strong coupling constant of the European organization for nuclear research (CERN). Also for the value of the strong coupling constant we have the equivalent expressions:

$$\alpha_s = e \cdot e^{-\pi} = e \cdot i^{2i} = i^{-2i/\pi} \cdot i^{2i} = i^{2i - (2i/\pi)} = i^{2i(n-1)/\pi}$$

In nuclear physics and particle physics, the weak interaction, which is also often called the weak force or weak nuclear force, is one of the four known fundamental interactions, with the others being electromagnetism, the strong interaction, and gravitation. It is the mechanism of interaction between subatomic particles that is responsible for the radioactive decay of atoms: The weak interaction participates in nuclear fission and nuclear fusion. The theory describing its behavior and effects is sometimes called quantum flavordynamics (QFD), however, the term QFD is rarely used, because the weak force is better understood by electroweak theory (EWT). The effective range of the weak force is limited to subatomic distances, and is less than the diameter of a proton. The weak interaction has such an incredibly short range that its strength must be evaluated in a different way than the electromagnetic force. The fact that both the strong force and the weak force initiate decays of particles gives a way to compare their strength. The lifetime of a particle is proportional to the inverse square of the coupling constant of the force which causes the decay. From the example of the decays of the delta and sigma baryons, the weak coupling constant can be related to the strong force coupling constant.

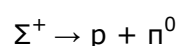
The strong interaction and weak interaction in [20] can be compared in a set of particle decays which yield the same final products. The Delta baryons (or  $\Delta$  baryons, also called Delta resonances) are a family of subatomic particles made of three up or down quarks (u or d quarks). Four closely related  $\Delta$  baryons exist:  $\Delta^{++}$  (constituent quarks: uuu),  $\Delta^+$  (uud),  $\Delta^0$  (udd), and  $\Delta^-$  (ddd), which respectively carry an electric charge of +2 e, +1 e, 0 e, and -1 e. The  $\Delta$  baryons have a mass of about 1.232 MeV/c<sup>2</sup>, a spin of 3/2, and an isospin of 3/2. Ordinary protons and neutrons, by contrast, have a mass of about 939 MeV/c<sup>2</sup>, a spin of 1/2, and an isospin of 1/2. The  $\Delta^+$  (uud) and  $\Delta^0$  (udd) particles are higher-mass excitations of the proton ( $N^+$ , uud) and neutron ( $N^0$ , udd), respectively. However, the  $\Delta^{++}$  and  $\Delta^-$  have no direct nucleon analogues. The decays of the delta baryons is:



The lifetime of the delta baryons is:

$$\tau_{\Delta} = 6 \times 10^{-24} \text{ s}$$

The sigma baryons are a family of subatomic hadron particles which have two quarks from the first flavor generation (up or down quarks), and a third quark from a higher flavor generation, in a combination where the wavefunction sign remains constant when any two quark flavors are swapped. They are thus baryons, with total isospin of 1, and can either be neutral or have an elementary charge of +2, +1, 0, or -1. They are closely related to the Lambda baryons, which differ only in the wavefunction's behavior upon flavor exchange. The decays of the sigma baryons is:



The lifetime of the delta baryons is:

$$\tau_{\Sigma} \approx 8 \times 10^{-11} \text{ s}$$

The extraordinary difference of 13 orders of magnitude in the lifetimes comes from the fact that the sigma decay does not conserve strangeness and therefore can proceed only by the weak interaction. The lifetime of a decay is proportional to the inverse square of the coupling constant between the initial and final products, and since the final products are identical, the difference in lifetime must come from the difference in coupling constants. The coupling constant ratio can then be estimated for this situation:

$$\begin{aligned} \frac{\alpha_w}{\alpha_s} &= \sqrt{\frac{\tau_{\Delta}}{\tau_{\Sigma}}} = 10^{-7} e \\ \frac{\alpha_w}{\alpha_s} &= 10^{-7} e \end{aligned} \quad (23)$$

From this expression and (22) we can result the world average value of the weak coupling constant  $\alpha_w$ :

$$\alpha_w = e \cdot \alpha_s \cdot 10^{-7}$$

$$\alpha_w = e^{2 \cdot n} \cdot 10^{-7}$$

$$\alpha_w = e \cdot e \cdot i^{2i} \cdot 10^{-7}$$

$$\alpha_w = e^2 \cdot i^{2i} \cdot 10^{-7}$$

So the recommended theoretical current world average value for the weak coupling constant is:

$$\alpha_w = (e \cdot i)^2 \cdot 10^{-7} \quad (24)$$

with numerical value:

$$\alpha_w = 3,19310 \cdot 10^{-8}$$

From expression (23) can result other equivalent expressions:

$$\alpha_w \cdot \alpha_s^{-1} = e \cdot 10^{-7}$$

$$\alpha_s \cdot \alpha_w^{-1} = e^{-1} \cdot 10^7$$

$$e \cdot \alpha_s = 10^7 \cdot \alpha_w \quad (25)$$

From this expression and (22) apply:

$$e^n \cdot \alpha_s \cdot \alpha_s = 10^7 \cdot \alpha_w$$

$$e^n \cdot \alpha_s^2 = 10^7 \cdot \alpha_w$$

$$\alpha_s^2 = 10^7 \cdot e^{-n} \cdot \alpha_w$$

$$\alpha_s^2 = i^{2i} \cdot 10^7 \cdot \alpha_w \quad (26)$$

From this expression and Euler's identity resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$(10^7 \cdot \alpha_s^{-2} \cdot \alpha_w)^i + 1 = 0$$

$$(10^{-7} \cdot \alpha_s^2 \cdot \alpha_w^{-1})^i + 1 = 0$$

$$10^{-7i} \cdot \alpha_s^{2i} \cdot \alpha_w^{-i} + 1 = 0$$

$$10^{-7i} \cdot \alpha_s^{2i} \cdot \alpha_w^{-i} = -1$$

$$\alpha_s^{2i} = -1 \cdot 10^{7i} \cdot \alpha_w^i$$

$$\frac{\alpha_s^{2i}}{\alpha_w^i} = -1 \cdot 10^{7i} \quad (27)$$

We reached the conclusion of the dimensionless unification of the strong nuclear and the weak nuclear forces:

$$e \cdot \alpha_s = 10^7 \cdot \alpha_w$$

$$\alpha_s^2 = -1 \cdot 10^7 \cdot \alpha_w$$

(Dimensionless unification of the strong nuclear and the weak nuclear force interactions)

#### 4. Dimensionless unification of the strong nuclear and the electromagnetic interactions

Based on Einstein's light quantum hypothesis, the duality of the photon was confirmed through quantum-mechanical experiments and examination. The photon is now regarded as a particle in fields related to the interaction of material with light that is absorbed and emitted; and regarded as a wave in regions relating to light propagation. It is known that among the four forces constituting the universe, the photon serves to convey electromagnetic force. The other three forces are gravitational force, strong force, and weak force. The photon plays an important role in the structure of the world where we live and is deeply involved with sources of matter and life. Through the work of Max Planck, Albert Einstein, Louis de Broglie, Arthur Compton, Niels Bohr, Erwin Schrödinger and many others, current scientific theory holds that all particles exhibit a wave nature and vice versa. This phenomenon has been verified not only for elementary particles, but also for compound particles like atoms and even molecules. For macroscopic particles, because of their extremely short wavelengths, wave properties usually cannot be detected.

Bohr regarded the "duality paradox" as a fundamental or metaphysical fact of nature. He regarded renunciation of the cause-effect relation, or complementarity, of the space-time picture, as essential to the quantum mechanical account. Werner Heisenberg considered the question further. He saw the duality as present for all quantum entities, but not quite in the usual quantum mechanical account considered by Bohr. He saw it in what is called second quantization, which generates an entirely new concept of fields that exist in ordinary space-time, causality still being visualizable. Jesús Sánchez in [17] explained that the fine-structure constant is one of the roots of the following trigonometric equation:

$$\cos a^{-1} = e^{-1} \quad (28)$$

Another elegant expression is the following exponential form equations:

$$e^{i/a} - e^{-1} = -e^{-i/a} + e^{-1}$$

$$e^{i/a} + e^{-i/a} = 2 \cdot e^{-1} \quad (29)$$

These expressions show the wave nature of light. The modern theory of quantum mechanics came to picture light as both a particle and a wave and, as a phenomenon which is neither a particle or a wave. Instead, modern physics sees light as something that can be described sometimes with mathematics appropriate to one type of macroscopic metaphor (particles) and sometimes another macroscopic metaphor (water waves), but is actually something that cannot be fully imagined. Also from [8] the fine-structure constant is one of the roots of the following trigonometric equation:

$$\cos(10^3 \cdot a^{-1}) = \phi^2 \cdot e^{-1}$$

$$e \cdot \cos(10^3 \cdot a^{-1}) = \phi^2 \quad (30)$$

Another elegant expression is the following exponential form equation:

$$e^{1000i/a} + e^{-1000i/a} = 2 \cdot \varphi^2 \cdot e^{-1} \quad (31)$$

From these expressions resulting the following equations:

$$\begin{aligned} \cos^{-1} a^{-1} \cdot \cos(10^3 \cdot a^{-1}) &= \varphi^2 \\ \cos(10^3 \cdot a^{-1}) &= \varphi^2 \cdot \cos a^{-1} \end{aligned} \quad (32)$$

We will use the expressions (22) and (28) to resulting the unity formulas that connects the strong coupling constant  $\alpha_s$  and the fine-structure constant  $\alpha$ :

$$\begin{aligned} \cos a^{-1} &= e^{-1} \\ \alpha_s &= e^{1-n} \\ \cos a^{-1} &= (e^n \cdot \alpha_s)^{-1} \\ \cos a^{-1} &= e^{-n} \cdot \alpha_s^{-1} \\ e^n \cdot \alpha_s \cdot \cos a^{-1} &= 1 \end{aligned} \quad (33)$$

Other forms of the equations are:

$$\begin{aligned} \cos a^{-1} &= (i^{-2i} \cdot \alpha_s)^{-1} \\ i^{-2i} \cdot \alpha_s \cdot \cos a^{-1} &= 1 \\ \cos a^{-1} &= i^{2i} \cdot \alpha_s^{-1} \\ \alpha_s \cdot \cos a^{-1} &= i^{2i} \end{aligned} \quad (34)$$

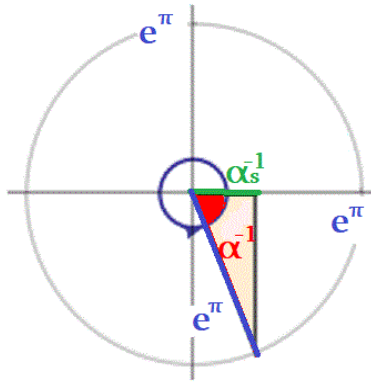
So the beautiful formulas for the strong coupling constant  $\alpha_s$  are:

$$\begin{aligned} \alpha_s &= e^{-n} \cdot \cos^{-1} a^{-1} \\ \alpha_s &= i^{2i} \cdot \cos^{-1} a^{-1} \end{aligned}$$

Now we need to study the following equivalent equations:

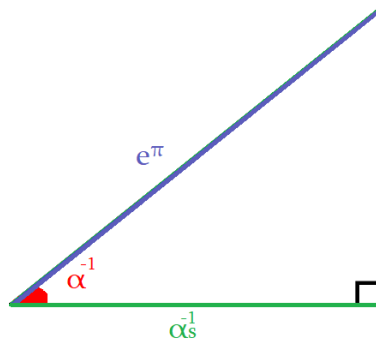
$$\begin{aligned} \cos \alpha^{-1} &= \frac{e^{-\pi}}{\alpha_s} \\ \cos \alpha^{-1} &= \frac{i^{2i}}{\alpha_s} \\ \cos \alpha^{-1} &= \frac{\alpha_s^{-1}}{e^{\pi}} \\ \cos \alpha^{-1} &= \frac{\alpha_s^{-1}}{i^{-2i}} \end{aligned}$$

The figure below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $e^n$ . The strong coupling constant  $\alpha_s$  and the fine-structure constant  $\alpha$  are in a right triangle with the variable acute angle  $\alpha^{-1}$  radians. The adjacent side is the variable side  $\alpha_s^{-1}$  while the hypotenuse is constant  $e^n$ . The fine structure constant is the ratio of the speed of the electron compared to the speed of light, in the first level of an atom. It is also related to the ratio of the Bohr radius of an atom to the Compton wavelength of an electron. We could try to relate it to the electromagnetic interactions in the atom.



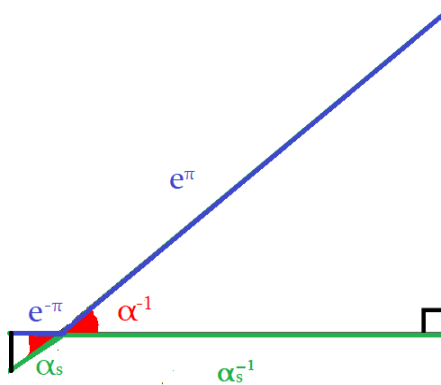
**Figure 2.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $e^\pi$ .

The nucleus emits a photon that has an intrinsic property associated with the electromagnetic interaction represented by a vector. This vector can rotate as the photon moves along. The electron has an additional property related to the electromagnetic interaction that is also represented by a vector.



**Figure 3.** The strong coupling constant  $\alpha_s$  and the fine-structure constant  $\alpha$  are in a right triangle with the variable acute angle  $\alpha^{-1}$  radians. The adjacent side is the variable side  $\alpha_s^{-1}$  while the hypotenuse is constant  $e^\pi$ .

Thus, when the photon reaches the electron, the electromagnetic interaction between them is related to the relative position of these vectors at the time of interaction. When we talk about relative placement between vectors, we can talk about a projection of one of them onto the other. This means that  $\cos \alpha^{-1}$  will be related to the interaction of these two properties of the photon and the electron. It would be related to their relative vector position at the time of interaction.



**Figure 4.** Geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.

The angle in  $\alpha^{-1}$  radians is not only the final interaction angle, but also includes, for example, the number of rotations the photon or electron vector has made before the interactions. From expressions (22) and (29) resulting the formulas that connects the strong coupling constant  $\alpha_s$  and the fine-structure constant  $\alpha$ :

$$e^{i/a} + e^{-i/a} = 2 \cdot e^{-1}$$

$$e^{i/a} + e^{-i/a} = 2 \cdot (e^n \cdot a_s)^{-1}$$

$$e^{i/a} - (e^n \cdot a_s)^{-1} = -e^{-i/a} + (e^n \cdot a_s)^{-1}$$

$$e^{i/a} + e^{-i/a} = 2 \cdot e^{-n} \cdot a_s^{-1}$$

$$e^n \cdot a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \tag{35}$$

Other forms of the equations are:

$$e^{i/a} + e^{-i/a} = 2 \cdot (i^{-2i} \cdot a_s)^{-1}$$

$$e^{i/a} + e^{-i/a} = 2 \cdot i^{2i} \cdot a_s^{-1}$$

$$e^{i/a} - i^{2i} \cdot a_s^{-1} = -e^{-i/a} + i^{2i} \cdot a_s^{-1}$$

$$a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i} \tag{36}$$

These equations are applicable for all energy scales. So the beautiful formulas for the strong coupling constant  $a_s$  are:

$$a_s = 2 \cdot e^{-n} \cdot (e^{i/a} + e^{-i/a})^{-1}$$

$$a_s = 2 \cdot i^{2i} \cdot (e^{i/a} + e^{-i/a})^{-1}$$

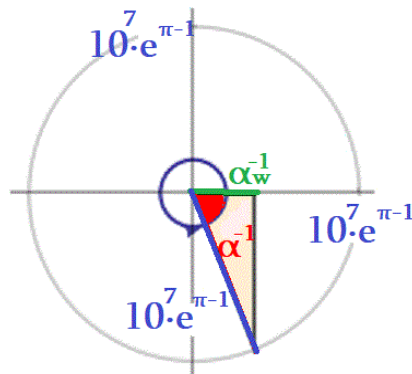
We reached the conclusion of the dimensionless unification of the strong nuclear and the electromagnetic forces:

$$a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i}$$

(Dimensionless unification of the strong nuclear and the electromagnetic interactions)

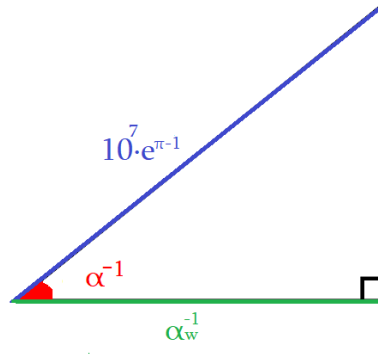
## 5. Dimensionless unification of the weak nuclear and electromagnetic interactions

In particle physics, the electroweak interaction or electroweak force is the unified description of two of the four known fundamental interactions of nature: electromagnetism and the weak interaction. The electroweak theory, in physics, is the theory that describes both the electromagnetic force and the weak force. Although these two forces appear very different at everyday low energies, the theory models them as two different aspects of the same force.



**Figure 5.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7 \cdot e^{\pi-1}$ .

The weak force acts only across distances smaller than the atomic nucleus, while the electromagnetic force can extend for great distances (as observed in the light of stars reaching across entire galaxies), weakening only with the square of the distance. Moreover, comparison of the strength of these two fundamental interactions between two protons, for instance, reveals that the weak force is some 10 million times weaker than the electromagnetic force. Yet one of the major discoveries of the 20th century has been that these two forces are different facets of a single, more-fundamental electroweak force.



**Figure 6.** Geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions

We will use the expressions (25) and (33) to resulting the unity formula that connect the weak coupling constant  $a_w$  and the fine-structure constant  $\alpha$ :

$$\begin{aligned}
 e \cdot a_s &= 10^7 \cdot a_w \\
 e^n \cdot a_s \cdot \cos \alpha^{-1} &= 1 \\
 e^n \cdot 10^7 \cdot a_w \cdot \cos \alpha^{-1} &= e \\
 e^{n-1} \cdot 10^7 \cdot a_w \cdot \cos \alpha^{-1} &= 1 \\
 10^7 \cdot a_w \cdot \cos \alpha^{-1} &= e^{1-n}
 \end{aligned} \tag{37}$$

Other forms of the equations are:

$$\begin{aligned}
 a_w \cdot \cos \alpha^{-1} &= e \cdot i^{2i} \cdot 10^{-7} \\
 10^7 \cdot a_w \cdot \cos \alpha^{-1} &= e \cdot i^{2i}
 \end{aligned} \tag{38}$$

The figure below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7 \cdot e^{n-1}$ . The strong coupling constant  $a_s$  and the fine-structure constant  $\alpha$  are in a right triangle with the variable acute angle  $\alpha^{-1}$  radians. The adjacent side is the variable side  $a_w^{-1}$  while the hypotenuse is constant  $10^7 \cdot e^{n-1}$ . So the formulas for the weak coupling constant  $a_w$  are:

$$\begin{aligned}
 a_w &= (e^{n-1} \cdot 10^7 \cdot \cos \alpha^{-1})^{-1} \\
 a_w &= e^{1-n} \cdot 10^{-7} \cdot \cos^{-1} \alpha^{-1} \\
 a_w &= e \cdot i^{2i} \cdot (10^7 \cdot \cos \alpha^{-1})^{-1} \\
 a_w &= e \cdot i^{2i} \cdot 10^{-7} \cdot \cos^{-1} \alpha^{-1}
 \end{aligned}$$

From (25) and (35) resulting the unity formulas that connects weak coupling constant  $a_w$  and the fine-structure constant  $\alpha$ :

$$\begin{aligned}
 e \cdot a_s &= 10^7 \cdot a_w \\
 e^n \cdot a_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) &= 2 \\
 e^n \cdot 10^7 \cdot a_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) &= 2 \cdot e \\
 e^{i/\alpha} + e^{-i/\alpha} &= 2 \cdot (e^{n-1} \cdot 10^7 \cdot a_w)^{-1}
 \end{aligned}$$

$$e^{i/a} - (e^{n-1} \cdot 10^7 \cdot a_w)^{-1} = -e^{-i/a} + (e^{n-1} \cdot 10^7 \cdot a_w)^{-1}$$

$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e^{1-n} \quad (39)$$

Other form of the equations is:

$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e \cdot i^{2i} \quad (40)$$

So the formulas for the weak coupling constant  $a_w$  are:

$$a_w = 2 \cdot [e^{n-1} \cdot 10^7 \cdot (e^{i/a} + e^{-i/a})]^{-1}$$

$$a_w = 2 \cdot e^{1-n} \cdot 10^{-7} \cdot (e^{i/a} + e^{-i/a})^{-1}$$

$$a_w = 2 \cdot e \cdot i^{2i} \cdot [10^7 \cdot (e^{i/a} + e^{-i/a})]^{-1}$$

$$a_w = 2 \cdot e \cdot i^{2i} \cdot 10^{-7} \cdot (e^{i/a} + e^{-i/a})^{-1}$$

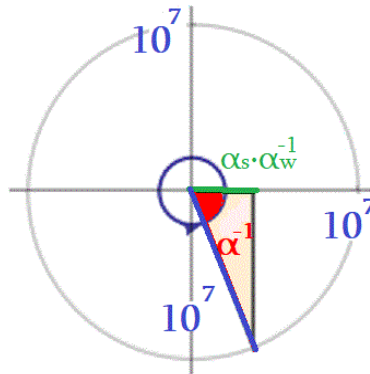
These equations are applicable for all energy scales. We reached the conclusion of the dimensionless unification of the weak nuclear and the electromagnetic forces:

$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e \cdot i^{2i}$$

(Dimensionless unification of the weak nuclear and the electromagnetic interactions)

## 6. Dimensionless unification of the strong nuclear, the weak nuclear and electromagnetic interactions

Quantum mechanics is a theoretical framework that only focuses on the three non-gravitational forces for understanding the universe in regions of both very small scale and low mass: subatomic particles, atoms, molecules, etc. Quantum mechanics successfully implemented the Standard Model that describes the three non-gravitational forces: strong nuclear, weak nuclear, and electromagnetic force—as well as all observed elementary particles. A Grand Unified Theory (GUT) is a model in particle physics in which, at high energies, the three gauge interactions of the Standard Model comprising the electromagnetic, weak, and strong forces are merged into a single force. Although this unified force has not been directly observed, many GUT models theorize its existence. If unification of these three interactions is possible, it raises the possibility that there was a grand unification epoch in the very early universe in which these three fundamental interactions were not yet distinct. Experiments have confirmed that at high energy the electromagnetic interaction and weak interaction unify into a single electroweak interaction. GUT models predict that at even higher energy, the strong interaction and the electroweak interaction will unify into a single electron nuclear interaction. This interaction is characterized by one larger gauge symmetry and thus several force carriers, but one unified coupling constant.



**Figure 7.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7$ .

We will use the expressions (25) and (28) to find the expression that connects the strong coupling constant  $a_s$ , the weak coupling constant  $a_w$  and the fine-structure constant  $\alpha$ :

$$e \cdot a_s = 10^7 \cdot a_w$$



$$\cos \alpha^{-1} = e^{-1}$$

$$\cos \alpha^{-1} = \alpha_s \cdot \alpha_w^{-1} \cdot 10^{-7}$$

So the unity formula that connects the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$  and the fine-structure constant  $\alpha$  is:

$$10^7 \cdot \alpha_w \cdot \cos \alpha^{-1} = \alpha_s \quad (41)$$

Now we need to study the following equivalent equations:

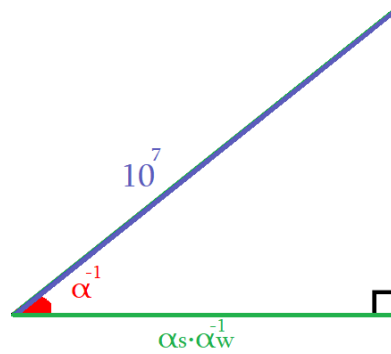
$$\cos \alpha^{-1} = \frac{10^{-7} \alpha_w^{-1}}{\alpha_s^{-1}}$$

$$\cos \alpha^{-1} = \frac{\alpha_s}{10^7 \alpha_w}$$

$$10^7 \cos \alpha^{-1} = \frac{\alpha_s}{\alpha_w}$$

$$\cos \alpha^{-1} = \frac{\alpha_s \alpha_w^{-1}}{10^7}$$

The figure below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7$ . The strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$  and the fine-structure constant  $\alpha$  are in a right triangle with the variable acute angle  $\alpha^{-1}$  radians. The adjacent side is the variable side  $\alpha_s \cdot \alpha_w^{-1}$  while the hypotenuse is constant  $10^7$ .



**Figure 8.** Geometric representation of the dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic interactions.

From expressions (25) and (29) resulting the beautiful formulas that connects the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$  and the fine-structure constant  $\alpha$ :

$$e \cdot \alpha_s = 10^7 \cdot \alpha_w$$

$$e^{i/\alpha} + e^{-i/\alpha} = 2 \cdot e^{-1}$$

$$e^{i/\alpha} + e^{-i/\alpha} = 2 \cdot 10^{-7} \cdot \alpha_s \cdot \alpha_w^{-1}$$

$$\alpha_w \cdot \alpha_s^{-1} \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot 10^{-7}$$

$$10^7 \cdot \alpha_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot \alpha_s \quad (42)$$

These equations are applicable for all energy scales. We reached the conclusion of the dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic forces:

$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot a_s$$

(Dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic interactions)

## 7. Dimensionless unification of the gravitational and the electromagnetic interactions

Gravity is a natural phenomenon by which all things with mass or energy, including planets, stars, galaxies, and even light, are brought toward one another. On Earth, gravity gives weight to physical objects, and the Moon's gravity causes the ocean tides. The gravitational attraction of the original gaseous matter present in the Universe caused it to begin coalescing and forming stars and caused the stars to group together into galaxies, so gravity is responsible for many of the large-scale structures in the Universe. Gravity has an infinite range, although its effects become increasingly weaker as objects get further away. Gravity is most accurately described by the general theory of relativity, which describes gravity not as a force, but as a consequence of masses moving along geodesic lines in a curved spacetime caused by the uneven distribution of mass. However, for most applications, gravity is well approximated by Newton's law of universal gravitation, which describes gravity as a force causing any two bodies to be attracted toward each other, with magnitude proportional to the product of their masses and inversely proportional to the square of the distance between them.

Gravity is the weakest of the four fundamental interactions of physics. As a consequence, it has no significant influence at the level of subatomic particles. In contrast, it is the dominant interaction at the macroscopic scale, and is the cause of the formation, shape and trajectory of astronomical bodies. Attempts to develop a theory of gravity consistent with quantum mechanics, a quantum gravity theory, which would allow gravity to be united in a common mathematical framework with the other three fundamental interactions of physics, are a current area of research.

A Planck length  $l_{pl}$  is about  $10^{-20}$  times the diameter of a proton, meaning it is so small that immediate observation at this scale would be impossible in the near future. The length Planck  $l_{pl}$  has dimension [L]. The length Planck  $l_{pl}$  can be defined by three fundamental natural constants, the speed of light at vacuum  $c$ , the reduced Planck constant and the gravity constant  $G$  as:

$$l_{pl} = \sqrt{\frac{\hbar G}{c^3}} = \frac{\hbar}{m_{pl} c} = \frac{h}{2\pi m_{pl} c} = \frac{m_p r_p}{4m_{pl}}$$

The 2018 CODATA recommended value of the Planck length is  $l_{pl} = 1,616255 \times 10^{-35}$  m with standard uncertainty  $0,000018 \times 10^{-35}$  m and relative standard uncertainty  $1,1 \times 10^{-5}$ . The Bohr radius  $a_0$  is a physical constant, approximately equal to the most probable distance between the nucleus and the electron in a hydrogen atom in its ground state. The Bohr radius  $a_0$  is defined as:

$$a_0 = \frac{\hbar}{\alpha m_e c} = \frac{r_e}{\alpha^2} = \frac{\lambda_c}{2\pi\alpha}$$

The 2018 CODATA recommended value of the Bohr radius is  $a_0 = 5,29177210903 \times 10^{-11}$  m with standard uncertainty  $0,00000000080 \times 10^{-11}$  m and relative standard uncertainty  $1,5 \times 10^{-10}$ .

The Planck constant, or Planck's constant, is a fundamental physical constant of foundational importance in quantum mechanics. The constant gives the relationship between the energy of a photon and its frequency, and by the mass-energy equivalence, the relationship between mass and frequency. Specifically, a photon's energy is equal to its frequency multiplied by the Planck constant. The constant is generally denoted by  $h$ . The reduced Planck constant, equal to the constant divided by  $2\pi$ , is denoted by  $\hbar$ . For the reduced Planck constant  $\hbar$  apply:

$$\hbar = a \cdot m_e \cdot a_0 \cdot c$$

So from these expressions we have:

$$\hbar^2 = a^2 \cdot m_e^2 \cdot a_0^2 \cdot c^2$$

$$(\hbar \cdot G / c^3) = a^2 \cdot m_e^2 \cdot a_0^2 \cdot (G / \hbar \cdot c)$$

$$(\hbar \cdot G / c^3) = a^2 \cdot a_0^2 \cdot (G \cdot m_e^2 / \hbar \cdot c)$$

$$l_{pl}^2 = a^2 \cdot a_0 \cdot G$$

So the new formula for the Planck length  $l_{pl}$  is:

$$l_{pl} = a\sqrt{a_G}\alpha_0 \quad (43)$$

Jeff Yee proposed in [10] that the mole and charge are related by deriving Avogadro's number from three constants, the Bohr radius, the Planck length and Euler's number. The fundamental unit of length in this unit system is the Planck length  $l_{pl}$ . Spacetime is proposed to be a lattice structure, in which its unit cells have sides of length  $a$ , marked below in the next figure. The lattice contains repeating cells with this structure, so it can be simplified to model a single unit cell of this repeating structure. These types of structures are commonly found in molecules. The center point of wave convergence is referred to here as a wave center. The separation length between granules in the unit cell is the diameter of a granule ( $2 \cdot l_{pl}$ ) multiplied by Euler's number ( $e$ ), which is the base of the natural logarithm. There are exactly Avogadro's number of unit cells in the radius of hydrogen. The Avogadro's number  $N_A$  can be calculated from the Planck length  $l_{pl}$ , the Bohr radius  $a_0$  and Euler's number  $e$ :

$$N_A = \frac{a_0}{2el_{pl}}$$

We will use this expression and the new formula for the Planck length  $l_{pl}$  to resulting the unity formula that connects the fine-structure constant  $\alpha$  and the gravitational coupling constant  $\alpha_G$ :

$$\begin{aligned} a_0 &= 2 \cdot e \cdot N_A \cdot l_{pl} \\ \alpha_0 &= 2eN_A\alpha\sqrt{\alpha_G}\alpha_0 \\ 2eN_A\alpha\sqrt{\alpha_G} &= 1 \end{aligned}$$

Therefore the unity formula that connects the fine-structure constant  $\alpha$ , the gravitational coupling constant  $\alpha_G$  and the Avogadro's number  $N_A$  is:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (44)$$

The unity formula is equally valid:

$$\alpha^2 \cdot \alpha_G = (2 \cdot e \cdot N_A)^{-2} \quad (45)$$

So the new formula for the Avogadro number  $N_A$  is:

$$N_A = \left( 2e\alpha\sqrt{\alpha_G} \right)^{-1} \quad (46)$$

It was presented in [13] the mathematical formulas that connects the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro's number  $N_A$ , the gravitational coupling constant  $\alpha_G$  of the electron and the gravitational coupling constant of the proton  $\alpha_G(p)$ :

$$\alpha_G(p) = \mu^2 \cdot \alpha_G \quad (47)$$

$$\alpha = \mu \cdot N_1 \cdot \alpha_G \quad (48)$$

$$\alpha \cdot \mu = N_1 \cdot \alpha_G(p) \quad (49)$$

$$\alpha^2 = N_1^2 \cdot \alpha_G \cdot \alpha_G(p) \quad (50)$$

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (51)$$

$$\mu^2 = 4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G(p) \cdot N_A^2 \quad (52)$$

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot a^3 \cdot N_A^2 \quad (53)$$

$$4 \cdot e^2 \cdot a \cdot \mu \cdot a_G^2 \cdot N_A^2 \cdot N_1 = 1 \quad (54)$$

$$\mu^3 = 4 \cdot e^2 \cdot a \cdot a_G(p)^2 \cdot N_A^2 \cdot N_1 \quad (55)$$

$$\mu^2 = 4 \cdot e^2 \cdot a_G \cdot a_G(p)^2 \cdot N_A^2 \cdot N_1^2 \quad (56)$$

$$\mu = 4 \cdot e^2 \cdot a \cdot a_G \cdot a_G(p) \cdot N_A^2 \cdot N_1 \quad (57)$$

Also from the expressions (28) and (44) resulting the expressions:

$$\cos(a^{-1}) = e^{-1}$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$$

$$4 \cdot a^2 \cdot a_G \cdot N_A^2 = e^{-2}$$

$$\cos^2 a^{-1} = 4 \cdot a^2 \cdot a_G \cdot N_A^2$$

$$a^{-2} \cdot \cos^2 a^{-1} = 4 \cdot a_G \cdot N_A^2 \quad (58)$$

This unity formula is equally valid:

$$\alpha^{-1} \cos \alpha^{-1} = 2N_A \sqrt{\alpha_G} \quad (59)$$

Also from the expressions (29) and (44) resulting another elegant exponential form equations:

$$e^{i/a} + e^{-i/a} = 2 \cdot e^{-1}$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$$

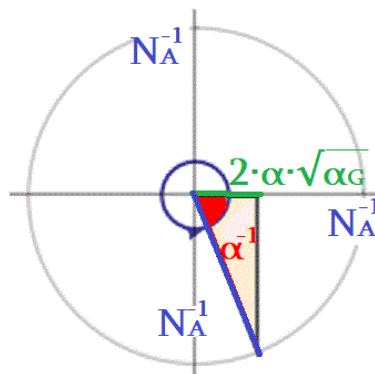
$$4 \cdot a^2 \cdot a_G \cdot N_A^2 = e^{-2}$$

$$16 \cdot a^2 \cdot a_G \cdot N_A^2 = (e^{i/a} + e^{-i/a})^2 \quad (60)$$

This unity formula is equally valid:

$$\alpha^{-1} \left( e^{\frac{i}{\alpha}} + e^{-\frac{i}{\alpha}} \right) = 4N_A \sqrt{\alpha_G} \quad (61)$$

The concept of power of two supports an idea of holographic concepts of the Universe or some of the fractal theories. Also it is used in wave mechanics, and it could be viewed in accordance with wave properties of the elementary particles in quantum physics.



**Figure 9.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $N_A^{-1}$ .

Also from the expressions (11),(44) and (60) resulting the expression with power of two:

$$2^{160} \cdot e^2 \cdot a^2 \cdot a_G = 1 \quad (62)$$

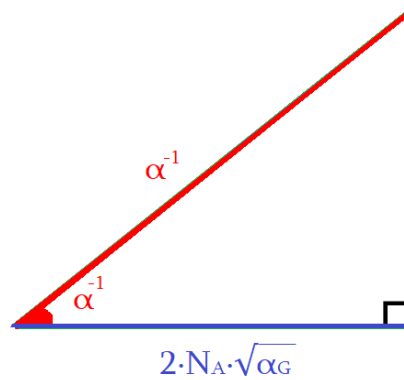
$$a^{-2} \cdot \cos^2 a^{-1} = 2^{160} \cdot a_G \quad (63)$$

$$2^{162} \cdot a^2 \cdot a_G = (e^{1/a} + e^{-1/a})^2 \quad (64)$$

Other form of the equations is:

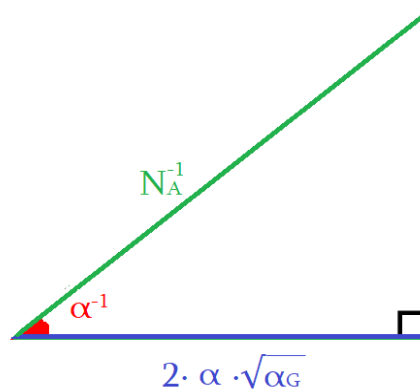
$$\alpha^{-1} \cos \alpha^{-1} = 2^{34} \sqrt{\alpha_G} \quad (65)$$

In his experiments of 1849–50, Michael Faraday was the first to search for a unification of gravity with electricity and magnetism. However, he found no connection. In 1900, David Hilbert published a famous list of mathematical problems. In Hilbert's sixth problem, he challenged researchers to find an axiomatic basis for all of physics.



**Figure 10.** First geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions

Gravity and electromagnetism are able to coexist as entries in a list of classical forces, but for many years it seemed that gravity could not be incorporated into the quantum framework, let alone unified with the other fundamental forces. For this reason, work on unification, for much of the twentieth century, focused on understanding the three forces described by quantum mechanics: electromagnetism and the weak and strong forces.



**Figure 11.** Second geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions

We reached the conclusion of the dimensionless unification of the gravitational and the electromagnetic forces:

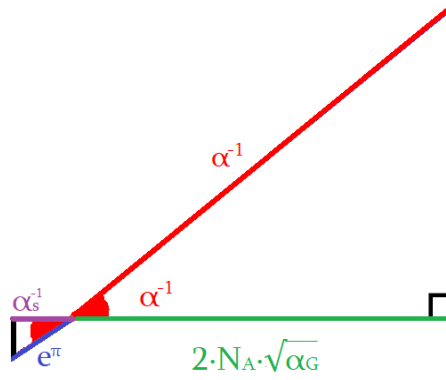
$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$$

$$16 \cdot a^2 \cdot a_G \cdot N_A^2 = (e^{1/a} + e^{-1/a})^2$$

(Dimensionless unification of the gravitational and the electromagnetic interactions)

## 8. Dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions

Although the Standard Model is believed to be theoretically self-consistent and has demonstrated huge successes in providing experimental predictions,it leaves some phenomena unexplained. It falls short of being a complete theory of fundamental interactions. For example,it does not fully explain baryon asymmetry,incorporate the full theory of gravitation as described by general relativity,or account for the universe's accelerating expansion as possibly described by dark energy. The model does not contain any viable dark matter particle that possesses all of the required properties deduced from observational cosmology. It also does not incorporate neutrino oscillations and their non-zero masses. Now we will find the equation that connect the coupling constants of the strong nuclear,the gravitational and the electromagnetic interactions.



**Figure 12.** Geometric representation of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions

We will use the expressions (22) and (44) to resulting the unity formulas that connect the strong coupling constant  $a_s$ ,the fine-structure constant  $a$  and the gravitational coupling constant  $a_G$ :

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$$

$$4 \cdot e^2 \cdot (e^n \cdot a_s)^2 \cdot a_G \cdot N_A^2 = 1$$

$$4 \cdot e^{2n} \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1 \quad (66)$$

Other form of the equation is:

$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i} \quad (67)$$

Also from the expression (22) and expression (51),(52),(53),(54),(55),(56),(57) resulting the mathematical formulas that connects the strong coupling constant  $a_s$ ,the proton to electron mass ratio  $\mu$ ,the fine-structure constant  $a$ ,the ratio  $N_1$  of electric force to gravitational force between electron and proton,the Avogadro number  $N_A$ ,the gravitational coupling constant  $a_G$  of the electron and the gravitational coupling constant of the proton  $a_{G(p)}$ :

$$4 \cdot e^{2n} \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1 \quad (68)$$

$$\mu^2 = 4 \cdot e^{2n} \cdot a_s^2 \cdot a^2 \cdot a_{G(p)} \cdot N_A^2 \quad (69)$$

$$\mu \cdot N_1 = 4 \cdot e^{2n} \cdot a_s^2 \cdot a^3 \cdot N_A^2 \quad (70)$$

$$4 \cdot e^{2n} \cdot a_s^2 \cdot a \cdot \mu \cdot a_G^2 \cdot N_A^2 \cdot N_1 = 1 \quad (71)$$

$$\mu^3 = 4 \cdot e^{2n} \cdot a_s^2 \cdot a \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (72)$$

$$\mu^2 = 4 \cdot e^{2n} \cdot a_s^2 \cdot a_G \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1^2 \quad (73)$$

$$\mu = 4 \cdot e^{2n} \cdot a_s^2 \cdot a \cdot a_G \cdot a_{G(p)} \cdot N_A^2 \cdot N_1 \quad (74)$$

Other equivalent forms of the equations are:

$$4 \cdot a s^2 \cdot a^2 \cdot a G \cdot N A^2 = i^{4i} \quad (75)$$

$$i^{4i} \cdot \mu = a s^2 \cdot a^2 \cdot a G(p) \cdot N A^2 \quad (76)$$

$$i^{4i} \cdot \mu \cdot N_1 = 4 \cdot a s^2 \cdot a^3 \cdot N A^2 \quad (77)$$

$$4 \cdot a s^2 \cdot a \cdot \mu \cdot a G^2 \cdot N A^2 \cdot N_1 = i^{4i} \quad (78)$$

$$i^{4i} \cdot \mu^3 = 4 \cdot a s^2 \cdot a \cdot a G(p)^2 \cdot N A^2 \cdot N_1 \quad (79)$$

$$i^{4i} \cdot \mu^2 = 4 \cdot e^{2n} \cdot a s^2 \cdot a G \cdot a G(p)^2 \cdot N A^2 \cdot N_1^2 \quad (80)$$

$$i^{4i} \cdot \mu = 4 \cdot a s^2 \cdot a \cdot a G \cdot a G(p) \cdot N A^2 \cdot N_1 \quad (81)$$

From the expressions (34) and (75) apply:

$$a s \cdot \cos a^{-1} = i^{2i}$$

$$2 \cdot N A \cdot a G^{1/2} = a^{-1} \cdot \cos a^{-1}$$

$$2 \cdot a s \cdot a \cdot a G^{1/2} \cdot N A = i^{2i}$$

$$2 \cdot a s \cdot a \cdot N A \cdot a G^{1/2} \cdot a s \cdot \cos a^{-1} = i^{2i} \cdot i^{2i}$$

$$2 \cdot a \cdot \cos a^{-1} \cdot a s^2 \cdot a G^{1/2} \cdot N A = i^{4i}$$

$$4 \cdot a^2 \cdot \cos^2 a^{-1} \cdot a s^4 \cdot a G \cdot N A^2 = i^{8i} \quad (82)$$

From the expressions (36) and (75) apply:

$$a s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i}$$

$$2 \cdot N A \cdot a G^{1/2} = a^{-1} \cdot (e^{i/a} + e^{-i/a})$$

$$2 \cdot a s \cdot a \cdot N A \cdot a G^{1/2} = i^{2i}$$

$$a s \cdot (e^{i/a} + e^{-i/a}) \cdot 2 \cdot a s \cdot a \cdot N A \cdot a G^{1/2} = 2 \cdot i^{2i} \cdot i^{2i}$$

$$a \cdot (e^{i/a} + e^{-i/a}) \cdot a s^2 \cdot a G^{1/2} \cdot N A = i^{4i}$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a s^4 \cdot a G \cdot N A^2 = i^{8i} \quad (83)$$

Also from the expressions (11),(75) and (83) resulting the expressions with power of two:

$$2^{80} \cdot a s \cdot a \cdot a G^{1/2} = i^{2i}$$

$$2^{160} \cdot a s^2 \cdot a^2 \cdot a G = i^{4i} \quad (84)$$

$$2^{80} \cdot a \cdot (e^{i/a} + e^{-i/a}) \cdot a s^2 \cdot a G^{1/2} = i^{4i}$$

$$2^{160} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a s^4 \cdot a G = i^{8i} \quad (85)$$

We reached the conclusion of the dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions:

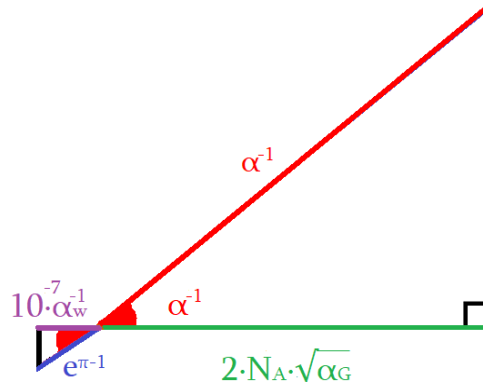
$$4 \cdot a s^2 \cdot a^2 \cdot a G \cdot N A^2 = i^{4i}$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a s^4 \cdot a G \cdot N A^2 = i^{8i}$$

(Dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions)

## 9. Dimensionless unification of the weak nuclear, the gravitational and the electromagnetic interactions

Now we will find the equation that connects the coupling constants of the weak nuclear, the gravitational and the electromagnetic interactions.



**Figure 13.** Geometric representation of the dimensionless unification of the weak nuclear, the gravitational and the electromagnetic interactions

We will use the expressions (25) and (68),(75) to resulting the unity formulas that connects the weak coupling constant  $\alpha_w$ , the fine-structure constant  $\alpha$  and the gravitational coupling constant  $\alpha_G$ :

$$e \cdot \alpha_s = 10^7 \cdot \alpha_w$$

$$2 \cdot e^\pi \cdot \alpha_s \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A = 1$$

$$2 \cdot \alpha_s \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A = i^{2i}$$

$$2 \cdot e^\pi \cdot 10^7 \cdot \alpha_w \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A = e$$

$$4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = e^2 \quad (86)$$

$$2 \cdot 10^7 \cdot \alpha_w \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A = i^{2i} \cdot e$$

$$4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \cdot e^2 \quad (87)$$

Also from the expression (22) and (68),(69),(70),(71),(72),(73),(74) resulting the mathematical formulas that connects the weak coupling constant  $\alpha_w$ , the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro's number  $N_A$ , the gravitational coupling constant  $\alpha_G$  of the electron and the gravitational coupling constant of the proton  $\alpha_{G(p)}$ :

$$4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = e^2 \quad (88)$$

$$e^2 \cdot \mu^2 = 4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_{G(p)} \cdot N_A^2 \quad (89)$$

$$e^2 \cdot \mu \cdot N_1 = 4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha^3 \cdot N_A^2 \quad (90)$$

$$4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = e^2 \quad (91)$$

$$e^2 \cdot \mu^3 = 4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (92)$$

$$e^2 \cdot \mu^2 = 4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha_G \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1^2 \quad (93)$$

$$e^2 \cdot \mu = 4 \cdot e^\pi \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_{G(p)} \cdot N_A^2 \cdot N_1 \quad (94)$$

From the expression (22) and (75),(76),(77),(78),(79),(80),(81) other equivalent forms of the equations are:



$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i} \cdot e^2 \quad (95)$$

$$i^{4i} \cdot e^2 \cdot \mu^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_{G(p)} \cdot NA^2 \quad (96)$$

$$i^{4i} \cdot e^2 \cdot \mu \cdot N_1 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^3 \cdot NA^2 \quad (97)$$

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot \mu \cdot a_{G^2} \cdot NA^2 \cdot N_1 = i^{4i} \cdot e^2 \quad (98)$$

$$i^{4i} \cdot e^2 \cdot \mu^3 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_{G(p)^2} \cdot NA^2 \cdot N_1 \quad (99)$$

$$i^{4i} \cdot e^2 \cdot \mu^2 = 4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a_G \cdot a_{G(p)^2} \cdot NA^2 \cdot N_1^2 \quad (100)$$

$$i^{4i} \cdot e^2 \cdot \mu = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_G \cdot a_{G(p)} \cdot NA^2 \cdot N_1 \quad (101)$$

From the expressions (26) and (82) apply:

$$a_w^{-1} \cdot a_s^2 = i^{2i} \cdot 10^7$$

$$a_s^2 = i^{2i} \cdot 10^7 \cdot a_w$$

$$2 \cdot a \cdot \cos a^{-1} \cdot a_s^2 \cdot a_G^{1/2} \cdot NA = i^{4i}$$

$$2 \cdot a \cdot \cos a^{-1} \cdot i^{2i} \cdot 10^7 \cdot a_w \cdot a_G^{1/2} \cdot NA = i^{4i}$$

$$2 \cdot 10^7 \cdot a \cdot \cos a^{-1} \cdot a_w \cdot a_G^{1/2} \cdot NA = i^{2i}$$

$$4 \cdot 10^{14} \cdot a^2 \cdot \cos^2 a^{-1} \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{4i} \quad (102)$$

From the expressions (26) and (83) apply:

$$a \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^2 \cdot a_G^{1/2} \cdot NA = i^{4i}$$

$$a \cdot (e^{i/a} + e^{-i/a}) \cdot i^{2i} \cdot 10^7 \cdot a_w \cdot a_G^{1/2} \cdot NA = i^{4i}$$

$$10^7 \cdot a \cdot (e^{i/a} + e^{-i/a}) \cdot a_w \cdot a_G^{1/2} \cdot NA = i^{2i}$$

$$10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{4i} \quad (103)$$

Also from the expressions (11),(102) and (103) resulting the expression with power of two:

$$2^{80} \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2} = i^{2i} \cdot e$$

$$2^{160} \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G = i^{4i} \cdot e^2 \quad (104)$$

$$2^{80} \cdot 10^7 \cdot a \cdot (e^{i/a} + e^{-i/a}) \cdot a_w \cdot a_G^{1/2} = i^{2i}$$

$$2^{160} \cdot 10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G = i^{4i} \quad (105)$$

We reached the conclusion of the dimensionless unification of the weak nuclear, the gravitational and electromagnetic forces:

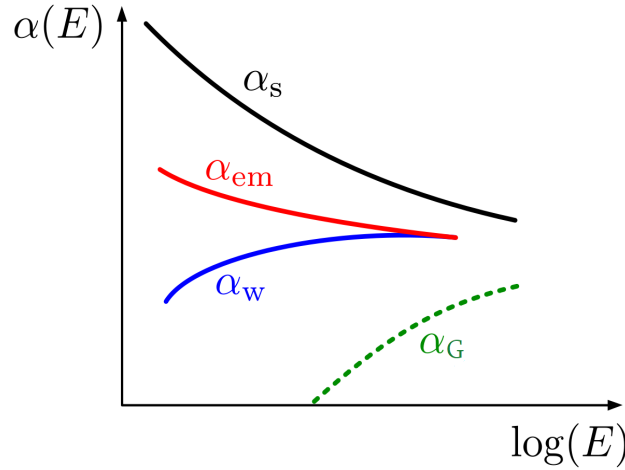
$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i} \cdot e^2$$

$$10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{8i}$$

(Dimensionless unification of the weak nuclear, the gravitational and the electromagnetic interactions)

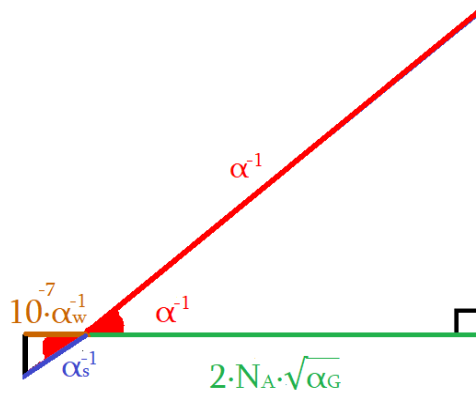
## 10. Dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions

A theory of everything is a hypothetical, singular, all-encompassing, coherent theoretical framework of physics that fully explains and links together all aspects of the universe. Finding a theory of everything is one of the major unsolved problems in physics. String theory and M-theory have been proposed as theories of everything.



**Figure 14.** Variation of the coupling constants of the four fundamental interactions of physics as a function of energy.

Over the past few centuries, two theoretical frameworks have been developed that, together, most closely resemble a theory of everything. These two theories upon which all modern physics rests are general relativity and quantum mechanics.



**Figure 15.** Geometric representation of the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions

General relativity is a theoretical framework that only focuses on gravity for understanding the universe in regions of both large scale and high mass: planets, stars, galaxies, clusters of galaxies etc. Now we will find the equation that connects the four coupling constants. We will use the expressions (26) and (75) to resulting the unity formulas that connects the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$ , the fine-structure constant  $\alpha$  and the gravitational coupling constant  $\alpha_G$ :

$$\alpha_w^{-1} \cdot \alpha_s^2 = i^{2i} \cdot 10^7$$

$$2 \cdot \alpha_s \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2} = i^{2i}$$

$$\alpha_w^{-1} \cdot \alpha_s^2 = 2 \cdot 10^7 \cdot \alpha_s \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2}$$

$$\alpha_w^{-1} \cdot \alpha_s = 2 \cdot 10^7 \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2}$$

$$2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha \cdot \alpha_G^{1/2} \cdot \alpha_s^{-1} = 1$$

$$\alpha_w \cdot \alpha \cdot \alpha_G^{1/2} \cdot \alpha_s^{-1} = (2 \cdot 10^7 \cdot N_A)^{-1} \tag{106}$$

$$2 \cdot 10^7 \cdot NA \cdot a_w \cdot a \cdot a_G^{1/2} = a_s$$

$$a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} = (2 \cdot 10^7 \cdot NA)^{-2} \quad (107)$$

So the beautiful unity formula that connects the strong coupling constant  $a_s$ , the weak coupling constant  $a_w$ , the fine-structure constant  $a$  and the gravitational coupling constant  $a_G$  is:

$$(2 \cdot 10^7 \cdot NA \cdot a_w \cdot a)^2 \cdot a_G = a_s^2$$

$$4 \cdot 10^{14} \cdot NA^2 \cdot a_w^2 \cdot a^2 \cdot a_G = a_s^2 \quad (108)$$

Sometimes the gravitational coupling constant for the proton  $a_{G(p)}$  is used instead of the gravitational coupling constant  $a_G$  for the electron:

$$a_{G(p)} = \mu^2 \cdot a_G$$

$$a_G^{1/2} = a_{G(p)}^{1/2} \cdot \mu^{-1}$$

$$a_s \cdot \mu \cdot (a_w \cdot a \cdot a_{G(p)}^{1/2})^{-1} = 2 \cdot 10^7 \cdot NA$$

$$a_s \cdot \mu = 2 \cdot 10^7 \cdot NA \cdot a_w \cdot a \cdot a_{G(p)}^{1/2}$$

$$2 \cdot 10^7 \cdot NA \cdot a_w \cdot a \cdot a_{G(p)}^{1/2} \cdot a_s^{-1} \cdot \mu^{-1} = 1$$

$$2 \cdot 10^7 \cdot NA \cdot a_w \cdot a \cdot a_{G(p)}^{1/2} \cdot a_s^{-1} = \mu \cdot a_s$$

$$a_w \cdot a \cdot a_{G(p)}^{1/2} \cdot a_s^{-1} = (2 \cdot 10^7 \cdot NA)^{-1} \cdot \mu \quad (109)$$

So the beautiful unity formula that connect the strong coupling constant  $a_s$ , weak coupling constant  $a_w$ , the fine-structure constant  $a$  and the gravitational coupling constant  $a_{G(p)}$  for the proton is:

$$(2 \cdot 10^7 \cdot NA \cdot a_w \cdot a)^2 \cdot a_{G(p)} = \mu^2 \cdot a_s^2$$

$$4 \cdot 10^{14} \cdot NA^2 \cdot a_w^2 \cdot a^2 \cdot a_{G(p)} = \mu^2 \cdot a_s^2 \quad (110)$$

From the expressions (58) and (106) apply:

$$\cos a^{-1} = 2 \cdot a \cdot a_G^{1/2} \cdot NA$$

$$2 \cdot 10^7 \cdot NA \cdot a_w \cdot a \cdot a_G^{1/2} = a_s$$

$$2 \cdot a \cdot a_G^{1/2} \cdot NA \cdot 2 \cdot 10^7 \cdot NA \cdot a_w \cdot a \cdot a_G^{1/2} = a_s \cos a^{-1}$$

$$4 \cdot 10^7 \cdot a^2 \cdot a_G \cdot a_w \cdot NA^2 = a_s \cos a^{-1}$$

$$a^{-1} \cdot (e^{i/a} + e^{-i/a}) = 4 \cdot NA \cdot a_G^{1/2}$$

$$2 \cdot 10^7 \cdot NA \cdot a_w \cdot a \cdot a_G^{1/2} = a_s$$

$$2 \cdot 10^7 \cdot NA \cdot a_w \cdot a \cdot a_G^{1/2} \cdot 4 \cdot NA \cdot a_G^{1/2} = a_s \cdot a^{-1} \cdot (e^{i/a} + e^{-i/a})$$

$$8 \cdot 10^7 \cdot NA^2 \cdot a_w \cdot a^2 \cdot a_G = a_s \cdot (e^{i/a} + e^{-i/a})$$

From the expressions (25) and (51),(52),(53),(54),(55),(56),(57) resulting the mathematical formulas that connects the strong coupling constant  $a_s$ , the weak coupling constant  $a_w$ , the proton to electron mass ratio  $\mu$ , the fine-structure constant  $a$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro's number  $NA$ , the gravitational coupling constant  $a_G$  of the electron and the gravitational coupling constant of the proton  $a_{G(p)}$ :

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2 \quad (111)$$

$$\mu^2 \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_{G(p)} \cdot NA^2 \quad (112)$$

$$\mu \cdot N_1 \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^3 \cdot NA^2 \quad (113)$$

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot \mu \cdot a_G^2 \cdot NA^2 \cdot N_1 \quad (114)$$

$$\mu^3 \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_G(p)^2 \cdot NA^2 \cdot N_1 \quad (115)$$

$$\mu \cdot a_s = 4 \cdot 10^{14} \cdot a_w^2 \cdot a_G \cdot G(p)^2 \cdot NA^2 \cdot N_1^2 \quad (116)$$

$$\mu \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_G \cdot a_G(p) \cdot NA^2 \cdot N_1 \quad (117)$$

Also from the expressions (11) and (108) resulting the expressions with power of two:

$$2^{80} \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2} \cdot a_s^{-1} = 1$$

$$2^{160} \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} = 1 \quad (118)$$

$$a_s = 2^{80} \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2}$$

$$a_s^2 = 2^{160} \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \quad (119)$$

We reached the conclusion of the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions:

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2$$

$$8 \cdot 10^7 \cdot NA^2 \cdot a_w \cdot a^2 \cdot a_G = a_s \cdot (e^{i/a} + e^{-i/a})$$

(Dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions)

## 11. Gravitational constant

The gravitational constant is an empirical physical constant that participates in the calculation of gravitational force between two bodies and is denoted by the letter G. It usually appears in Isaac Newton's law of universal gravitation and Albert Einstein's general theory of relativity. The physicist Sir Isaac Newton in 1687 published his book "Philosophiae Naturalis Principia Mathematica" where he presented the law of universal gravity to describe and calculate the mutual attraction of particles and huge objects in the universe. In this paper, Isaac Newton concluded that the attraction between two bodies is proportional to the product of their masses and inversely proportional to the square of the distance separating them. However, these must be adjusted by introducing the gravity constant G. The gravitational constant G occupies an anomalous position among the other constants of physics. The mass M of any celestial object cannot be determined independently of the gravitational attraction that it exerts. Thus, the combination G·M, not the separate value of M, is the only meaningful property of a star, planet, or galaxy. According to general relativity and the principle of equivalence, G does not depend on material properties but is in a sense a geometric factor. The gravitational constant is defined as:

$$G = \alpha_G \frac{\hbar c}{m_e^2}$$

The 2018 CODATA recommended value of gravitational constant is  $G = 6,67430 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$  with standard uncertainty  $0,00015 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$  and relative standard uncertainty  $2,2 \times 10^{-5}$ . Now we will find the formulas for the gravitational constant G using the unity formulas for the coupling constants that we calculated. From expression (44) the gravitational coupling constant  $a_G$  can be written in the form:

$$4 \cdot e^2 \cdot NA^2 \cdot a^2 \cdot a_G = 1$$

$$a_G = (2 \cdot e \cdot a \cdot NA)^{-2} \quad (120)$$

Therefore from this expression the formula for the gravitational constant is:

$$G = (2e\alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (121)$$

From equivalent expressions (68) and (75) the gravitational coupling constant  $\alpha_G$  can be written in the forms:

$$4 \cdot e^{2n} \cdot \alpha_s^2 \cdot a^2 \cdot \alpha_G \cdot N_A^2 = 1$$

$$\alpha_G = (2 \cdot e^n \cdot \alpha_s \cdot a \cdot N_A)^{-2} \quad (122)$$

$$4 \cdot \alpha_s^2 \cdot a^2 \cdot \alpha_G \cdot N_A^2 = i^{4i}$$

$$\alpha_G = i^{4i} \cdot (2 \cdot \alpha_s \cdot a \cdot N_A)^{-2} \quad (123)$$

Therefore from these expressions the equivalent formulas for the gravitational constant are:

$$G = (2e^\pi \alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (124)$$

$$G = i^{4i} (2\alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (125)$$

From equivalent expressions (88) and (95) the gravitational coupling constant  $\alpha_G$  can be written in the form:

$$4 \cdot 10^{14} \cdot e^{2n} \cdot \alpha_w^2 \cdot a^2 \cdot \alpha_G \cdot N_A^2 = e^2$$

$$\alpha_G = (2 \cdot e^{n-1} \cdot 10^7 \cdot \alpha_w \cdot a \cdot N_A)^{-2} \quad (126)$$

$$4 \cdot 10^{14} \cdot \alpha_w^2 \cdot a^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \cdot e^2$$

$$\alpha_G = i^{4i} \cdot e^2 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot a \cdot N_A)^{-2} \quad (127)$$

Therefore from these expressions the equivalent formulas for the gravitational constant are:

$$G = (2e^{\pi-1} 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (128)$$

$$G = i^{4i} e^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (129)$$

From expression (111) the gravitational coupling constant  $\alpha_G$  can be written in the form:

$$4 \cdot 10^{14} \cdot N_A^2 \cdot \alpha_w^2 \cdot a^2 \cdot \alpha_G = \alpha_s^2$$

$$\alpha_G = \alpha_s^2 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot a \cdot N_A)^{-2} \quad (130)$$

Therefore from this expression the formula for the gravitational constant is:

$$G = \alpha_s^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (131)$$

## 12. Gravitational fine-structure constant

The relevant constant in atomic physics is the fine-structure constant  $\alpha$ , which plays a fundamental role in atomic physics and quantum electrodynamics. The analogous constant in cosmology is the gravitational fine-structure constant  $\alpha_g$ . It plays a fundamental role in cosmology. The mysterious value of the gravitational fine-structure constant  $\alpha_g$  is an equivalent way to express the biggest issue in theoretical physics. The new formula for the Planck length  $l_{pl}$  is:

$$l_{pl} = a \sqrt{a_G \alpha_0}$$

The fine-structure constant equals:

$$\alpha^2 = \frac{r_e}{a_0}$$

From these expressions we have:

$$l_{pl} = \frac{\alpha \sqrt{\alpha_G r_e}}{\alpha^2}$$

$$l_{pl} = \frac{\sqrt{\alpha_G}}{\alpha} r_e$$

$$\frac{l_{pl}^3}{r_e^3} = \frac{\sqrt{\alpha_G^3}}{\alpha^3}$$

The gravitational fine structure constant  $\alpha_g$  is defined as:

$$\alpha_g = \frac{l_{pl}^3}{r_e^3}$$

$$\alpha_g = \frac{\sqrt{\alpha_G^3}}{\alpha^3}$$

$$\alpha_g = \sqrt{\frac{\alpha_G^3}{\alpha^6}} \quad (132)$$

with numerical value:

$$\alpha_g = 1,886837 \times 10^{-61}$$

Also equals:

$$\alpha_g^2 \cdot \alpha^6 = \alpha_G^3$$

$$\alpha_g^2 = \alpha_G^3 \cdot \alpha^{-6}$$

$$\alpha_g^2 = \left( \frac{\alpha_G}{\alpha^2} \right)^3$$

Now we will try to find the best mathematical expression of the gravitational fine structure constant  $\alpha_g$  with the mathematical constants. In trying to do this we found surprising coincidences and various approaches for the math constants. A approach for Archimedes constant  $\pi$  is:

$$\pi^6 \simeq \frac{2^{300}}{6 \cdot 7^{103}} \quad (133)$$

A approach for the Gelfond's constant  $e^\pi$  is:

$$e^\pi \simeq \frac{55}{\pi} \sqrt{\frac{2}{\ln \pi}} \quad (134)$$

A approximation expression that connects the golden ratio  $\phi$ , the Archimedes constant  $\pi$  and the Euler's number  $e$  is:

$$2^2 11^2 e \simeq 3^4 \phi^5 \sqrt[3]{\pi} \quad (135)$$

Two approximations expressions that connects the golden ratio  $\phi$ , the Archimedes constant  $\pi$ , the Euler's number  $e$  and the Euler's constant  $\gamma$  are:

$$4e^2 \gamma \ln^2(2\pi) \simeq \sqrt{3^3} \phi^5 \quad (136)$$

$$\sqrt{3^5} e \gamma \ln(2\pi) \sqrt[3]{\pi} \simeq 11^2 \quad (137)$$

The expression that connects the gravitational fine-structure constant  $\alpha_g$  with the Archimedes constant  $\pi$ , the Euler's number  $e$  and the Euler's constant  $\gamma$  is:

$$\alpha_g = [e \cdot \gamma \cdot \ln^2(2 \cdot \pi)]^{-1} \times 10^{-60} = 1,886837 \times 10^{-61} \quad (138)$$

The expression that connects the gravitational fine-structure constant  $\alpha_g$  with the golden ratio  $\phi$  and the Euler's number  $e$  is:

$$\alpha_g = \frac{4e}{3\sqrt{3}\phi^5} \times 10^{-60} = 1,886837 \times 10^{-61} \quad (139)$$

The expression that connects the gravitational fine-structure constant  $\alpha_g$  with the Archimedes constant  $\pi$  is:

$$\alpha_g = \frac{\sqrt{3^5} \sqrt[3]{\pi}}{11^2} \times 10^{-60} = 1,886837 \times 10^{-61} \quad (140)$$

The expression that connects the gravitational fine-structure constant  $\alpha_g$  with the golden ratio  $\phi$  and the Euler's constant  $\gamma$  is:

$$\alpha_g = \frac{7\phi\gamma^2}{2} \times 10^{-60} = 1,886826 \times 10^{-61} \quad (141)$$

The expression that connects the gravitational fine-structure constant  $\alpha_g$  with the Archimedes constant and the golden ratio  $\phi$  is:

$$\alpha_g = \frac{2\pi}{3\phi^5} \times 10^{-60} = 1,888514 \times 10^{-61} \quad (142)$$

From the expressions (120) and (132) resulting the unity formula for the gravitational fine-structure constant  $\alpha_g$ :

$$\alpha_g = (2 \cdot e \cdot a^2 \cdot NA)^{-3} \quad (143)$$

Also apply the expressions:

$$(2 \cdot e \cdot a^2 \cdot NA)^3 \cdot \alpha_g = 1$$

$$8 \cdot e^3 \cdot a^6 \cdot \alpha_g \cdot NA^3 = 1$$

From the expressions (123) and (132) resulting the unity formula for the gravitational fine-structure constant  $\alpha_g$ :

$$\alpha_g = i^{6i} \cdot (2 \cdot a_s \cdot a^2 \cdot NA)^{-3} \quad (144)$$

Also apply the expression:

$$(2 \cdot a_s \cdot a^2 \cdot NA)^3 \cdot a_g = i^{6i}$$

$$8 \cdot a_s^3 \cdot a^6 \cdot a_g \cdot NA^3 = i^{6i}$$

From the expressions (127) and (132) resulting the unity formula for the gravitational fine-structure constant  $a_g$ :

$$a_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot NA)^{-3} \quad (145)$$

Also apply the expression:

$$(2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot NA)^3 \cdot a_g = i^{6i} \cdot e^3$$

$$8 \cdot 10^{21} \cdot a_w^3 \cdot a^9 \cdot a_g \cdot NA^3 = i^{6i} \cdot e^3$$

From the expressions (130) and (132) resulting the unity formulas for the gravitational fine-structure constant  $a_g$ :

$$a_g = (10^7 \cdot a_w \cdot a_G^{1/2} \cdot e^{-1} \cdot a_s^{-1} \cdot a^{-1})^3 \quad (146)$$

Also apply the expressions:

$$a_g = 10^{21} \cdot a_w^3 \cdot a_G^{3/2} \cdot a_s^{-3} \cdot a^{-3} \cdot e^{-3}$$

$$a_g \cdot a_s^3 \cdot a^3 \cdot e^3 = 10^{21} \cdot a_w^3 \cdot a_G^{3/2}$$

So the unity formula for the gravitational fine-structure constant  $a_g$  is:

$$a_g^2 = (10^{14} \cdot a_w^2 \cdot a_G \cdot e^{-2} \cdot a_s^{-2} \cdot a^{-2})^3 \quad (147)$$

Also apply the expressions:

$$a_g^2 = 10^{42} \cdot a_w^6 \cdot a_G^3 \cdot e^{-6} \cdot a_s^{-6} \cdot a^{-6}$$

$$e^6 \cdot a_s^6 \cdot a^6 \cdot a_g^2 = 10^{42} \cdot a_w^6 \cdot a_G^3$$

$$a_g^2 \cdot (e \cdot a_s \cdot a)^6 = (10^{14} \cdot a_w^2 \cdot a_G)^3$$

From the expressions (130) and (132) resulting the unity formula for the gravitational fine-structure constant  $a_g$ :

$$a_g = i^{6i} \cdot (10^7 \cdot a_w \cdot a_G^{1/2} \cdot a_s^{-2} \cdot a^{-1})^3$$

$$a_g = 10^{21} \cdot i^{6i} \cdot (a_w \cdot a_G^{1/2} \cdot a_s^{-2} \cdot a^{-1})^3$$

$$a_g = 10^{21} \cdot i^{6i} \cdot a_w^3 \cdot a_G^{3/2} \cdot a_s^{-6} \cdot a^{-3} \quad (148)$$

Also apply the expressions:

$$a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2} = i^{2i} \cdot 10^7$$

$$a_g \cdot a_s^6 \cdot a^3 = 10^{21} \cdot i^{6i} \cdot a_w^3 \cdot a_G^{3/2}$$

So the unity formulas for the gravitational fine-structure constant  $a_g$  are:

$$a_g^2 = i^{6i} \cdot (10^{14} \cdot a_w^2 \cdot a_G \cdot a_s^{-4} \cdot a^{-2})^3$$

$$a_g^2 = 10^{42} \cdot i^{12i} \cdot (a_w^2 \cdot a_G \cdot a_s^{-4} \cdot a^{-2})^3$$

$$a_g^2 = 10^{42} \cdot i^{12i} \cdot a_w^6 \cdot a_G^3 \cdot a_s^{-12} \cdot a^{-6} \quad (149)$$

Also apply the expressions:



$$\alpha_g^2 \cdot \alpha_s^{12} \cdot a^6 \cdot a_w^{-6} \cdot a_G^{-3} = i^{12i} \cdot 10^{42}$$

$$(\alpha_s^6 \cdot a^3 \cdot \alpha_g)^2 = (10^{14} \cdot i^{4i} \cdot a_w^2 \cdot a_G)^3$$

$$\alpha_s^{12} \cdot a^6 \cdot \alpha_g^2 = 10^{42} \cdot i^{12i} \cdot a_w^6 \cdot a_G^3$$

So the unity formulas for the gravitational fine-structure constant  $\alpha_g$  are:

$$\alpha_g = \left( \frac{10^7 \alpha_w \sqrt{\alpha_G}}{e \alpha_s a} \right)^3 \quad (150)$$

$$\alpha_g^2 = 10^{42} \left( \frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 a^2} \right)^3 \quad (151)$$

$$\alpha_g = 10^{21} i^{6i} \left( \frac{\alpha_w \sqrt{\alpha_G}}{\alpha_s^2 a} \right)^3 \quad (152)$$

$$\alpha_g^2 = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha_s^2 a^2} \right)^3 \quad (153)$$

This expression connects the gravitational fine-structure constant  $\alpha_g$  with the four coupling constants. Perhaps the gravitational fine structure constant  $\alpha_g$  is the coupling constant for the fifth force. Some speculative theories have proposed a fifth force to explain various anomalous observations that do not fit existing theories. The characteristics of this fifth force depend on the hypothesis being advanced. Many postulate a force roughly the strength of gravity with a range of anywhere from less than a millimeter to cosmological scales. Another proposal is a new weak force mediated by W and Z bosons. The search for a fifth force has increased in recent decades due to two discoveries in cosmology which are not explained by current theories. It has been discovered that most of the mass of the universe is accounted for by an unknown form of matter called dark matter. Most physicists believe that dark matter consists of new, undiscovered subatomic particles, but some believe that it could be related to an unknown fundamental force. Second, it has also recently been discovered that the expansion of the universe is accelerating, which has been attributed to a form of energy called dark energy. Some physicists speculate that a form of dark energy called quintessence could be a fifth force.

### 13. Dimensionless unification of atomic physics and cosmology

In the context of cosmology the cosmological constant is a homogeneous energy density that causes the expansion of the universe to accelerate. Originally proposed early in the development of general relativity in order to allow a static universe solution it was subsequently abandoned when the universe was found to be expanding. Now the cosmological constant is invoked to explain the observed acceleration of the expansion of the universe. The cosmological constant is the simplest realization of dark energy, which is the more generic name given to the unknown cause of the acceleration of the universe. Its existence is also predicted by quantum physics, where it enters as a form of vacuum energy, although the magnitude predicted by quantum theory does not match that observed in cosmology.

The cosmological constant  $\Lambda$  is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. Nobody really knows what the cosmological constant is exactly, but it is required in cosmological equations in order to reconcile theory with our observations of the universe. One potential explanation for the cosmological constant lies in the realm of modern particle physics. Experiments have verified that empty space is permeated by countless virtual particles constantly popping in and out of existence. It is commonly believed that the cosmological constant problem can only be solved ultimately in a unified theory of quantum gravity and the standard model of electroweak and strong interactions, which is still absent so far. But connecting vacuum energy to the cosmological constant is not straightforward. Based on their observations of supernovas, astronomers estimate that dark energy should have a small and sedate value, just enough

to push everything in the universe apart over billions of years. Yet when scientists try to calculate the amount of energy that should arise from virtual particle motion, they come up with a result that's 120 orders of magnitude greater than what the supernova data suggest. The cosmological constant has the same effect as an intrinsic energy density of the vacuum,  $\rho_{vac}$  and an associated pressure. In this context, it is commonly moved onto the right-hand side of the equation, and defined with a proportionality factor of  $\Lambda = 8 \cdot \pi \cdot \rho_{vac}$  where unit conventions of general relativity are used (otherwise factors of  $G$  and  $c$  would also appear, i.e.:

$$\Lambda = 8\pi\rho_{vac}\frac{G}{c^4} = \kappa\rho_{vac}$$

where  $\kappa$  is Einstein's rescaled version of the gravitational constant  $G$ . The cosmological constant has been introduced in gravitational field equations by Einstein in 1917 in order to satisfy Mach's principle of the relativity of inertia. Then it was demonstrated by Cartan in 1922 that the Einstein field tensor including a cosmological constant  $\Lambda$ :

$$E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}$$

is the most general tensor in Riemannian geometry having null divergence like the energy momentum tensor  $T_{\mu\nu}$ . This theorem has set the general form of Einstein's gravitational field equations as  $E_{\mu\nu} = \kappa \cdot T_{\mu\nu}$  and established from first principles the existence of  $\Lambda$  as an unvarying true constant. The cosmological constant problem dates back to the realization that it is equivalent to a vacuum energy density. One of the main consequences in cosmology of a positive cosmological constant is an acceleration of the expansion of the universe. Such an acceleration has been first detected in 1981 in the Hubble diagram of infrared elliptical galaxies, yielding a positive value close to the presently measured one, but with still large uncertainties. Accurate measurements of the acceleration of the expansion since 20 years have reinforced the problem. The cosmological constant  $\Lambda$ , as it appears in Einstein's equations, is a curvature. As such, besides being an energy density, it is also the inverse of the square of an invariant cosmic length  $L$ .

In the early-mid 20th century Dirac and Zel'dovich were among the first scientists to suggest an intimate connection between cosmology and atomic physics. Though a revolutionary proposal for its time, Dirac's Large Number Hypothesis (1937) adopted a standard assumption of the non-existence of the cosmological constant term  $\Lambda=0$ . Zel'dovich insight (1968) was to realize that a small but nonzero cosmological term  $\Lambda>0$  allowed the present day radius of the Universe to be identified with the de Sitter radius which removed the need for time dependence in the fundamental couplings. Thus, he obtained the formula:

$$\Lambda = \frac{m_p^6 G^2}{\hbar^6}$$

where  $m$  is a mass scale characterizing the relative strengths of the gravitational and electromagnetic interactions, which he identified with the proton mass  $m_p$ .

Laurent Nottale in [18] which, instead, suggests the identification  $m = m_e/\alpha$ . He assumed that the cosmological constant  $\Lambda$  is the sum of a general-relativistic term and of the quantum, scale-varying, gravitational self-energy of virtual pairs. A renormalization group approach is used to describe its scale-dependence. We argue that the large scale value of  $\Lambda$  is reached at the classical electron scale. This reasoning provides with a large-number relation:

$$\alpha \frac{m_{pl}}{m_e} = \left( \frac{L}{l_{pl}} \right)^{\frac{1}{3}}$$

The cosmological constant  $\Lambda$  has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length  $L$ :

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d = \sqrt{\frac{3}{\Lambda}}$$

From this equation resulting the expressions for the gravitational fine structure constant  $\alpha_g$ :

$$\alpha \frac{m_{pl}}{m_e} = \left( l_{pl} \sqrt{\Lambda} \right)^{-\frac{1}{3}}$$

$$\alpha_g = l_{pl} \sqrt{\Lambda}$$

$$\alpha_g = \sqrt{\frac{G\hbar\Lambda}{c^3}}$$

So the cosmological constant  $\Lambda$  equals:

$$\Lambda = \alpha_g^2 l_{pl}^{-2}$$

$$\Lambda = \frac{l_{pl}^4}{r_e^6}$$

$$\Lambda = \alpha_g^2 \frac{c^3}{G\hbar}$$

$$\Lambda = \frac{G}{\hbar^4} \left( \frac{m_e}{a} \right)^6$$

From the expression (143) resulting the dimensionless unification of the atomic physics and the cosmology:

$$\alpha_g = (2 \cdot e \cdot a^2 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = (2 \cdot e \cdot a^2 \cdot N_A)^{-6} \quad (154)$$

$$(2 \cdot e \cdot a^2 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = 1 \quad (155)$$

Now we will use the unity formulas of the dimensionless unification of atomic physics and cosmology to find the equations of the cosmological constant. For the cosmological constant equals:

$$\Lambda = \left( 2e\alpha^2 N_A \right)^{-6} \frac{c^3}{G\hbar} \quad (156)$$

From the expression (144) resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g = i^{6i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-6} \quad (157)$$

$$(2 \cdot a_s \cdot a^2 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = i^{12i} \quad (158)$$

For the cosmological constant equals:

$$\Lambda = i^{12i} (2\alpha_s a^2 N_A)^{-6} \frac{c^3}{G\hbar} \quad (159)$$

From the expression (145) resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-3}$$

$$|pl|^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot NA)^{-6} \quad (160)$$

$$(2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot NA)^6 \cdot |pl|^2 \cdot \Lambda = i^{12i} \cdot e^6 \quad (161)$$

For the cosmological constant equals:

$$\Lambda = i^{12i} e^6 (2 \cdot 10^7 a_w a^3 N_A)^{-6} \frac{c^3}{G\hbar} \quad (162)$$

From the expression (146) resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g^2 = 10^{42} \left( \frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2} \right)^3$$

$$l_{pl}^2 \Lambda = 10^{42} \left( \frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2} \right)^3 \quad (163)$$

$$e^6 \cdot a_s^6 \cdot a^6 \cdot |pl|^2 \cdot \Lambda = 10^{42} \cdot a_G^3 \cdot a_w^6 \quad (164)$$

For the cosmological constant equals:

$$\Lambda = 10^{42} \left( \frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2} \right)^3 \frac{c^3}{G\hbar} \quad (165)$$

From the expression (147) resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g^2 = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3$$

$$l_{pl}^2 \Lambda = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \quad (166)$$

$$a_s^{12} \cdot a^6 \cdot |pl|^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot a_G^3 \cdot a_w^6 \quad (167)$$

This unity formula is a simple analogy between atomic physics and cosmology. For the cosmological constant equals:

$$\Lambda = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \frac{c^3}{G\hbar} \quad (168)$$

The Equation of the Universe is:

$$\frac{\Lambda G\hbar}{c^3} = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \quad (169)$$

## 14. Cosmological parameters

The Hubble constant  $H_0$  is one of the most important numbers in cosmology because it is required to estimate the size and age of the universe. This number indicates the rate at which the universe is expanding. The Hubble constant can be used to determine the inherent brightness and masses of stars in nearby galaxies, examine the same properties in more distant galaxies and galaxy clusters, infer the amount of dark matter in the universe, and obtain the scale size of distant clusters as far as clusters test for theoretical cosmological models. In 1929, American astronomer Edwin

Hubble announced his discovery that galaxies, in all directions, seemed to be moving away from us and have greater displacement for attenuated galaxies. However, the true value for  $H_0$  is very complicated. Astronomers need two measurements:

- a) First, spectroscopic observations reveal the redshift of the galaxy, showing its radial velocity.
- b) The second measurement, the most difficult value, is the exact distance of the galaxy from Earth.

The unit of the Hubble constant is  $1 \text{ km/s/Mpc}$ . The 2018 CODATA recommended value of the Hubble constant is  $H_0 = 67,66 \pm 0,42 \text{ (km/s)/Mpc} = (2,1927664 \pm 0,0136) \times 10^{-18} \text{ s}^{-1}$ . Hubble length or Hubble distance is a unit of distance in cosmology, defined as the speed of light multiplied by Hubble time  $L_H = c \cdot H_0^{-1}$ . This distance is equivalent to 4,550 million parsecs, or 14,4 billion light-years, 13,8 billion years. Hubble's distance would be the distance between the Earth and the galaxies currently falling away from us at the speed of light, as shown by the substitution  $r = c \cdot H_0^{-1}$  in the equation for Hubble's law,  $u = H_0 \cdot r$ .

The critical density is the average density of matter required for the Universe to just halt its expansion, but only after an infinite time. A Universe with a critical density is said to be flat. In his theory of general relativity, Einstein demonstrated that the gravitational effect of matter is to curve the surrounding space. In a Universe full of matter, both its overall geometry and its fate are controlled by the density of the matter within it. If the density of matter in the Universe is high (a closed Universe), self-gravity slows the expansion until it halts, and ultimately re-collapses. In a closed Universe, locally parallel light rays converge at some extremely distant point. This is referred to as spherical geometry. If the density of matter in the Universe is low (an open Universe), self-gravity is insufficient to stop the expansion, and the Universe continues to expand forever (albeit at an ever decreasing rate). In an open Universe, locally parallel light rays ultimately diverge. This is referred to as hyperbolic geometry. Balanced on a knife edge between Universes with high and low densities of matter, there exists a Universe where parallel light rays remain parallel. This is referred to as a flat geometry, and the density is called the critical density. In a critical density Universe, the expansion is halted only after an infinite time.

To date, the critical density is estimated to be approximately five atoms per cubic meter, whereas the average density of ordinary matter in the Universe is believed to be 0,2–0,25 atoms per cubic meter. A much greater density comes from the unidentified dark matter; both ordinary and dark matter contribute in favor of contraction of the universe. However, the largest part comes from so-called dark energy, which accounts for the cosmological constant term. Although the total density is equal to the critical density the dark energy does not lead to contraction of the universe but rather may accelerate its expansion. Therefore, the universe will likely expand forever. An expression for the critical density is found by assuming  $\Lambda$  to be zero and setting the normalized spatial curvature,  $k$ , equal to zero. When the substitutions are applied to the first of the Friedmann equations we find:

$$\rho_c = \frac{3H_0^3}{8\pi G}$$

It should be noted that this value changes over time. The critical density changes with cosmological time, but the energy density due to the cosmological constant remains unchanged throughout the history of the universe. The amount of dark energy increases as the universe grows, while the amount of matter does not. The density parameter  $\Omega$  is defined as the ratio of the actual density  $\rho$  to the critical density  $\rho_c$  of the Friedmann universe. The relation between the actual density and the critical density determines the overall geometry of the universe, when they are equal, the geometry of the universe is flat (Euclidean). The galaxies we see in all directions are moving away from the Earth, as evidenced by their red shifts. Hubble's law describes this expansion. Remarkably, study of the expansion rate has shown that the universe is very close to the critical density that would cause it to expand forever.

The density parameter  $\Omega$  is defined as the ratio of the average density of matter and energy in the Universe  $\rho$  to the critical density  $\rho_c$  of the Friedmann universe. The relation between the actual density and the critical density determines the overall geometry of the universe; when they are equal, the geometry of the universe is flat (Euclidean). In earlier models, which did not include a cosmological constant term, critical density was initially defined as the watershed point between an expanding and a contracting Universe. The density parameter is given by:

$$\Omega_0 = \frac{\rho}{\rho_c}$$

where  $\rho$  is the actual density of the Universe and  $\rho_c$  the critical density. Although current research suggests that  $\Omega_0$  is very close to 1, it is still of great importance to know whether  $\Omega_0$  is slightly greater than 1, less than 1, or exactly equal to 1, as this reveals the ultimate fate of the Universe. If  $\Omega_0$  is less than 1, the Universe is open and will continue to expand forever. If  $\Omega_0$  is greater than 1, the Universe is closed and this will eventually halt its expansion and recollapse. If  $\Omega_0$  is exactly equal to 1 then the Universe is flat and contains enough matter to halt the expansion but

not enough to recollapse it. It is important to note that the  $\rho$  used in the calculation of  $\Omega_0$  is the total mass/energy density of the Universe. In other words, it is the sum of a number of different components including both normal and dark matter as well as the dark energy suggested by recent observations. We can therefore write:

$$\Omega_0 = \Omega_B + \Omega_D + \Omega_\Lambda$$

$$\Omega_0 = \Omega_m + \Omega_\Lambda$$

$$\Omega_0 = \Omega_B + \Omega_{D+\Lambda}$$

where:

$\Omega_B$  is the density parameter for normal baryonic matter,

$\Omega_D$  is the density parameter for dark matter,

$\Omega_\Lambda$  is the density parameter for dark energy,

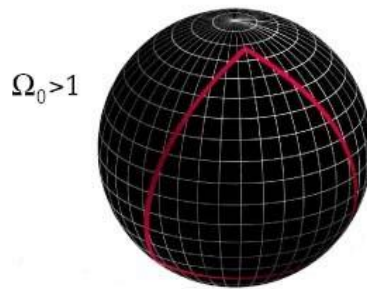
$\Omega_m$  is the sum of the density parameter for normal baryonic matter and the density parameter for dark matter,

$\Omega_{D+\Lambda}$  is the sum of the density parameter for the density parameter for dark matter and the density parameter for dark energy.

The sum of the contributions to the total density parameter  $\Omega_0$  at the current time is:

$$\Omega_0 = 1,02 \pm 0,02$$

Current observations suggest that we live in a dark energy dominated Universe with  $\Omega_\Lambda = 0,73$ ,  $\Omega_D = 0,23$  and  $\Omega_B = 0,04$ . To the accuracy of current cosmological observations, this means that we live in a flat,  $\Omega_0 = 1$  Universe. Instead of the cosmological constant  $\Lambda$  itself, cosmologists often refer to the ratio between the energy density due to the cosmological constant and the critical density of the universe, the peak point of a density sufficient to prevent the universe from expanding forever, at one level of the universe is the ratio between the energy of the universe due to the cosmological constant  $\Lambda$  and the critical density of the universe, that is what we would call the fraction of the universe consisting of dark energy.



**Figure 16.** If  $\Omega_0$  is greater than 1, the Universe is closed and this will eventually halt its expansion and recollapse.

The assessment of baryonic matter at the current time was assessed by WMAP to be  $\Omega_B = 0,044 \pm 0,004$ . From the dimensionless unification of the fundamental interactions the density parameter for normal baryonic matter is:

$$\Omega_B = 10^{-7} \cdot a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2}$$

$$\Omega_B = 2 \cdot 10^7 \cdot N_A \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2}$$

$$\Omega_B = 2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a})$$

$$\Omega_B = 2 \cdot N_A \cdot a_s \cdot a \cdot a_G^{1/2}$$

$$\Omega_B = 2^{-1} \cdot a_s \cdot (e^{i/a} + e^{-i/a})$$

$$\Omega_B = a_w^{-1} \cdot a_s^2 \cdot 10^{-7}$$

$$\Omega_B = e^{-1} \cdot a_s$$

$$\Omega_B = e^{-\pi}$$

$$\Omega_B = i^{2i} \quad (170)$$

$$\Omega_B = 0,043214$$

$$\Omega_B = 4,32\%$$

From the dimensionless unification of the fundamental interactions the density parameter for dark matter is:

$$\Omega_D = 6 \cdot \Omega_B = 6 \cdot e^{-\pi} = 6 \cdot i^{2i} = 0,2592835 = 25,92\%$$

The fraction of the effective mass of the universe attributed to dark energy or the cosmological constant is  $\Omega_\Lambda = 0,73 \pm 0,04$ . With 73% of the influence on the expansion of the universe in this era, dark energy is viewed as the dominant influence on that expansion. The previous history of the big bang is viewed as being at first radiation dominated, then matter dominated, and now having passed into the era where dark energy is the dominant influence. The density parameter for dark energy is defined as:

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}$$

From the dimensionless unification of the fundamental interactions the density parameter for the dark energy is:

$$\Omega_\Lambda = 17 \cdot \Omega_B = 17 \cdot e^{-\pi} = 17 \cdot i^{2i} = 0,73463661 = 73,46\%$$

The sum of the density parameter for normal baryonic matter and the density parameter for the dark matter is:

$$\Omega_m = 7 \cdot e^{-\pi} = 7 \cdot i^{2i} = 0,3024974 = 30,25\%$$

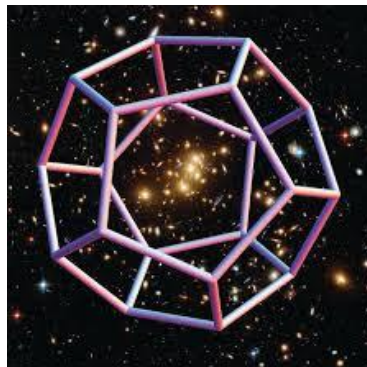
The sum of the density parameter for normal baryonic matter and the density parameter for the dark energy is:

$$\Omega_{D+\Lambda} = 23 \cdot \Omega_B = 23 \cdot e^{-\pi} = 23 \cdot i^{2i} = 0,99392 = 99,39\%$$

The sum of the density parameter for normal baryonic matter and the density parameter for the dark energy is:

$$\Omega_0 = \Omega_B + \Omega_D + \Omega_\Lambda = i^2 + 6 \cdot i^{2i} + 17 \cdot i^{2i} = 24 \cdot i^{2i} = 1,037134 \quad (171)$$

A positively curved universe is described by elliptic geometry, and can be thought of as a three-dimensional hypersphere, or some other spherical 3-manifold (such as the Poincaré dodecahedral space), all of which are quotients of the 3-sphere. Poincaré dodecahedral space is a positively curved space, colloquially described as "soccer ball-shaped", as it is the quotient of the 3-sphere by the binary icosahedral group, which is very close to icosahedral symmetry, the symmetry of a soccer ball. This was proposed by Jean-Pierre Luminet and colleagues in 2003 and an optimal orientation on the sky for the model was estimated in 2008. When the universe expands sufficiently, the cosmological constant  $\Lambda$  becomes more important than the energy density of matter in determining the fate of the universe. If  $\Lambda > 0$  there will be an approximately exponential expansion. This seems to be happening now in our universe.



**Figure 17.** Perhaps the shape of the universe is Poincaré dodecahedral space.

In cosmology, the equation of state of a perfect fluid is characterized by a dimensionless number  $w$ , equal to the ratio of its pressure  $p$  to its energy density  $\rho$ :

$$w = \frac{p}{\rho}$$

Stable  $w$  of the state equation is the ratio of the pressure exerted by dark energy on the universe to the energy per unit volume. This ratio is  $w=-1$  for a real cosmological constant and is generally different for alternating time changes of vacuum energy forms quintessence. This ratio is often used by scientists. The state equation  $w$  has value  $w=-1,028 \pm 0,032$ . This number means how quickly the dark energy density changes as the universe expands. If  $w=-1$ , the density is strictly constant, if  $w>-1$ , the density decreases, and if  $w<-1$ , the density actually increases with time. Einstein's cosmological constant is just the idea that there is a fixed minimum energy density everywhere in the universe; this vacuum energy would correspond to  $w=-1$ . It's easy enough to get an energy density that slowly diminishes, with  $w>-1$ , all you need to do is invent some scalar field slowly rolling down a very gentle potential, so that the energy is nearly constant but in fact gradually diminishes. If  $w<-1$ , corresponding to a gradually increasing energy density. It's not what you would typically expect; the expansion of the universe tends to dilute energy, not increase it. So for some time cosmologists who put observational limits on the value of  $w$  would exclude  $w<-1$  by hand. The energy density thus tends to increase, implying  $w<-1$  called "phantom energy" because the Phantom Menace had just come out and also because negative-kinetic-energy fields also appear in the context of quantized gauge theories, where they are called "ghost" fields. If  $w$  is less than  $-1$  and constant, the energy density grows without bound and everything in the universe is ripped to shreds at some finite point in the future. From the dimensionless unification of the fundamental interactions the state equation  $w$  has value:

$$w = -24 \cdot e^{-\pi} = -24 \cdot i^{2i} = -1,037134 \quad (172)$$

For as much as  $w<-1$ , the density actually increases with time.

The famous formula  $E=m \cdot c^2$  of Einstein is better replaced by  $E=K \cdot m \cdot c^2$ . In this  $E$  becomes the sum of two types of energy, the measured normal energy density of the universe  $E(O)$  and the sum of the dark energy and the dark matter density of the universe  $E(D)$ . This reveals hitherto unsuspected quantum roots for the equation  $E=m \cdot c^2$ . Einstein's equation  $E=m \cdot c^2$  is actually the sum of two parts of quantum relativity  $E(O)$  from the quantum particle and  $E(D)$  from the quantum wave. From the dimensionless unification of the fundamental interactions for the measurable ordinary energy  $E(O)$  apply:

$$E(O) = i^{2i} \cdot m \cdot c^2$$

Also from the dimensionless unification of the fundamental interactions for the sum of the dark energy with the dark matter density of the universe  $E(D)$  apply:

$$E(D) = 23 \cdot i^{2i} \cdot m \cdot c^2$$

So for the total energy  $E$  apply:

$$E = K \cdot m \cdot c^2$$

$$E = E(O) + E(D)$$

$$E = i^{2i} \cdot m \cdot c^2 + 23 \cdot i^{2i} \cdot m \cdot c^2$$

$$E = (i^{2i} + 23 \cdot i^{2i}) \cdot m \cdot c^2$$

$$E = 24 \cdot i^{2i} \cdot m \cdot c^2 \quad (173)$$

Other forms of the equation are:

$$E = 12 \cdot i^{2i} \cdot m \cdot c^2 + 12 \cdot i^{2i} \cdot m \cdot c^2$$

$$E = 12 \cdot i^{2i} \cdot m \cdot c^2 - i^2 \cdot 12 \cdot i^{2i} \cdot m \cdot c^2$$



$$E=12 \cdot i^{2i} \cdot m \cdot c^2 - 12 \cdot i^{2i} \cdot m \cdot (i \cdot c)^2$$

$$12 \cdot i^{2i} \cdot m \cdot (i \cdot c)^2 + E = 12 \cdot i^{2i} \cdot m \cdot c^2 \quad (174)$$

The second theory is that the density parameter for normal baryonic matter is:

$$\Omega_B = e^{-n} = i^{2i} = 0,043214 = 4,32\%$$

The density parameter for dark matter is:

$$\Omega_D = 6 \cdot e^{-n} = 6 \cdot i^{2i} = 0,25928 = 25,928\%$$

The density parameter for dark energy is:

$$\Omega_\Lambda = 16 \cdot e^{-n} = 16 \cdot i^{2i} = (4 \cdot i^i)^2 = 0,69142 = 69,142\%$$

The sum of the density parameter for normal baryonic matter and the density parameter for the dark matter is:

$$\Omega_m = 7 \cdot e^{-n} = 7 \cdot i^{2i} = 0,3024974 = 30,25\%$$

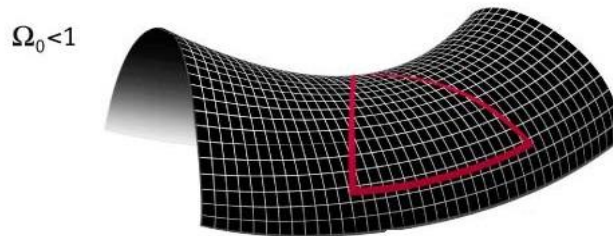
The sum of the density parameter for normal baryonic matter and the density parameter for the dark energy is:

$$\Omega_{D+\Lambda} = 22 \cdot e^{-n} = 22 \cdot i^{2i} = 0,95070 = 95,07\%$$

The sum of the density parameter for normal baryonic matter and the density parameter for the dark energy is:

$$\Omega_0 = \Omega_B + \Omega_D + \Omega_\Lambda = i^2 + 6 \cdot i^{2i} + 16 \cdot i^{2i} = 23 \cdot i^{2i} = 0,99392 = 99,39\%$$

If  $\Omega_0$  is greater than 1, the Universe is closed and this will eventually halt its expansion and recollapse. A hyperbolic universe, one of a negative spatial curvature, is described by hyperbolic geometry, and can be thought of locally as a three-dimensional analog of an infinitely extended saddle shape. There are a great variety of hyperbolic 3-manifolds, and their classification is not completely understood.



**Figure 18.** If  $\Omega_0$  is less than 1, the Universe is open and will continue to expand forever.

Those of finite volume can be understood via the Mostow rigidity theorem. For hyperbolic local geometry, many of the possible three-dimensional spaces are informally called "horn topologies", so called because of the shape of the pseudosphere, a canonical model of hyperbolic geometry. An example is the Picard horn, a negatively curved space colloquially described as "funnel-shaped". The state equation  $w$  has value:

$$w = -23 \cdot e^{-n} = -23 \cdot i^{2i} = -0,99392$$

The thirty theory is that the density parameter for normal baryonic matter is:

$$\Omega_B = e^{-n} = i^{2i} = 0,043214 = 4,32\%$$

The density parameter for dark matter is:

$$\Omega_D = 5 \cdot e^{-n} = 5 \cdot i^{2i} = 0,216069 = 21,60\%$$

The density parameter for dark energy is:

$$\Omega_{\Lambda}=17 \cdot e^{-n}=17 \cdot i^{2i}=0,73463661=73,46\%$$

The sum of the density parameter for normal baryonic matter and the dark matter is:

$$\Omega_m=6 \cdot e^{-n}=6 \cdot i^{2i}=0,25928=25,92\%$$

The sum of the density parameter for normal baryonic matter and the dark energy is:

$$\Omega_{D+\Lambda}=22 \cdot e^{-n}=22 \cdot i^{2i}=0,95070=95,07\%$$

The sum of the density parameter for normal baryonic matter and the density parameter for the dark energy is:

$$\Omega_0=\Omega_B+\Omega_D+\Omega_{\Lambda}=i^2+5 \cdot i^{2i}+17 \cdot i^{2i}=23 \cdot i^{2i}=0,99392$$

The state equation  $w$  has value:

$$w=-23 \cdot e^{-n}=-23 \cdot i^{2i}=-0,99392$$

## 15. Maximum and minimum values for natural quantities

Since time philosophers,poets,and scientists have pondered the relationship between the microcosm and the macrocosm. This theory was started by Pythagoras who saw the universe and the body as a harmonious unity. The microcosm and the macrocosm. Since Newton,the scales of the largest and the smallest have extended by ten orders of magnitude in both directions. It was only in the late 1910s,however,that the first physical fact was discovered that could provide a quantitative clue to the interconnection between the micro-and mega-worlds. It was a mathematician, Hermann Weyl,who made this discovery. His discovery later gave rise to such different ideas as the hypothetical variation of the gravitational constant and the anthropic principle. Although this link between the micro-and mega-worlds is regarded as an empirical fact,its recognition was intertwined with developments in advanced theoretical physics. The laws of physics have a set of fundamental constants,and it is generally admitted that only dimensionless combinations of constants have physical significance. The most fully developed version of the idea in antiquity was made by Plato,but fragmentary evidence indicates that philosophers before him articulated some version of it. The idea may have begun as an archetypal theme of mythology that the pre-Socratic philosophers reworked into a more systematic form. Unfortunately,it is impossible to reconstruct their thinking in much detail,and clear references attributing the doctrine to Democritus and Pythagoras are quite late,dating to the fifth and ninth centuries C.E.,respectively. Among extant Greek texts,the term first appears in the Physics of Aristotle,where it occurs in an incidental remark. Plato did not use the terminology when he developed the idea.

There have been theories of the shortest and largest natural quantities. F. Dieterlen in [22] presented a law. Physics can be summed up in a few limiting statements. They imply that in nature every physical observable is bounded by a value close to the Planck value. The speed limit is equivalent to special relativity,the force limit to general relativity,and the action limit to quantum theory. The newly discovered maximum force principle makes it possible to summarize special relativity, quantum theory and general relativity into a fundamental limiting principle each.

We present the law of the gravitational fine-structure constant  $\alpha_g$  followed by ratios of maximum and minimum theoretical values for natural quantities. This theory uses quantum mechanics,cosmology,thermodynamics,and special and general relativity. Length  $l$ ,time  $t$ ,speed  $u$  and temperature  $T$  have the same max/min ratio which is.

$$\alpha_g = \frac{l_{min}}{l_{max}} = \frac{t_{min}}{t_{max}} = \frac{v_{min}}{v_{max}} = \frac{T_{min}}{T_{max}} \quad (175)$$

Energy  $E$ , mass  $M$ , action  $A$ ,momentum  $P$  and entropy  $S$  have another max/min ratio,which is the square of  $\alpha_g$ .

$$\alpha_g^2 = \frac{E_{min}}{E_{max}} = \frac{M_{min}}{M_{max}} = \frac{A_{min}}{A_{max}} = \frac{P_{min}}{P_{max}} = \frac{S_{min}}{S_{max}} \quad (176)$$

Force  $F$  has max/min ratio which is  $\alpha_g^4$ :

$$\alpha_g^4 = \frac{F_{min}}{F_{max}} \quad (177)$$

Mass density has max/min ratio which is  $\alpha_g^5$ :

$$\alpha_g^5 = \frac{\rho_{min}}{\rho_{max}} \quad (178)$$

A smallest length in nature thus implies that there is no way to define exact boundaries of objects or elementary particles. Length  $l$  has the max/min ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{max}} \quad (179)$$

The maximum distance  $l_{max}$  corresponds to the distance of the universe:

$$l_{max} = LH = c \cdot H_0^{-1} = \alpha_g \cdot l_{min}$$

Perhaps for the minimum distance  $l_{min}$  apply:

$$l_{min} = 2 \cdot e \cdot |p| \quad (180)$$

So for the Bohr radius  $a_0$  apply:

$$a_0 = 2 \cdot e \cdot NA \cdot |p|$$

$$a_0 = NA \cdot l_{min}$$

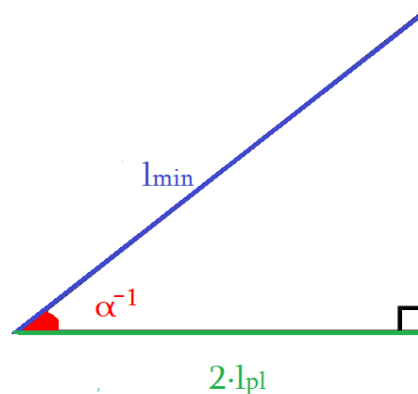
From expressions (22) and (179) apply:

$$\cos \alpha^{-1} = e^{-1}$$

$$\cos \alpha^{-1} \cdot l_{min} = 2 \cdot |p|$$

$$\cos \alpha^{-1} = \frac{2l_{pl}}{l_{min}} \quad (181)$$

A precise discussion shows that measurement errors increase when the characteristic measurement energy approaches the Planck energy. In that domain, the measurement errors of any observable are comparable to the measurement values. Limited measurement precision implies that at Planck energy it is impossible to speak about points, instants, events or dimensionality. Limited precision implies that no observable can be described by real numbers. Limited measurement precision also implies that at Planck length it is impossible to distinguish positive and negative time values: particles and antiparticles are thus not clearly distinguished at Planck scales. The fundamental unit of length in this unit system is the Planck length  $l_{pl}$ . Spacetime is proposed to be a lattice structure, in which its unit cells have sides of length  $a$ , marked below in the next figure.



**Figure 19.** Geometric representation of the fundamental unit of length.

Gravitons do indeed have mass, and their motions generate kinetic energy. Thus, they have both energy and mass, and they obey the law of conservation of energy and matter. If gravitons did not have mass there would be no physics that we could understand. Other particles have mass, but they are much larger, much less numerous, and cannot substitute for the gravitational effects which generate space curvature. The great mystery of so-called force at a distance is explained by the mass of gravitons. Where things get really interesting is in the smallest dimensions. Even an incredibly small and nearly massless particle can have great adjacent gravitational powers, as long as the centers of two attracted particles are sufficiently close. Dark energy and dark matter are thus aspects of a phenomenon. That phenomenon is the flow of gravitons on a Planck scale, expressed as spacetime foam. Gravitons flow on a massive scale among universe bubbles and the matter between. Given enough flowing gravitons in the spacetime foam, on a scale the human mind can hardly comprehend, there is apparent force at a distance, expressed as the bending of space. Mass M have max/min ratio, which is the square of  $\alpha_g$ :

$$\alpha_g^2 = \frac{M_{min}}{M_{max}} \quad (182)$$

Also apply the expressions:

$$M_{max} = \alpha_g^2 \cdot M_{min}$$

$$l_{max}^2 \cdot M_{min} = l_{min}^2 \cdot M_{max}$$

$$M_{min} \cdot M_{max} = m_{pl}^2$$

The following applies to the minimum mass  $M_{min}$ :

$$M_{min} c^2 = \frac{\hbar}{t_{max}}$$

$$M_{min} c^2 = \hbar H_0$$

$$M_{min} = \frac{\hbar H_0}{c^2}$$

$$M_{min} = \frac{\hbar}{c l_{max}}$$

So apply the expressions:

$$M_{min} = \frac{\hbar}{c} \sqrt{\Lambda} \quad (183)$$

$$M_{min} = \frac{m_{pl}^2}{M_{max}} \quad (184)$$

$$M_{min} = \frac{m_{pl}^2}{M_{max}} \quad (185)$$

Therefore for the minimum mass  $M_{min}$  apply:

$$M_{min} = \alpha_g m_{pl} \quad (186)$$

$$M_{min} = \frac{\alpha_G}{\alpha^3} m_e \quad (187)$$

$$M_{min} = \frac{\sqrt[3]{\alpha_g^2}}{\alpha} m_e \quad (188)$$

R. Adler in [21] calculated the energy ratio in cosmology, the ratio of the dark energy density to the Planck energy density. Atomic physics has two characteristic energies, the rest energy of the electron  $E_e$ , and the binding energy of the hydrogen atom  $E_H$ . The rest energy of the electron  $E_e$  is defined as:

$$E_e = m_e c^2$$

The binding energy of the hydrogen atom  $E_H$  is defined as:

$$E_H = \frac{m_e e^4}{2\hbar^2}$$

Their ratio is equal to half the square of the fine-structure constant:

$$\frac{E_H}{E_e} = \frac{\alpha^2}{2}$$

Cosmology also has two characteristic energy scales, the Planck energy density  $\rho_{pl}$ , and the density of the dark energy  $\rho_\Lambda$ . The Planck energy density is defined as:

$$\rho_{pl} = \frac{E_{pl}}{l_{pl}} = \frac{c^7}{\hbar G^2}$$

To obtain an expression for the dark energy density in terms of the cosmological constant we recall that the cosmological term in the general relativity field equations may be interpreted as a fluid energy momentum tensor of the dark energy according to so the dark energy density  $\rho_\Lambda$  is given by:

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G}$$

The ratio of the energy densities is thus the extremely small quantity:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{\alpha_g^2}{8\pi}$$

So with expression (139) for the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2\varphi^{-5}}{3^3\pi\varphi^5} \times 10^{-120} \quad (189)$$

Some authors consider the small value of the ratio to be arguably one of the most mysterious problems in present day physics. The understanding of atomic structure required the discovery of the fundamental dynamical constant  $\hbar$ . Viewed in this way the cosmological analog of  $\hbar$  is  $\Lambda$ , but any dynamical role it may play is not yet apparent. It is amusing to note that in the presence of two length scales, and their dimensionless ratio, dimensional analysis becomes problematic, a dimensional estimate can contain an arbitrary function of the ratio, for example a power or a logarithm. In the case of cosmology it is clear that dimensional estimates, with two disparate length scales, may be much worse than useless.

In 1961, Dicke observed that a dimensionless number must necessarily be large to make the lifetime of stars long enough to produce heavy chemical elements such as carbon. Knowing that carbon is the most essential element for biological materials, this is the first claim called "Human Coincidence", which infers that the connection between physical constants is necessary for the existence of life in the universe. Thermonuclear combustion is necessary for the production of elements heavier than hydrogen. Again it takes several billion years for this to occur. type of conversion inside a star. According to the general theory of relativity no universe can provide several billion years of time unless it is several light-years in extent. Serious criticisms and interpretations have been made on the issue of the large number hypothesis and the existence of intelligent beings or life. One of the most difficult issues in

understanding consciousness is understanding how information is synthesized to form our subjective experience. The widely accepted hypothesis is that gamma currents ranging from many places in the brain combine to create a unified subjective experience. In this way, neurons performing different tasks in separate areas of the brain are divided into a single instantaneous activity. From [23] the gamma rhythm is a pattern of neuronal oscillations whose frequency ranges from 25 Hz to 100 Hz although 40 Hz is typical. Gamma frequency oscillations are present during wakefulness and REM sleep. Changes in electrical membrane potential generate neuronal action potentials. Oscillatory activity of neurons is connected to these spikes. The oscillation of the single neuron can be observed in fluctuations at the threshold of the membrane potential. The time quantum in the brain  $t_B$ , the smallest unit of time that related to the 40 Hz oscillation of the gamma rate:

$$\frac{t_B}{t_{pl}} = \sqrt[3]{\alpha_g^2} \quad (190)$$

The Planck time depends on the fundamental constants such as the Planck constant  $h$ , the gravitational constant  $G$  and the speed of light  $c$ , while it is not clear whether the shorter time scale in the brain also depends on these fundamental constants. Thus, the observer who can observe a universe tuned to the various fundamental constants must have synchronous activity of gamma oscillations of about 40 Hz in his nervous system. This is what we find from the experimental results in modern neuroscience.

## 16. Conclusions

It presented the dimensionless unification of the fundamental interactions. We reached the conclusion of the simple unification of the nuclear and the atomic physics:

$$10 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha})^{1/2} = 13 \cdot i$$

We calculated the unity formulas that connect the coupling constants of the fundamental forces. The dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e \cdot \alpha_s = 10^7 \cdot \alpha_w$$

$$\alpha_s^2 = i^{2i} \cdot 10^7 \cdot \alpha_w$$

The dimensionless dimensionless unification of the strong nuclear and electromagnetic interactions:

$$\alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot i^{2i}$$

The dimensionless dimensionless unification of the weak nuclear and electromagnetic interactions:

$$10^7 \cdot \alpha_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot e \cdot i^{2i}$$

The dimensionless unification of the strong nuclear, the weak nuclear and electromagnetic interactions:

$$10^7 \cdot \alpha_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot \alpha_s$$

The dimensionless unification of the gravitational and the electromagnetic interactions:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1$$

$$16 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = (e^{i/\alpha} + e^{-i/\alpha})^2$$

The dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions:

$$4 \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i}$$

$$\alpha^2 \cdot (e^{i/\alpha} + e^{-i/\alpha}) \cdot \alpha_s^4 \cdot \alpha_G \cdot N_A^2 = i^{8i}$$

The dimensionless unification of of the weak nuclear, the gravitational and the electromagnetic interactions:

$$4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \cdot e^2$$

$$10^{14} \cdot \alpha^2 \cdot (e^{i/\alpha} + e^{-i/\alpha})^2 \cdot \alpha_w^2 \cdot \alpha_G \cdot N_A^2 = i^{8i}$$

The dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions:

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2$$

$$8 \cdot 10^7 \cdot N_A^2 \cdot \alpha_w \cdot \alpha^2 \cdot \alpha_G = \alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha})$$

We found the formula for the Gravitational constant:

$$G = \alpha_s^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

We calculated the expression that connects the gravitational fine structure constant with the four coupling constants:

$$\alpha_g^2 = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3$$

Perhaps the gravitational fine structure constant is the coupling constant for the fifth force. It presented that the gravitational fine structure constant is a simple analogy between atomic physics and cosmology. The conclusion of the dimensionless unification of atomic physics and cosmology:

$$\alpha_s^{12} \cdot \alpha^6 \cdot |p|^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot \alpha_G^3 \cdot \alpha_w^6$$

We found the formula for the cosmological constant:

$$\Lambda = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \frac{c^3}{G \hbar}$$

The Equation of the Universe is:

$$\frac{\Lambda G \hbar}{c^3} = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3$$

We proposed a possible solution for the cosmological parameters. From the dimensionless unification of the fundamental interactions the density parameter for normal baryonic matter is:

$$\Omega_B = e^{-n} = i^{2i} = 0,043214 = 4,32\%$$

The density parameter for dark matter is:

$$\Omega_D = 6 \cdot e^{-n} = 6 \cdot i^{2i} = 0,2592835 = 25,92\%$$

The density parameter for the dark energy is:

$$\Omega_\Lambda = 17 \cdot e^{-n} = 17 \cdot i^{2i} = 0,73463661 = 73,46\%$$

The sum of the density parameter for normal baryonic matter and the density parameter for the dark energy is:

$$\Omega_0 = 24 \cdot e^{-n} = 24 \cdot i^{2i} = 1,037134$$

A positively curved universe is described by elliptic geometry, and can be thought of as a three-dimensional

hypersphere, or some other spherical 3-manifold, such as the Poincaré dodecahedral space, all of which are quotients of the 3-sphere. The state equation  $w$  has value:

$$w = -24 \cdot e^{-\pi} = -24 \cdot i^{2i} = -1,037134$$

For as much as  $w < -1$ , the density actually increases with time. Finally we presented the law of the gravitational fine-structure constant  $\alpha_g$  followed by ratios of maximum and minimum theoretical values for natural quantities. Perhaps for the minimum distance  $l_{\min}$  apply:

$$l_{\min} = 2 \cdot e \cdot |p|$$

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