Unified Modeling that Explains Dark Matter Data, Dark Energy Effects, and Galaxy Formation Stages

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Abstract

Physics lacks a confirmed description of dark matter, has yet to develop an adequate understanding of dark energy, and includes unverified conjectures regarding new elementary particles. This essay features modeling that addresses those problems and explains otherwise unexplained data. Our modeling starts from five bases – multipole expansions for the electromagnetic and gravitational fields associated with an object, the list of known elementary particles, some aspects of mathematics for isotropic harmonic oscillators, concordance cosmology, and a conjecture that the universe includes six isomers of most elementary particles. The multipole expansions – which have use in conjunction with Newtonian kinematics modeling, special relativity, and general relativity – lead to a catalog of kinematics properties such as charge, magnetic moment, mass, and repulsive gravitational pressure. The multipole expansions also point to all known elementary particles, some properties of those particles, and properties of some would-be elementary bosons and elementary fermions. The harmonic-oscillator mathematics points to Gauge symmetries regarding some elementary bosons. The would-be elementary fermions lack charge and would measure as dark matter. The conjecture regarding six isomers of most elementary particles rounds out and dominates our specification for dark matter. Five of the isomers form the basis for most dark matter. Our modeling explains ranges of observed ratios of dark matter effects to ordinary matter effects – for the universe, galaxy clusters, two sets of galaxies observed at high redshifts, three sets of galaxies observed at modest redshifts, and one type of depletion of cosmic microwave background radiation. Our description of repulsive gravitational pressure points toward resolution for tensions – between data and modeling – regarding the recent rate of expansion of the universe, resolution for possible tensions regarding large-scale clumping, and resolution for possible tensions regarding interactions between neighboring galaxies. Our work regarding gravity, dark matter, and elementary particles suggests characterizations for eras that might precede the inflationary epoch, a mechanism that might have produced baryon asymmetry, mechanisms that govern the rate of expansion of the universe, and insight about galaxy formation and evolution.

Keywords: beyond the Standard Model, dark matter, galaxy formation, neutrino masses, evolution of the universe

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1. Introduction

1.1. An overview of our work

This essay develops new modeling and uses the modeling to suggest new elementary particles, a specification for dark matter, and insight regarding dark energy. The specification for dark matter and the modeling pertaining to dark energy explain astrophysics data and cosmology data that other physics modeling seems not to explain.

Some relevant data includes ranges of observed ratios of dark matter presence or effects to ordinary matter presence or effects. The following phrases associate with some such ratios - densities of the universe, presences within galaxy clusters, presences within galaxies, and specific depletion of CMB (or, cosmic microwave background radiation). Our work seems to explain four ranges of ratios of presences within galaxies and the other ranges of ratios to which the previous sentence alludes.

Other relevant data pertains to large-scale aspects such as the rate of expansion of the universe. Our work seems to point toward resolutions of tensions between data and popular models. (We use the word popular - and related phrases such as popular modeling - to refer to modeling that other people developed.)

Bases for our work include the list of known elementary particles; aspects of the elementary particle Standard Model; concordance cosmology; new modeling (based on Diophantine equations and a new type of multipole expansion) regarding long-range forces, properties of objects, and elementary particles; a hypothesis that nature includes six isomers of all elementary particles except the photon and other would-be carriers of long-range forces; and modeling based on mathematics for isotropic multidimensional harmonic oscillators.

Our modeling seems to unite and extend aspects of general physics, elementary particle physics, astrophysics, and cosmology.

1.2. One way in which our work adds to popular modeling

We preview one way in which our work contrasts with and adds to popular modeling.

We discuss modeling that has bases in multipole expansions.

Each one of some popular models and each one of some of our models uses the word multipole when discussing aspects of fields - such as the electromagnetic field - that an object produces. (Reference [1] discusses multipole expansions regarding electromagnetism. Reference [2] discusses a multipole expansion - that associates with general relativity - regarding gravitation.)

1.2.1. Popular modeling that associates with the word multipole

We associate the acronym ODP - as in the three-word phrase one distributed property - with some popular modeling uses of multipole expansions.

Regarding an object A, ODP modeling considers one property - such as charge - of the object. The property models as extending over a nonzero spatial volume. ODP modeling points to an approximate characterization of a contribution to a field - such as the electromagnetic field - that associates with object A. For an object A that models as not moving, the contribution based on charge is to a potential that associates with the electric field. For popular modeling that associates with Newtonian physics, the characterization features a sum of terms, with each term including a factor r^{-n}, in which r denotes the distance from a specific point that associates with object A and n is a positive integer. For $n \geq 2$, a term includes dependence on angular coordinates.

Some applications of ODP modeling consider objects A that consist of clumps, which each clump having properties that are similar to the properties of the other clumps.

Regarding electromagnetism, each one of the clumps might have the same charge as has each other clump. Regarding a case that associates with exactly one clump, the potential associates with one term and the radial factor r^{-1} pertains for that term. Popular modeling associates the word monopole with this case. Regarding a case that associates with exactly two clumps, the potential associates with more than one term and the radial factor r^{-2} pertains for one term. Popular modeling associates the word dipole with this case.

Regarding acoustics, reference [3] discusses cases in which the words monopole, dipole, and quadrupole associate with various numbers of similar speakers, with the geometric arrangement of the speakers, and with the relative phases of the sounds that each speaker emits.

1.2.2. A preview of new modeling that associates with the word multipole

Popular modeling associates the notion of a spin-1 boson - the photon - with the electromagnetic field. Popular modeling includes the notion that one can model a photon in terms of integer units of circular polarization. (For this discussion, we de-emphasize modeling that has bases in linear polarizations. Our work does not run counter to the notion that - for popular modeling - modeling based on linear polarizations can pertain.) One unit of circular polarization associates with an angular momentum of magnitude $\hbar$. A photon associates with two modes. One mode associates with a nonnegative integer number of units of left circular polarization. One mode associates with a nonnegative integer number of units of right circular polarization. In popular models, the two modes model as having no coupling between each other. For example, two units of left circular polarization plus one unit of right circular polarization do not combine to net to one unit of left circular polarization.

We consider modeling that has inspiration in a notion of adding and subtracting unequal integer units of circular polarization. For example, mathematically, subtracting one unit of right circular polarization from two units of left circular polarization yields one unit of left circular polarization. For this modeling, the individual quantities that contribute to a sum do not necessarily associate directly with measurable aspects of nature. A list of items that contribute to a sum can associate directly with aspects of nature. A value of a sum can associate directly with aspects of nature.

We (arbitrarily) associate positive sums with left circular polarization and, thereby, associate negative sums with right circular polarization.

The symbol Σ denotes the magnitude of the value of a sum. We posit that $\Sigma = 1$ associates with electromagnetism and that $\Sigma = 2$ associates with gravity.

Each so-called solution (or, sum) associates with a symbol of the form ΣgΓ. Here, the symbol Γ denotes a list of the magnitudes of the integers that contribute to the sum. We show such lists in ascending order. For example, the symbol 1g1'2 associates with the following two solutions - $+1 = -1 + 2$ (which associates with one unit of left circular polarization) and $-1 = +1 - 2$ (which associates with one unit of right circular polarization). We use the one-element term solution-pair to associate with a pair of solutions for which one solution associates with reversing each sign that associates with the other solution. For $n_\Gamma \geq 4$, for each one of some ΣgΓ, more than one solution-pair pertains. We use the letter x and the symbol ΣgΓx to denote the notion that more than one solution-pair pertains.

The symbol n_Γ denotes the number of items in a sum. For $n_\Gamma = 1$, one solution-pair pertains. The word monopole associates with $n_\Gamma = 1$. For $n_\Gamma = 2$, two solution-pairs pertain. The word dipole associates with $n_\Gamma = 2$. For $n_\Gamma = 3$, four solution-pairs pertain. The word quadrupole associates with $n_\Gamma = 3$. For $n_\Gamma = 4$, eight solution-pairs pertain. The word octupole associates with $n_\Gamma = 4$.

We posit that our modeling pertains regarding kinematics that associate with the popular physics two-word phrases Newtonian kinematics, special relativity, and general relativity. Regarding applications - of our modeling - that associate with popular physics Newtonian modeling, we posit that the spatial dependence of potential is the product - r^{-n_Γ} - of one factor of r^{-1} for each of the n_Γ items in the sum.

In our modeling, the solution-pair that associates with the symbol 1g1 associates with the intrinsic property of charge (of an object A), with $n_\Gamma = 1$, and with a monopole contribution to the electromagnetic field.

Our modeling includes two uses of the solution-pair that associates with the symbol 1g1'2 and with $n_\Gamma = 2$.

- One use associates with a contribution - to the electromagnetic field - that associates with nonzero translational (or, instantaneously linear) motion of (the charge of) object A. This contribution can associate with a three-vector that associates with translational velocity.

- One use associates with a contribution - to the electromagnetic field - that associates with the magnetic moment of object A. For other than point-like modeling regarding object A, the following notion can - but does not necessarily - pertain. The object models as having a uniform distribution of charge. The distribution models as rotating around an axis (that runs through the center of object A), with one value of angular velocity pertaining regarding all components of the object. For this other-than-point-like example, the $n_\Gamma = 2$ contribution can associate with a three-vector that associates with angular velocity.

The first use of the solution-pair that associates with 1g1'2 associates with the two-word term extrinsic property (of object A). For 1g1'2, the extrinsic property is current of charge (or, charge-current). The second use of the solution-pair that associates with 1g1'2 associates with the two-word term intrinsic property (of object A). For 1g1'2, the intrinsic property is (nominal) magnetic moment.

We associate the acronym MCP - as in multiple concentrated properties - with some aspects of our modeling. Regarding an object A, modeling can treat object A as having zero volume. An MCP model for contributions that associate with fields can feature a sum of terms, with each term associating with one of the words monopole, dipole, quadrupole, and so forth. Each term can associate with one or more properties of object A. For an MCP model regarding electromagnetism, properties can include intrinsic properties (such as charge and magnetic moment) of the object and extrinsic properties (such as current of charge) of the object. A term that associates with (static) charge associates with the word monopole. A term that associates with a current of charge or with intrinsic magnetic moment associates with the word dipole. For an MCP model regarding gravitation, properties can include intrinsic properties (such as mass - or rest energy - and aspects of stress-energy) of the object and extrinsic properties (such as current of rest energy) of the object.

An MCP model can feature both electromagnetic properties of the object and gravitational properties of the object.

We posit that MCP modeling can transcend the notions of point-like object and point-like property, We posit that MCP modeling can use notions of volume-like regions and property densities that pertain to those regions. Here, volume-like refers to a limited range with respect to a temporal coordinate and with respect to spatial coordinates.

We posit that, for MCP models, an upper limit of 32-pole pertains.

For some circumstances, one might need - to achieve adequately accurate results - to use a combination of ODP modeling and MCP modeling. Combining work based on ODP modeling notions of multipole and work based on MCP notions of multipole does not necessarily lead to problems.

1.2.3. A preview of the scope of results from - and unity within - our modeling

Our modeling includes the following aspects.

Solution-pairs for which $\Sigma = 1$ and $\Sigma \in \Gamma$ associate with electromagnetic properties (such as charge) of objects and with properties of electromagnetic fields.

Solution-pairs for which $\Sigma = 2$ and $\Sigma \in \Gamma$ associate with gravitational or mechanical properties (such as mass) of objects and with properties of gravitational fields.

Solution-pairs for which $\Sigma = 3$ and $\Sigma \in \Gamma$ and solution-pairs for which $\Sigma = 4$ and $\Sigma \in \Gamma$ might associate with properties of objects and with properties of fields that would be similar to electromagnetic fields and gravitational fields.

Solution-pairs for which $\Sigma = 0$ and $n_\Gamma \geq 3$ associate with all known elementary particles, except for the photon, and with possible elementary particles that people have yet to find.

Solution-pairs that associate with known elementary bosons might point to Gauge symmetries that popular modeling associates with known elementary bosons.

Solution-pairs that associate with known elementary fermions point to the notion - that popular modeling associates with known elementary fermions - of three flavours. Solution-pairs that associate with unfound possible elementary fermions point to the notion of three flavours.

The notion that associates with $\Sigma = 0$ and $n_\Gamma = 0$ points to the possibility that nature includes six isomers (or, near copies) of each elementary particle that associates with a solution-pair for which $\Sigma = 0$ and $n_\Gamma \geq 3$. (In mathematics, the symbol $\emptyset$ denotes a set with no members. We use the notation $\Gamma = \emptyset$ to denote the empty list Γ. We use the notation 0g$\emptyset$ to denote the case for which $\Sigma = 0$ and $n_\Gamma = 0$.)

We use the acronym LRI (for the two-element phrase long-range interaction) to associate with fields that associate with solution-pairs for which $\Sigma \in \Gamma$ and to associate with elementary particles that mediate interactions that associate with those fields. We use the word simple and the two-word phrase simple particles to associate with elementary particles that associate with solution-pairs for which $\Sigma = 0$ and $\Gamma \neq \emptyset$.

We use the notion that nature includes six isomers of each simple particle to help explain data - including data about dark matter effects and dark energy effects - that popular modeling seems not to explain. All of the stuff associating with five isomers and some stuff that associates with the other isomer measures as dark matter.

Solution-pairs for which $\Sigma = 1$ and $\Sigma \in \Gamma$ and for which $\Sigma = 2$ and $\Sigma \in \Gamma$ associate with popular models for electromagnetism, gravity, and kinematics. (Each one of some popular kinematics modeling techniques associates with one of the two-word terms Newtonian physics, special relativity, and general relativity.) Regarding each of various specific popular modeling techniques, associations with solution-pairs for which $\Sigma \in \Gamma$ point to ranges of situations for which the modeling technique might be adequately accurate and to ranges of situations in which the modeling technique might not be adequately accurate.

Solution-pairs for which $1 \leq \Sigma \leq 4$ and $\Sigma \notin \Gamma$ might provide bases for useful modeling regarding anomalous properties of objects.

1.3. Possible confirmation for our work

People report seemingly prevalent ranges of ratios of dark matter effects to ordinary matter effects. Our work seems to explain the ranges. (Elsewhere, we cite references that report relevant data. See discussion related to table 17, table 18, and table 19.)

We use symbols of the form DM:OM to associate with ranges. The acronym DM associates with dark matter and with the three-word phrase dark matter effects. The acronym OM associates with ordinary matter and with the three-word phrase ordinary matter effects. People infer ratios of DM effects to OM effects. Each DM:OM range associates with inferred ratios. For each DM:OM range, each of the DM number and the OM number associates (approximately) with a small positive integer. This essay does not discuss numeric bounds for individual ranges.

Some of the DM:OM ranges associate with $5^+ : 1$. These ranges pertain for densities of the universe, amounts of stuff in some galaxy clusters, and amounts of stuff in many galaxies.

One DM:OM range associates with $\approx 1 : 1$. This range pertains regarding some specific depletion of CMB (or, cosmic microwave background radiation) and might pertain regarding the overall intensity of the cosmic optical background.

Other DM:OM ranges (that are not the range $5^+ : 1$ that associates with many galaxies) associate with galaxies. The range $0^+ : 1$ pertains for many early galaxies and for some later galaxies. (Here, the word early tends to associate with times that popular modeling associates with redshifts z of at least - and perhaps somewhat less than - seven. Here, the word later tends to associate with z that are significantly less than seven.) The range $\sim 4 : 1$ pertains for some later galaxies. The range $1 : 0^+$ pertains for some later galaxies. (Regarding $1 : 0^+$, the three-word term dark matter galaxy pertains.) Also, we suggest that the range $1 : 0^+$ pertains for many early galaxies. (However, current techniques might not suffice to detect early galaxies for which the range $1 : 0^+$ pertains.)

We know of no other such seemingly possibly prevalent ranges of ratios.

In the context of our modeling, our explanations regarding DM:OM ranges seem to require results that we develop regarding - at least - general physics, particle physics, and astrophysics.

Popular modeling seems not to explain the ranges.

1.4. Associations between our work, data, and popular modeling

We discuss associations between our work and other work. Other work includes observational research and popular modeling. We discuss briefly aspects of other work. We provide references - regarding other work - to review articles and other information. (For example, reference [4] provides an overview of concordance cosmology and related topics regarding general physics, dark matter, and elementary particles.) We suggest context for associating our work with other work. We do not necessarily explore thoroughly relationships between our work and other work.

1.4.1. Physics constants and physics properties

People discuss possibilities for relationships between electromagnetism and gravity. For example, reference [5] explores notions of a coupling between electromagnetism and gravity. People discuss possibilities for modeling that blends modeling used regarding electromagnetism and modeling used regarding gravity. Reference [6] and reference [7] discuss Einstein-Maxwell equations that suggest combining electromagnetic stress-energy tensors and the Einstein field equations, which have origins in modeling regarding gravitation.

People discuss, at least in the context of popular modeling notions that associate with general relativity, possible relationships between mass and angular momentum. (See reference [8] and articles to which reference [8] alludes.) Our work regarding simple bosons suggests a relationship between mass, angular momentum, and charge. (See equation (41).)

Our work seems to interrelate some physics constants. (See table 7 and table 9.) Our work seems to interrelate some properties, including via modeling that catalogs physics properties. (See table 12.)

Our work offers new approaches to estimating some physics properties. This essay points to masses - that would comport with recent experimental results and that would have smaller standard deviations than standard deviations that associate with recent experiments - for each of the tau elementary fermion and the Higgs boson. (See respectively table 9 and table 7.) Our work suggests - regarding the anomalous magnetic dipole moment of the tau elementary fermion - a possible estimate that might approximate a Standard Model estimate. (See discussion related to table 8 and table 9.)

1.4.2. Elementary particles

Our approach to predicting and describing elementary particles differs from popular modeling approaches; suggests some new elementary particles that popular modeling suggests; suggests some new elementary particles that popular modeling does not suggest; seems not to suggest some new elementary particles that popular modeling suggests; suggests new details about neutrino masses and some properties of other known elementary particles; and seems to be compatible with data.

1.4.2.1. Popular modeling that tries to suggest new elementary particles.

Reference [9] lists some types of modeling that people have considered regarding trying to extend the elementary particle Standard Model, including trying to suggest elementary particles that people have yet to find. Reference [10] provides information about some of these types of modeling. References [11], [12], and [13] provide some information about modeling and about experimental results. Reference [14] provides other information about modeling and about experimental results. (See reviews numbered 86, 87, 88, 89, 90, and 94.)

1.4.2.2. Possible particles that popular modeling and our modeling suggest.

Reference [15] suggests the notions of dark matter charges and dark matter photons. We suggest dark matter isomers of charged elementary particles and, in effect, dark matter components - such as components associating with electrostatics and magnetostatics - of electromagnetism. (See discussion related to table 12.)

Reference [16] suggests the notion of a inflaton field. We suggest an inflaton elementary particle. (See table 5 and note the 0I boson.)

People suggest the notion of a graviton. (See, for example, reference [17].) We suggest a graviton. (See table 6.)

Reference [18] discusses notions of sterile neutrinos and heavy neutrinos. We suggest possible elementary particles that might associate with notions of heavy neutrinos. (See table 5.)

1.4.2.3. Possible elementary particles that popular modeling might rule out.

Reference [19] notes that modeling based on QFT (or, quantum field theory) suggests that massless elementary particles cannot have spins that exceed two. Our work suggests a possible spin-three analog to the photon and the possible graviton. (See table 6.) Our work suggests a possible spin-four analog to the photon and the possible graviton. (See table 6.) Discussions related to subsequent citations to reference [19] and discussions related to table 20 suggest that our work might not be incompatible with popular modeling notions that nature does not include zero-mass elementary bosons that have spins that exceed two.

1.4.2.4. Possible elementary particles that our modeling seems not to suggest.

A symmetry regarding Maxwell's equations suggests that nature might include magnetic monopoles. Reference [20] discusses theory. Reference [13] reviews modeling and experiments regarding magnetic monopoles. We suggest that nature might not include an interaction that would associate with magnetic monopoles. (See table 12.) Reference [21] discusses a search - for magnetic monopoles - that did not detect magnetic monopoles.

Reference [11] reviews modeling and experiments regarding axions. Reference [11] notes modeling that suggests that nature might include axions. We suggest that nature might not include axions. (See table 5.) We suggest that phenomena that popular modeling might attribute to axions might not associate with axions. One such phenomenon could be electromagnetic interactions between ordinary matter and dark matter based on, for example, aspects that associate with one-some use of the 1g1‘2‘4 solution-pair component of electromagnetism. (See table 12.)

Reference [12] reviews modeling and experiments regarding leptoquarks. We suggest that nature might not include leptoquarks. (See table 5.)

1.4.2.5. Neutrino masses.

Reference [18] discusses modeling and data about neutrino masses and neutrino oscillations.

We suggest neutrino masses. (See table 10.) As far as we know, our modeling is not incompatible with data that reference [18] discusses. Future experimentation might help validate or refute aspects of our work regarding neutrinos.

1.4.2.6. Gravitation.

Reference [22] discusses experimental tests of theories of gravity.

We suggest effects - associating with isomers of elementary particles and with reaches of components of gravity - that suggest that popular modeling regarding gravity would not be adequately accurate for some circumstances. (See table 12, table 14, and table 16.) We are uncertain as to the extent to which aspects that reference [22], reference [23], and reference [24] discuss would tend to validate or refute aspects of our modeling that pertains to gravitation.

We use modeling - regarding gravity - that has some similarities to popular modeling that associates with the term gravitoelectromagnetism. (References [25] and [26] discuss gravitoelectromagnetism.) Our modeling regarding gravity has some similarities to models that use classical physics perturbations regarding Newtonian gravity. (Reference [2] deploys modeling that associates with non-spherical distributions of mass.)

1.4.3. Cosmology

We think that - with some exceptions - our work does not necessarily suggest significant changes - to concordance cosmology - regarding the large-scale evolution of the universe. (References [27], [28], [29], [30], and [4] review aspects of concordance cosmology. Reference [31] discusses attempts to explain the rate of expansion of the universe.)

Each exception that this essay discusses associates either with a possible aspect of nature for which people have no observations or with a known gap between observations and concordance cosmology.

One exception pertains regarding before inflation. One exception pertains regarding recent changes in the rate of expansion of the universe. In each case, we suggest noteworthy contributions by a gravitational force component for which each instance (of the component) has a reach that is greater than one isomer. (See table 12 and table 14.) For times associating with between the two cases, we suggest dominance by gravitational force components that have reaches of one isomer. For times associating with between the two cases, we do not propose significant incompatibilities between our work and large-scale concordance cosmology.

1.4.3.1. Possibilities regarding aspects before inflation.

We think that no direct observations pertain to phenomena that occurred before inflation. We suggest two eras before inflation. (See table 14.) The first of those two eras features aspects that the Standard Model and concordance cosmology do not include. (Reference [32] discusses possibilities leading up to a Big Bang. References [33] and [28] discuss inflation.) One aspect is the jay boson. (See table 5 and table 14.) The other aspect is the set of 2g1‘2‘3‘4‘8x components of gravity. (See table 14.) An instance of each component has a reach of six isomers. For purposes of discussion, we assume that the universe transited those two eras. We assume that concordance cosmology can embrace the jay boson. For the first of those two eras, an extrapolation of concordance cosmology techniques might underestimate the strength of the key driver - the 2g1‘2‘3‘4‘8x components of gravity - by a factor of six.

1.4.3.2. Phenomena that affect the current multi-billion-years era.

People suggest that concordance cosmology underestimates recent increases in the rate of expansion of the universe. (References [29], [34], [35], [36], and [37] discuss relevant notions.)

We think that we point to a basis for the underestimates. Regarding times before that lead-up, we suggest dominance by an attractive quadrupole gravitational force component - 2g1‘2‘3. (See table 14.) Each instance of that force component has a reach of one isomer. Before and during the recent multi-billion-years era, the 2g2‘4 gravitational force component gains prominence and then becomes dominant. Each instance of 2g2‘4 has a reach of two isomers. We suggest that concordance cosmology models that work well regarding times for which reach-one dominance pertains would not necessarily work well after those times. We suggest that extrapolating based on such concordance cosmology modeling would underestimate (conceptually by a factor of two) the strength of the driver for increases in the rate of expansion. We suggest that - to get good results via concordance cosmology modeling - people might adjust the equation of state. In general, for each relevant density, components of pressure that associate with repulsion need to increase.

Our suggested resolution regarding the underestimate seems to differ from possible resolutions based on concordance cosmology modeling. Our suggested resolution focuses on phenomena that would pertain at the times for which concordance cosmology modeling seems not to be adequate. Other possible resolutions might focus on phenomena early in the history of the universe. (See reference [29].)

1.4.4. Astrophysics

We think that our modeling is not necessarily incompatible with astrophysics data or with results based on concordance cosmology modeling. (Here, we assume that the two-word term concordance cosmology includes aspects that associate with dark matter, astrophysics, and effects of gravity on scales as small as one galaxy.)

1.4.4.1. Properties of dark matter.

Reference [38] suggests the following notions. Most dark matter comports with notions of cold dark matter. Models that associate with the two-word term modified gravity might pertain; but - to the extent that the models suggest long-range astrophysical effects - such models might prove problematic. Popular modeling suggests limits on the masses of basic dark matter objects. Observations suggest small-scale challenges to the notion that all dark matter might be cold dark matter. People use laboratory techniques to try to detect dark matter. People use astrophysical techniques to try to infer properties of dark matter. (Reference [39] discusses astrophysical and cosmological techniques.)

We think that our modeling regarding dark matter comports with such notions. For astrophysical phenomena (and not necessarily regarding the rate of expansion of the universe), components - that have reaches other than six - of gravity play roles locally; however, the impacts do not extend to cosmological scales. The one dark matter isomer that might evolve similarly to ordinary matter might provide bases for resolving some of the small-scale challenges.

1.4.4.2. Observations and models regarding galaxy formation.

Reference [40] discusses galaxy formation and evolution, plus contexts in which galaxies form and evolve. Reference [40] discusses parameters by which popular modeling classifies and describes galaxies.

We suggest that - regarding galaxies - observations of ratios of dark matter effects to ordinary matter effects might tend to cluster near some specific ratios. (See table 17.) Our modeling seems to explain such ratios. (See table 16 and table 17.)

Our modeling suggests that ratios of dark matter effects to ordinary matter effects might reflect fundamental aspects - of nature - that concordance cosmology modeling does not include. Here, a key aspect is that of isomers. (See table 16 and table 17.)

Reference [40] seems not to preclude galaxies that have few ordinary matter stars. Reference [40] seems not to preclude galaxies that have little ordinary matter.

We think that ratios - of dark matter effects to ordinary matter effects - that our modeling suggests are not necessarily incompatible with verified concordance cosmology modeling.

1.4.4.3. Observations and models regarding interactions between galaxies.

Reference [41] suggests that concordance cosmology modeling might not adequately explain gravitational interactions between neighboring galaxies. We suggest that notions pertaining to reaches and isomers might help to bridge the gap between observations and concordance cosmology modeling.

We think that our work points to a possible opportunity to study harmony between results based on established kinematics models and results based on our notions of components of gravity.

2. Methods

2.1. A Diophantine equation that underlies our modeling

We anticipate using mathematics that features sums of terms, in which each term is the multiplicative product of an integer k and an integer s_k for which the magnitude is one of one and zero.

Equation (1) shows the set of all k that our modeling considers.

$$K_{-2,8} \equiv \{-2,-1,0,1,2,3,4,6,8\} \tag{1}$$

Equation (2) shows the range for each s_k.

$$s_k = -1,\ 0,\ \text{or}\ +1 \tag{2}$$

Equation (3) defines a set of nonnegative integers k. For each nonnegative integer n, equation (4) defines symbols for subsets of K_8.

$$K_8 \equiv \{0,1,2,3,4,6,8\} \tag{3}$$

$$K_n \equiv \{k|k \in K_8 \text{ and } k \leq n\} \tag{4}$$

Equation (5) provides a symbol for a set K_n and one set of choices regarding values of relevant s_k.

$$K(K_n, \{s_k\}) \tag{5}$$

Equation (6) shows the relevant sum (and the relevant Diophantine equation).

$$s = \sum_{k \in K_n} k s_k \tag{6}$$

The following items provide information regarding a $K(K_n, \{s_k\})$. Regarding equation (8) and our displaying a list Γ, we use the notation k_a'k_b'...'k_{max} and the convention that $k_a < k_b < ... < k_{max}$.

$$k_{max} \equiv \max\{k|k \in K_n \text{ and } |s_k| = 1\} \tag{7}$$

$$\Gamma = \text{ the ascending-order list of } k \text{ for which } k \geq 1 \text{ and } |s_k| = 1 \tag{8}$$

$$n_\Gamma \equiv \text{ number of } k \in \Gamma \tag{9}$$

$$n_0 \equiv \text{ number of } k \in \{k|k \in K_8,\ 1 \leq k \leq k_{max}, \text{ and } s_k = 0\} \tag{10}$$

We consider a list Γ. Each k for which $k \in \Gamma$ associates with two possibilities - $s_k = -1$ and $s_k = +1$. Equation (11) shows the number of relevant solutions that associate with equation (6).

$$2^{n_\Gamma} \tag{11}$$

We can pair each solution with a solution for which, for each s_k in the first solution, $-s_k$ associates with the second solution. We associate the one-element term solution-pair with such a pair. Each solution-pair associates with a value of Σ, per equation (12).

$$\Sigma \equiv |s| \tag{12}$$

Equation (13) shows the number of solution-pairs that associate with a value of n_Γ.

$$2^{n_\Gamma - 1} \tag{13}$$

For a solution-pair, equation (14) denotes a symbol that we use. (We choose the letter g in anticipation that 1gΓ solution-pairs associate with electromagnetism and that 2gΓ solution-pairs associate with gravity. One might think of g as in gamma rays and g as in gravity.)

$$\Sigma \mathrm{g} \Gamma \tag{14}$$

For $n_\Gamma \geq 4$, each one of some combinations of Γ and Σ associates with more than one solution-pair. For a combination of Γ and Σ that associates with more than one solution-pair, equation (15) shows a symbol that we use.

$$\Sigma \mathrm{g} \Gamma \mathrm{x} \tag{15}$$

Based on equation (13), we make the following (mathematical) associations. The word monopole associates with $n_\Gamma = 1$. The word dipole associates with $n_\Gamma = 2$. The word quadrupole associates with $n_\Gamma = 3$. The word octupole associates with $n_\Gamma = 4$. The one-element construct 16-pole associates with $n_\Gamma = 5$.

For any one list Γ for which $n_\Gamma \geq 2$, more than one value of Σ has relevance. To the extent that the notion of multipole associates with mathematical notions of a space, Σ might be an integer variable that associates with the space.

Table 1 alludes to all $s = \sum_{k \in K_n} (k s_k)$ expressions for which $1 \leq k_{max} \leq 4$. (See discussion related to equation (6).)

We introduce the one-word term cascade. The notion of cascade associates with adding - to the Γ_y that associates with one solution-pair - one new element so as to produce a Γ_z that associates with at least one other solution-pair. Sometimes, we specify that we consider only Γ_z-based solution-pairs for

Table 1: $\Sigma = |s| = |\sum_{k \in K_n}(ks_k)|$ solution-pairs for which $1 \leq k_{max} \leq 4$. The columns labeled $1 \cdot s_1$ through $4 \cdot s_4$ show contributions that associate with terms of the form ks_k. The number $n_{\Sigma g\Gamma}$ equals $2^{n_\Gamma - 1}$ and states the number of solution-pairs. The column for which the one-element label is $\cdots$pole associates mathematically with the number of solutions. For a row for which exactly one solution pertains, the column shows the word monopole. For a row for which exactly two solutions pertain, the column shows the word dipole. For a row for which exactly four solutions pertain, the column shows the word quadrupole. For a row for which exactly eight solutions pertain, the column shows the word octupole. For the case of octupole, each one of $\Sigma = 2$ and $\Sigma = 4$ associates with two solution-pairs. Regarding $\Sigma = 2$, $|-1+2-3+4| = 2 = |-1-2-3+4|$. Regarding $\Sigma = 4$, $|-1-2+3+4| = 4 = |+1+2-3+4|$.

k_{max}	Γ	$1 \cdot s_1$	$2 \cdot s_2$	$3 \cdot s_3$	$4 \cdot s_4$	Σ	n_0	n_Γ	$n_{\Sigma g\Gamma}$	$\cdots$pole
1	1	± 1	-	-	-	1	0	1	1	Monopole
2	2	0	± 2	-	-	2	1	1	1	Monopole
2	1‘2	± 1	± 2	-	-	1,3	0	2	2	Dipole
3	3	0	0	± 3	-	3	2	1	1	Monopole
3	1‘3	± 1	0	± 3	-	2,4	1	2	2	Dipole
3	2‘3	0	± 2	± 3	-	1,5	1	2	2	Dipole
3	1‘2‘3	± 1	± 2	± 3	-	0,2,4,6	0	3	4	Quadrupole
4	4	0	0	0	± 4	4	3	1	1	Monopole
4	1‘4	± 1	0	0	± 4	3,5	2	2	2	Dipole
4	2‘4	0	± 2	0	± 4	2,6	2	2	2	Dipole
4	3‘4	0	0	± 3	± 4	1,7	2	2	2	Dipole
4	1‘2‘4	± 1	± 2	0	± 4	1,3,5,7	1	3	4	Quadrupole
4	1‘3‘4	± 1	0	± 3	± 4	0,2,6,8	1	3	4	Quadrupole
4	2‘3‘4	0	± 2	± 3	± 4	1,3,5,9	1	3	4	Quadrupole
4	1‘2‘3‘4	± 1	± 2	± 3	± 4	0,2,2,4,4,6,8,10	0	4	8	Octupole

which the value of Σ equals the value of Σ that pertains for the Γ_y-based solution-pair. The notion of cascade also associates with the set of solution-pairs that cascade, via one step or more than one step, from one solution-pair.

We associate the symbol Σg with solutions of the form ΣgΓ. We associate the symbol Σg’ with Σg solutions for which $\Sigma \in (\Gamma \cup \{0\})$. (Regarding $k = 0$, the following notions pertain. Per equation (8), $k = 0$ is never a member of Γ. Per equation (4), for each K_n, $k = 0$ is a member of K_n.) We associate the symbol Σg” with Σg solutions for which $\Sigma \notin (\Gamma \cup \{0\})$.

2.2. Some physics modeling that has bases in the Diophantine equation

We consider two objects - object A and object C. Object A has active properties, such as charge or mass. At some instant (which respect to some set of temporal and spatial coordinates), object C senses effects of long-range fields (such as the electromagnetic field or the gravitational field) that associate with object A. We imagine a hypothetical zero-mass boson (such as a photon or a graviton) that associates with object C sensing object A. The hypothetical boson has nonzero integer spin (in units of $\hbar$) of s. We use the two-word phase boson B to associate with the hypothetical boson.

We assume the following regarding boson B. $s > 0$ associates with the popular modeling notion of left circular polarization and with the physics popular modeling notion of an angular momentum of magnitude of $|s|\hbar$. $s < 0$ associates with the popular modeling notion of right circular polarization and with the popular modeling notion of an angular momentum of magnitude of $|s|\hbar$.

Values of s_k do not necessarily associate with popular modeling. (Informally, we say the following. For $k \geq 1$, $ks_k > 0$ associates with left circular polarization and a magnitude of $k\hbar$. For $k \geq 1$, $ks_k < 0$ associates with right circular polarization and a magnitude of $k\hbar$.)

We associate 1g’ solution-pairs with electromagnetism and with photons. We associate 2g’ solution-pairs with gravitation and a notion of (as yet, hypothetical) gravitons. We use the two-element term LRI interactions (or, the two-element term LRI forces) to refer to interactions that associate with ΣgΓ solution-pairs for which $\Sigma \geq 1$. The acronym LRI abbreviates the two-element phrase long-range interaction.

For point-like objects A and C and regarding popular modeling Newtonian kinematics, equation (16) characterizes the RSDP (or, radial spatial dependence of potential) that object C senses (based on the field that associates with boson B) regarding object A. Here, r is a distance from object A to object C. For $n_\Gamma \geq 2$, angular dependence also pertains.

$$V(r) \propto r^{-n_\Gamma} \tag{16}$$

The series consisting of monopole, dipole, and so forth that associates with Newtonian kinematics associates with the series monopole, dipole, and so forth that associates with table 1 and with math that underlies our modeling. (Reference [2] discusses a - different - multipole-expansion application regarding gravitation and modeling that associates with general relativity. Reference [3] discusses an application - of notions of monopole, dipole, and so forth - regarding acoustics.)

Table 1 and equation (16) anticipate that our modeling associates - with each other - aspects of electromagnetism and aspects of gravitation. (Popular modeling discusses possibilities for relationships between electromagnetism and gravity. For example, reference [5] explores notions of a coupling between electromagnetism and gravity. Reference [6] and reference [7] discuss Einstein-Maxwell equations that suggest combining electromagnetic stress-energy tensors and the Einstein field equations, which have origins in modeling regarding gravitation.)

2.3. Isotropic-harmonic-oscillator mathematics that underlies our modeling

Popular modeling includes the notion of quantum transitions that excite or de-excite boson fields. Popular modeling includes uses of harmonic oscillator mathematics to model excitations and de-excitations of boson fields. Our modeling includes harmonic oscillator mathematics applications that model excitations and de-excitations of boson fields and that associate Gauge symmetries with some elementary bosons.

We discuss some notions related to mathematics that associate with the three-word term isotropic harmonic oscillator.

Modeling for a j-dimensional isotropic harmonic oscillator can feature j linear coordinates $x_{k'}$ - each with a domain $-\infty < x_{k'} < \infty$ - and an operator that is the sum - over k' - of j operators of the form that equation (17) shows. The number C is positive and is common to all j uses of equation (17). The word isotropic associates with the commonality - across all j uses of equation (17) - of the number C.

$$-\frac{\partial^2}{\partial(x_{k'})^2} + C \cdot (x_{k'})^2 \tag{17}$$

For $j \geq 2$, one can split the overall operator into pieces. Equation (18) associates with a split into two pieces. Here, each of j_1 and j_2 is a positive integer.

$$j = j_1 + j_2 \tag{18}$$

We discuss aspects that associate with D-dimensional isotropic harmonic oscillators. Regarding equation (18), depending on the context in which one applies notions below, D can be any one of j, j_1, and j_2.

For any integer D that exceeds one, mathematics includes notions that link modeling for D one-dimensional harmonic oscillators and modeling for one D-dimensional harmonic oscillator, assuming that the D one-dimensional oscillators associate with notions of equal strengths (or, of a common value of C). For an integer D that exceeds one and a D-dimensional isotropic quantum harmonic oscillator, mathematics associates a symmetry that associates with the mathematics group $SU(D)$ with the ground state of the D-dimensional oscillator. (See reference [42].)

We use an expression of the form gen(*group*) to denote the number of generators for a group. For $D \geq 2$, mathematics provides that equation (19) pertains.

$$\text{gen}(SU(D)) = D^2 - 1 \tag{19}$$

Popular modeling associates a symmetry that associates with the mathematics group $U(1)$ with excitations and de-excitations of a one-dimensional harmonic oscillator.

Mathematics provides that equation (20) pertains.

$$\text{gen}(U(1)) = 1 \tag{20}$$

We discuss solutions to a generalization that is based on popular modeling uses of isotropic harmonic oscillator equations.

For $D \geq 2$, popular modeling related to isotropic harmonic oscillators can feature partial differential equations, a radial coordinate, and $D-1$ angular coordinates. Equation (21) defines a radial coordinate.

$$x = (\sum_{k'} (x_{k'})^2)^{1/2} \tag{21}$$

Our modeling replaces x via the expression that equation (22) shows. Here, r denotes the radial coordinate and has dimensions of length. The parameter η has dimensions of length. The parameter η

Table 2: Terms associating with a partial differential equation (assuming that $(\xi'/2) = 1$ and $\eta = 1$). Equation (23) and equation (24) show the partial differential equation. In table 2, the second column provides a symbol for the term to which the first column alludes.

Term/ $\exp(-r^2/2)$	Symbol	Change in power of r	Nonzero unless ...	Note
$-r^{\nu+2}$	T_{+2}	$+2$	-	$= -V_{+2}$
$(D+\nu)r^{\nu}$	T_{0a}	0	$D+\nu = 0$	-
νr^{ν}	T_{0b}	0	$\nu = 0$	-
$-\nu(\nu+D-2)r^{\nu-2}$	T_{-2}	-2	$\nu = 0$ or $(\nu+D-2) = 0$	$= -V_{-2}$
$\Omega r^{\nu-2}$	V_{-2}	-2	$\Omega = 0$	$= -T_{-2}$
$r^{\nu+2}$	V_{+2}	$+2$	-	$= -T_{+2}$

is a nonzero real number. The magnitude $|\eta|$ associates with a scale length. For popular modeling, the domain $0 \leq r < \infty$ pertains. (For this work, r does not necessarily associate with uses of r elsewhere - for example, in equation (16) - in this essay.)

$$x = r/\eta \tag{22}$$

Equations (23) and (24) associate with popular modeling. Each of ξ and ξ' is an as-yet unspecified constant. The symbol $\phi_R(r)$ denotes a function of r. The symbol ${\nabla_r}^2$ denotes a Laplacian operator. In popular modeling, Ω associates with aspects that associate with angular coordinates. (For $D = 3$, reference [43] shows a representation for Ω in terms of an operator that is a function of spherical coordinates.)

$$\xi\phi_R(r) = (\xi'/2)(-\eta^2{\nabla_r}^2 + (\eta)^{-2}r^2)\phi_R(r) \tag{23}$$

$${\nabla_r}^2 = r^{-(D-1)}(\partial/\partial r)(r^{D-1})(\partial/\partial r) - \Omega r^{-2} \tag{24}$$

Our modeling branches from popular modeling. We assume that the symbol Ω is a constant. We do not necessarily require that D is a positive integer for which $D \geq 2$. Also, we include solutions that pertain for the domain that equation (25) shows. (Some aspects of popular modeling associate with the following notions. D is a nonnegative integer. ϕ_R associates with a radial factor that is part of a representation of a wave function. For $D = 1$, equation (24) might not be appropriate. For $D > 1$, a representation of a wave function may need to include a factor for which angular coordinates play roles. The domain for a representation of such a wave function needs to include $r = 0$. For our work, ϕ_R does not necessarily associate with the notion of a factor in a representation for a wave function and does not necessarily need to have a definition for $r = 0$.) With respect to the domain $0 \leq r < \infty$, ϕ_R associates with the mathematics notion of having a definition almost everywhere.

$$0 < r < \infty \tag{25}$$

We consider solutions of the form that equation (26) shows. (In popular modeling, solutions that associate with equation (17) and with $D = 1$ have the form $H(x)\exp(-x^2)$, in which $H(x)$ is a Hermite polynomial. As we are about to show, mathematics that our modeling uses allows for an adequately useful set of solutions for which each solution associates with - in effect - a one-term polynomial.)

$$\phi_R(r) \propto (r/\eta)^{\nu}\exp(-r^2/(2\eta^2)), \text{ with } \eta^2 > 0 \tag{26}$$

Table 2 provides details that lead to solutions that equations (27) and (28) characterize. We consider equations (23), (24), and (26). The table assumes, without loss of generality, that $(\xi'/2) = 1$ and that $\eta = 1$. More generally, we assume that each of the four terms $T_{\_}$ and each of the two terms $V_{\_}$ includes appropriate appearances of $(\xi'/2)$ and η. The term V_{+2} associates with the rightmost term in equation (23). The term V_{-2} associates with the rightmost term in equation (24). The four $T_{\_}$ terms associate with the other term to the right of the equals sign in equation (24). The sum of the two $T_{0\_}$ terms associates with the factor $D + 2\nu$ in equation (27) below.

Equations (27) and (28) characterize solutions. The parameter η does not appear in these equations.

$$\xi = (D + 2\nu)(\xi'/2) \tag{27}$$

$$\Omega = \nu(\nu + D - 2) \tag{28}$$

We explore the topic of normalization regarding $\phi_R(r)$.

In popular modeling, $\phi_R(r)$ normalizes if and only if equation (29) pertains. The symbol $(\phi_R(r))^*$ denotes the complex conjugate of $\phi_R(r)$.

$$\int_0^\infty (\phi_R(r))^* \phi_R(r) r^{D-1} dr < \infty \tag{29}$$

Our work embraces somewhat the same concept - as popular modeling embraces - regarding normalization. The difference in the domain for r (that is, $0 < r < \infty$ for our modeling versus $0 \leq r < \infty$ for popular modeling) is not material for this essay. For essentially the entire remainder of this essay, we assume that equation (30) pertains. (For a complex number z, the expression $z = \Re(z) + i\Im(z)$ pertains. The expression $\Re(z)$ denotes the real part of z. The expression $\Im(z)$ denotes the imaginary part of z. The symbol i denotes the positive square root of the number -1.) We take the liberty to assume that the normalization criterion that equation (29) defines pertains for any real number D. (This essay does not explore applications of notions that popular modeling might associate with the expression $\lim_{D \to j}(\cdots)$, in which j is an integer.)

$$\Im(D) = 0 \tag{30}$$

For essentially the entire remainder of this essay, we assume that equation (31) pertains.

$$\Im(\nu) = 0 \tag{31}$$

Equation (32) associates with the domains of D and ν for which normalization pertains for $\phi_R(r)$. For $D + 2\nu = 0$, normalization pertains in the limit $\eta^2 \to 0^+$. Regarding mathematics relevant to normalization for $D+2\nu = 0$, the delta function that equation (33) shows pertains. Here, $(x')^2$ associates with r^2 and 4ϵ associates with η^2. (Reference [44] provides equation (33).) The difference in domains, between $-\infty < x' < \infty$ and equation (25), is not material here. (Our use of this type of modeling features normalization. Considering normalization leads to de-emphasizing possible concerns, regarding singularities as r approaches zero, regarding some $\phi_R(r)$.)

$$D + 2\nu \geq 0 \tag{32}$$

$$\delta(x') = \lim_{\epsilon \to 0^+} (1/(2\sqrt{\pi\epsilon})) e^{-(x')^2/(4\epsilon)} \tag{33}$$

We use the one-element term volume-like to describe solutions for which $D+2\nu > 0$. The term volume-like pertains regarding behavior with respect to the coordinate or coordinates that underlie modeling. (For popular modeling, generally, the word coordinates - as in r plus angular coordinates - can be appropriate.) We use the one-element term point-like to describe solutions for which $D + 2\nu = 0$. For a point-like solution, $\phi_R(r)$ is effectively zero for all $r > 0$. The term point-like pertains regarding behavior with respect to the coordinate or coordinates that underlie modeling.

2.4. Some modeling that has bases in Diophantine and harmonic-oscillator mathematics

Discussion related to equation (16) and table 1 points to the notion that the solution-pair 1g1 associates with a contribution - to the electric field - based on the charge of an object A. In popular modeling, moving charge contributes to the magnetic field. The contribution associates with the word dipole. We posit that use of the solution-pair 1g1‘2 can associate with moving charge.

Compared to the Γ for 1g1, the Γ for 1g1‘2 includes one additional element. We posit that we can associate an $SU(2)$ symmetry with the ground state for a two-dimensional isotropic harmonic oscillator for which one dimension associates with $s_2 = -1$ and one dimension associates with $s_2 = +1$. Per equation (19), three generators pertain. We posit that the three generators associate with the three degrees of freedom that associate with the (translational) velocity of object A.

The following notions pertain regarding modeling for charge and moving charge and pertain regarding other modeling.

We consider a so-called one-some use of a solution-pair that associates with a Σ_ygΓ_y. We consider a solution-pair Σ_zgΓ_z that associates with a one-step cascading from the solution-pair that associates with Σ_ygΓ_y. We use the term three-some to associate - in this context - with the solution-pair Σ_zgΓ_z. We use the term four-some to pertain to usage of the combination of a one-some term and a same-Σ three-some term that cascades from the one-some term. The three-some term associates with three additional (compared to the one-some term) degrees of freedom. The term 3-vector pertains.

Table 3: Cascades that include Σg' solution pairs. The leftmost column alludes to the source from which the item in the second column cascades. The symbol † alludes to the previous row. A ΣgΓ solution pair for which the rightmost column does not provide a note can serve in a one-some role and can serve in a three-some role. A ΣgΓ solution pair for which the rightmost column shows the acronym NNC (for the three-word phrase no next cascade) can serve in a three-some role, does not further cascade, and cannot serve in a one-some role. The symbol ‡ points to a set of ΣgΓ solution pairs that terminates at least two cascades. The acronym PNR (for the three-word phrase possibly not relevant) associates with the notions that $6 \in \Gamma$ and that NNC does not pertain. (Regarding the notion of $6 \in \Gamma$, see discussion related to table 6.)

Cascades from	ΣgΓ solution-pairs	ΣL	Note
0g∅	1g1	1L	-
†	1g1'2	1L	-
†	1g1'2'4	1L	-
†	1g1'2'4'8	1L	-
†	1g1'2'4'6'8x ‡	1L	NNC
1g1'2'4	1g1'2'4'6x	1L	PNR
†	1g1'2'4'6'8x ‡	1L	NNC
-	1g1'4'6	1L	PNR
†	1g1'4'6'8	1L	PNR
†	1g1'2'4'6'8x ‡	1L	NNC
0g∅	2g2	2L	-
†	2g2'4	2L	-
†	2g2'4'8	2L	NNC
1g1'2	2g1'2'3	2L	-
†	2g1'2'3'4x	2L	-
†	2g1'2'3'4'8x	2L	-
†	2g1'2'3'4'6'8x ‡	2L	NNC
-	2g1'2'3'6'8x	2L	PNR
†	2g1'2'3'4'6'8x ‡	2L	NNC
0g∅	3g3	3L	-
†	3g3'6	3L	NNC
2g2'4	3g2'3'4	3L	-
†	3g2'3'4'6	3L	PNR
†	3g2'3'4'6'8 ‡	3L	NNC
3g2'3'4	3g2'3'4'8	3L	-
†	3g2'3'4'6'8 ‡	3L	NNC
0g∅	4g4	4L	-
†	4g4'8	4L	NNC
2g1'2'3	4g1'2'3'4x	4L	-
†	4g1'2'3'4'6x	4L	PNR
†	4g1'2'3'4'6'8x ‡	4L	NNC
4g1'2'3'4x	4g1'2'3'4'8x	4L	-
†	4g1'2'3'4'6'8x ‡	4L	NNC

For cases in which the one-some solution-pair associates with a scalar property, the three-some solution pair associates with translational velocity, Σ_y equals Σ_z, and one deploys modeling based on special relativity, the four-some associates with the popular modeling notion of 4-vector.

We anticipate making various uses of modeling based on notions of four-somes. For some uses, the popular modeling notion of three degrees of freedom pertains regarding the three-some. For some uses, the popular modeling notion of three degrees of freedom does not necessarily pertain regarding the three-some and a different notion of three aspects (such as three fermion flavours) pertains regarding the three-some.

Table 3 shows cascades that associate with Σg' solution-pairs for which $1 \leq \Sigma \leq 4$.

2.5. Some modeling that has bases in harmonic-oscillator mathematics

Popular modeling associates the two-word term ground state with the expression that equation (34) shows.

$$\nu = 0 \tag{34}$$

Based on equation (27), popular modeling associates a ground state energy with some positive multiplier times a factor of $D/2$. For example, for $D = 3$, the factor is 3/2. Popular modeling associates notions of excited states with positive integers ν. For example, for $D = 3$ and $\nu = 1$, the factor becomes $5/2$ and popular modeling says that the energy for a first excitation is proportional to $1 = (5/2) - (3/2)$.

Our modeling associates the two-word term base state with the expression that equation (35) shows. Equation (35) echoes the normalization-centric limit that associates with equation (32).

$$\nu = -D/2 \tag{35}$$

Based on equation (27), one might say that - regarding popular modeling - a base state associates with zero energy.

For a positive even integer D, retrofitting - as additional oscillators - the base state into some aspects of popular modeling associates with $D + 2\nu = 0$ and might associate with zero change regarding the popular notion of ground state energy. Such a retrofit might pertain regarding popular modeling wave functions.

We discuss - in the sense of equation (18), the notions of $j_0 = j = 2$, $j_1 = 1$, and $j_2 = 1$.

We assume that the j_1 oscillator associates with boson excitations. (Regarding popular modeling, equation (20) pertains regarding excitations and does not necessarily pertain regarding a relevant ground state symmetry. Popular modeling does not seem to include our notion of base state.) One base state associates with $D_0 = 2$ and $\nu_0 = -1$. The j_1 ground state associates with $D_1 = 1$ and $\nu_1 = 0$. The j_1 base state associates with $D_1 = 1$ and $\nu_1 = -1/2$. The j_2 ground state associates with $D_2 = 1$ and $\nu_2 = 0$. The j_2 base state associates with $D_2 = 1$ and $\nu_2 = -1/2$.

We anticipate that - for nonzero spin simple bosons - equation (36) might associate with a relevant j_2 ground state Gauge-like symmetry.

$$D_2 = 1,\, \nu_2 = 0 \ \rightarrow \ U(1) \text{ ground state symmetry} \tag{36}$$

Here, the two-element term Gauge-like symmetry might associate with the popular modeling notion of Gauge symmetry. More generally, we anticipate exploring the extent to which some modeling based on even integer values of D might associate with popular modeling Gauge symmetries. (See discussion related to equation (55).)

3. Results - General physics

3.1. Electromagnetic fields and electromagnetic properties of objects

We associate solution-pairs for which $\Sigma = |s| = 1$ with modeling regarding electromagnetism.

We show some mathematics regarding the combination of $\Sigma = |s| = 1$ and K_8. (Below, the two-word phrase in part points to the notion that we show one of two relevant solutions. Also, each of the solution-pairs - other than 1g1 - cascades from the 1g1 solution-pair.)

- The following are the only monopole, dipole, and quadrupole solution-pairs - 1g1 (based in part on $1 = |+1|$), 1g1‘2 (based in part on $1 = |-1+2|$), and 1g1‘2‘4 (based in part on $1 = |-1-2+4|$).
- The following notions point to three possibly relevant octupole solution-pairs - 1g1‘2‘4‘6x (based in part on $1 = |+1-2-4+6|$ and $1 = |-1-2-4+6|$), and 1g1‘2‘4‘8 (based in part on $1 = |-1-2-4+8|$).

For now, we focus on Newtonian modeling for electromagnetic interactions between point-like objects.

We consider contributions that a nonzero-charge object A makes to the electromagnetic field.

We associate the solution-pair 1g1 with the following notions - the position (at some time) of object A, the charge (at the same time) of object A, and MCP monopole modeling. Notions that associate with 1g1 comport with equation (16). These notions associate with modeling - within the topic of electromagnetism - that associates with the notion of producing a physics-monopole contribution to the electric field.

Popular modeling includes the notion that moving charge associates with the notion of producing a physics-dipole contribution to the magnetic field. Regarding modeling that comports with special relativity, the notion of a charge and current four-vector pertains. Some useful physics modeling does not comport with special relativity. We do not want to limit applications of our work to modeling that must comport with special relativity.

For our modeling, we extend the notion of a charge “one-some” to a charge-and-current “four-some.” (For modeling that would be compatible with special relativity, the notion of charge-and-current four-some associates with the popular modeling notion of charge-and-charge-current four-vector.)

Within the context of $\Sigma = 1$, the only solution-pair for which $n_\Gamma = 2$ (and for which the mathematical notion of dipole pertains) is 1g1‘2.

We associate the two (in the Γ for 1g1‘2) with (translational or linear) velocity. The one (in the Γ for 1g1‘2) continues to associate with position and charge. Here, Γ associates with a dipole potential (per equation (16)) and associates with a contribution to the magnetic field. We associate the one-element term three-some with this use of the solution-pair 1g1‘2.

Regarding modeling that has basis in point-like objects A that have zero intrinsic magnetic moments, we posit that one-some use of the solution-pair 1g1, three-some use of the solution-pair 1g1‘2, and the notion that the associated four-some can be a four-vector suffice to associate with Maxwell's equations.

Popular modeling includes the notion of nonzero (intrinsic) magnetic moments for objects - such as electrons - that popular modeling models as point-like. The property of magnetic moment moves along with the associated object A. We posit that a one-some use of 1g1‘2 and a three-some use of 1g1‘2‘4 associate with modeling for the motion of the property (of object A) of intrinsic magnetic moment. Here, the one-some item models as a three-vector (that associates with the magnitude of the magnetic moment and with the direction of an axis that associates with the magnetic moment). The three-some item associates with the one-some three-vector and with a second three-vector that associates with translational motion.

We posit that modeling that associates 1g1‘2‘4 with a property and makes one-some use of 1g1‘2‘4 has uses. Here, the four (as in $4 \in \Gamma$) associates with angular motion within object A. (For the Earth, $2 \in \Gamma$ associates with the magnitude and axis of the magnetic field, $4 \in \Gamma$ associates with the magnitude and axis of rotation, and the two axes do not align with each other.) Three-some use of 1g1‘2‘4‘8 associates with linear motion. Over time, the internal configuration changes and the one-some 1g1‘2‘4 property changes. We posit that modeling can feature a notion that internal stress-energy associates with the present configuration and one-some 1g1‘2‘4 property. Stress-energy dissipation associates with the changes of configuration and property.

Popular modeling includes the notion of the three-item series position, velocity, and acceleration. In our work and regarding an object A that has nonzero charge, 1g1 associates with position of charge and 1g1‘2 associates with velocity of charge. We posit that - regarding charge - 1g1‘2‘4 associates with acceleration. Popular modeling associates acceleration of an object A with forces (such as electromagnetic forces and gravitational forces) that associate with fields that associate with objects other than object A. Notions that combine object A and its environment become relevant. With respect to the 1g1 property of charge, the notion of quadrupole components of the 1g (or, electromagnetic) field associates - in some sense - with a partial loss of identity for object A. The popular modeling notion of stress-energy can pertain. We posit that - in concert with the notion of partial loss of identity regarding object A - the notion that the stress-energy can associate with intrinsic aspects (such as charge) of object A, extrinsic aspects (such as velocity) of object A, and aspects (such as the electromagnetic field) of the environment in which object A exists.

Elsewhere, this essay continues discussion regarding one-some uses and three-some uses of components of LRI fields (including the electromagnetic field) and regarding properties of objects. (See, for example, discussion related to table 12.)

3.2. Gravitational fields and gravitational properties of objects

We associate solution-pairs for which $\Sigma = |s| = 2$ with modeling regarding gravitation.

We associate the solution-pair 2g2 with two notions. One notion is the position of mass (for which we might say there is an object A). One notion is that of monopole physics. These two notions associate with modeling - within the topic of gravitation - that associates with the notion of a monopole component of a gravitational field.

We generate part of a cascade - that associates with 2g2 - by doubling integers that appear in a cascade that associates with 1g1.

Regarding one-somes and related three-somes, the following notions can pertain. 2g2 can associate with mass (or, scalar energy) and one-some. 2g2‘4 can associate with a current of mass (or energy) and three-some. Regarding special relativity, the combination of one-some 2g2 and three-some 2g2‘4 associates with an energy-and-momentum four-vector.

Regarding one-somes (and de-emphasizing three-somes), the following aspects pertain. 2g2‘4 associates with rotating mass (or energy). 2g2‘4‘8 might not associate with a useful (for the purposes of kinematics modeling) one-some, because - within K_8 - there is no way to represent an associated three-some.

We consider a cascade that associates with 2g1‘2‘3. (Regarding equation (6), $+2 = +1 - 2 + 3$.) Per remarks above regarding quadrupole components, we expect that - for some modeling - the quadrupole nature of 2g1‘2‘3 can associate with notions of stress-energy.

Compared to 2g2, 2g1‘2‘3 associates with two instances of three degrees of freedom (or three choices). Regarding modeling based on Newtonian physics and one-some uses of solution-pairs, we associate one instance (of three choices) with a possible magnitude and axis of rotation around a minimal moment of inertia and the other instance with a possible magnitude and axis of rotation around a maximal moment of inertia. (Some objects might model as having a spherically symmetric distribution of energy and not having either of the two axes. Some objects might model as having a unique axis for one moment and not having a unique axis for the other moment.)

Regarding popular modeling based on general relativity and a stress-energy tensor, we posit the following notions. One-some use of the 2g2 solution-pair associates with the one energy-density component of the stress-energy tensor. One-some use of the 2g2‘4 solution-pair associates with the three pressure components of the stress-energy tensor. One-some use of the 2g1‘2‘3 solution-pair associates with the twelve other components of the stress-energy tensor and with six independent values. Three values pertain to the three components with which popular physics modeling associates the two-word term momentum density. The same three values pertain to the three components with which popular physics modeling associates the two-word term energy flux. Three values pertain to the three components with which popular physics modeling associates the two-word term shear stress. The same three values pertain to the three components with which popular physics modeling associates the two-word term momentum flux.

Within the confines of K_4, two solution-pairs cascade from 2g1‘2‘3. Regarding one solution-pair, the equality $+2 = -1 + 2 - 3 + 4$ pertains. Regarding the other solution-pair, $+2 = +1 + 2 + 3 - 4$ pertains. We denote the pair of solution-pairs by the symbol 2g1‘2‘3‘4x.

Expanding from K_4 to K_8 leads to another set of possibilities regarding matched one-somes and three-somes. Solution-pairs of the form 2g1‘2‘3‘4‘8x might associate with the notion of one-some. Solution-pairs of the form 2g1‘2‘3‘4‘6‘8x might associate with the notion of three-some.

3.3. Attractive components and repulsive components of the gravitational field

We start by discussing modeling that associates with electromagnetism and special relativity.

We assume that - as sensed by an object C, an object A has nonzero charge. We consider three cases. For each of the first two cases, we assume that object A is not moving.

As a baseline case, we assume that object C is at rest.

As a second case, we assume that object C moves. The charge-and-current four-some (or, for this case, four-vector) that object C senses for object A associates - compared to the baseline case - with a larger magnitude of charge and with a nonzero current of charge. The electric field that associates with active properties of object A equals the electric field for the baseline case. Thus, object C senses a smaller (compared to the baseline case) ratio of electric field (associated with active properties of object A) to charge (that object C senses regarding object A).

As a third case, object A moves and object C does not move. Based on notions that associate with relativity, the following notion carries over from the second case. Object C senses a smaller (compared to the baseline case) ratio of electric field (associated with active properties of object A) to charge (that object C senses regarding object A).

We extrapolate to gravity.

One-some uses of monopole, quadrupole, and 16-pole solution-pairs can associate with no translational motion (by object A). One-some uses of monopole, quadrupole, and 16-pole solution-pairs associate with gravitational attraction (by object A). Three-some uses of dipole solution-pairs associate with dilutions to attraction (that associates with one-some use of the monopole solution-pair) and, hence, with repulsion. (Reference [45] discusses notions of repulsive components of gravity.) Three-some uses of octupole solution-pairs associate with dilutions to attraction (that associates with one-some use of the quadrupole solution-pair) and, hence, with repulsion. One-some uses of dipole and octupole solution-pairs associate with dilutions to attraction and, hence, with repulsion. (Within K_8, there are no 64-pole solution-pairs. The notion of one-some uses of 32-pole solution-pairs might not pertain.)

3.4. Eras regarding gravitational interactions between objects

We discuss gravitational interactions between two objects.

Table 4 shows components of gravitational interactions.

We discuss aspects regarding two objects that generally always move away from each other.

Table 4: Components of gravitational interactions. The column with the one-element label RSDP associates with popular modeling Newtonian models.

Interaction	Character	One-some 2g' solution-pairs	RSDP
16-pole	Attractive	2g1'2'3'4'8x	r^{-5}
Octupole	Repulsive	2g1'2'3'4x	r^{-4}
Quadrupole	Attractive	2g1'2'3	r^{-3}
Dipole	Repulsive	2g2'4	r^{-2}
Monopole	Attractive	2g2	r^{-1}

We consider popular modeling Newtonian models. As the two objects move away from each other, the relative effect of an RSDP $r^{-(n+1)}$ component decreases compared to the effect of an RSDP r^{-n} component. (Perhaps, compare with equation (16).) One might associate the two-word phrase time period with a time range in which an RSDP r^{-n_p} component provides dominant effects. Assuming that objects move away from each other and that one time period associates with $r^{-(n+1)}$ and another time period associates with r^{-n}, the time period that associates with $r^{-(n+1)}$ comes before the time period that associates with r^{-n}.

We posit - for popular modeling Newtonian models, special relativity, and general relativity - that, as two objects move away from each other, sub-sequences of the sequence 16-pole dominates, octupole dominates, $\cdots$, and monopole dominates pertain.

Two smaller objects (such as galaxies) transit similar time periods more quickly than do two larger objects (such as galaxy clusters).

3.5. Left- and right- regarding handednesses, solutions, isomers, and circular polarizations

3.5.1. Left-handedness and right-handedness

For each known elementary particle that has nonzero charge, popular modeling associates the notion of left-handed with the notions of particle and matter and associates the notion of right-handed with the notions of antiparticle and antimatter. Each such elementary particle associates with nonzero spin. The charge of an antimatter particle has the same magnitude and the opposite sign of the charge of the matter particle. The handedness associates with aspects of how the elementary particle contributes to the overall angular momentum of a system (of objects) that includes an instance of the particle or antiparticle.

3.5.2. Left-solution and right-solution

Our modeling includes the notion that each solution-pair associates with two solutions. Our modeling includes the notions of left-solution and right-solution. Across all known elementary particles that have nonzero charge, left-solution pertains for each case in which left-handedness pertains and right-solution pertains for each case in which right-handedness pertains. In our modeling, notions of left-solution and right-solution have relevance regarding all elementary particles, including elementary particles that have nonzero spin and no charge, including the Higgs boson (which has zero spin and zero charge), and including the possible inflaton (which would have zero spin, zero charge, and zero mass). Our notions of left-solution and right-solution do not run counter to the popular modeling notion that an elementary particle with zero charge can be its own antiparticle.

3.5.3. Left-isomer and right-isomer

Our modeling includes six isomers of all simple elementary particles. (Perhaps, preview discussion that relates to equation (45).) We use the two-word term isomer zero to name the isomer that includes ordinary matter stuff. We name the other five isomers isomer one, ..., and isomer five.

Our modeling points to three isomer-pairs. Regarding the stuff that associates with each isomer-pair, our modeling suggests that the number of left-solution simple particles equals the number of right-solution simple particles. We number the isomers so that each one of the isomer-pairs associates with a different one of the three following pairs - isomer zero and isomer three, isomer one and isomer four, and isomer two and isomer five.

Popular modeling associates with the notion that isomer zero currently includes much more left-solution simple particle stuff than right-solution simple particle stuff. Our modeling suggests that isomer three currently includes much more right-solution simple particle stuff than left-solution simple particle stuff.

With caution, we point to possible use of the one-element term left-isomer to associate with an isomer that associates with much more left-solution simple particle stuff than right-solution simple particle stuff and we point to possible use of the one-element term right-isomer to associate with an isomer that associates with much more right-solution simple particle stuff than left-solution simple particle stuff. Notions of caution associate with the following two sentences. Popular modeling and our modeling suggest that, early in the history of the universe, isomer zero had equal numbers of left-solution simple particles and right-solution simple particles. Regarding isomers one, two, three, four, five, and six, we know of no data about numbers of left-solution simple particles and numbers of right-solution simple particles.

3.5.4. Left-circular polarization and right-circular polarization

Regarding models regarding the photon, popular modeling includes models that feature the notions of one mode that associates with left-circular polarization and one mode that associates with right-circular polarization. (Alternative modeling features two modes that associate with linear polarization.) Our notion of left-solution associates with left-circular polarization regarding the photon. Our notion of right-solution associates with right-circular polarization regarding the photon. Our notions of left-solution and right-solution do not run counter to popular modeling that features linear polarization modes. Our notions of left-solution and right-solution do not run counter to the popular modeling notion that the notion of two photon modes does not necessarily associate directly with the notion that the photon is its own antiparticle.

Our models regarding each LRI elementary particle include notions of left-circular polarization and right-circular polarization. Our models do not necessarily associate LRI elementary particles with notions of left-handedness and right-handedness. Our models do not necessarily associate instances of LRI elementary particles with individual isomers.

3.6. Modeling that associates with color charge

Modeling above associates with notions that, for $0 \leq k_{max} \leq 4$, popular modeling notions of expressions for RSDP of $r^{-k_{max}}$, $r^{-(k_{max}-1)}$, $\cdots$, and r^{-1} can pertain. We anticipate discussing modeling for which the notion of RSDP r^0 has relevance. (See discussion - about isomers of simple particles - that leads to equation (45).)

Popular modeling associates the RSDP r^{+1} with the strong interaction and the notion of asymptotic freedom. Our modeling posits uses for the notion that k_{-1} associates with the RSDP r^{+1} and with aspects of the strong interaction. Our modeling includes the notion of s_{-1}.

Our modeling associates the three color charges with the three generators of a group $SU(2)$ that associate with a symmetry of the ground state of a two-dimensional harmonic oscillator that associates with $s_{-1} = -1$ and $s_{-1} = +1$. The color charge clear (or, white) might associate with the one generator that associates with a $U(1)$ symmetry that associates with a one-dimensional oscillator that might associate with $s_{-1} = 0$.

4. Results - Elementary particles

4.1. Symbols for families of elementary particles

We use the following notions to catalog elementary particles. A symbol of the form $S\Phi$ associates with a so-called family of elementary particles. Each elementary particle associates with one family. Each family associates with one of one, three, or eight elementary particles. For a family, the value S denotes the spin (in units of $\hbar$) for each elementary particle in the family. S associates with the popular modeling expression $S(S+1)\hbar^2$ that associates with angular momentum. Regarding popular modeling, known values of S include 0, 0.5, and 1. For each one of the numbers 0, 0.5, 1, 2, 3, and 4, our work points to at least one possible elementary particle for which S would equal that number. The symbol Φ associates with a symbol of the form X_Q, in which X is a capital letter and Q is the magnitude of charge (in units of $|q_e|$, in which q_e denotes the charge of an electron) for each particle in the family. For cases for which $Q = 0$, this essay omits - from the symbols for families - the symbol Q.

4.2. Simple particles

We anticipate that the following notions pertain regarding simple particles to which this essay points.

- Each simple particle associates with a one-some use of one solution-pair for which $\Sigma = 0$ or with one-some uses of two solution-pairs for which $\Sigma = 0$. (Regarding one-some solution-pairs, quarks associate with one-some uses of two solution-pairs and all other simple particles associate with one-some uses of one solution-pair. For quarks, one-some use of one of the solution-pairs associates with

$Q = 1$ and one-some use of the other solution-pair associates with $Q = 0$. Each quark associates with a Q of $(2/3)1 + (1/3)0$ or with a Q of $(1/3)1 + (2/3)0$.)

- Each simple particle associates with (a) a one-some use of a $\Sigma = 0$ solution-pair that associates with one of the Higgs, Z, and W bosons or (b) one-some uses of $\Sigma = 0$ solution-pairs that cascade from one-some $\Sigma = 0$ solution-pairs that associate with the Higgs, Z, and W bosons. (This essay de-emphasizes the notion that some simple particles might not associate with these cascades.)
- Each simple particle associates with at least one three-some use of a solution-pair for which $\Sigma = 0$.
- Each simple particle is a fermion $\Leftrightarrow$ $6 \in \Gamma$ for all relevant one-some solution-pairs. (The symbol $\Leftrightarrow$ denotes the four-word phrase if and only if.)
- Each simple particle can model as free (as in a popular modeling use of the word free) $\Leftrightarrow$ $8 \notin \Gamma$ for each relevant one-some solution-pair.
- For one-some uses of solution-pairs, $\{1, 3, 4\} \subset K_n$ $\Leftrightarrow$ the solution-pair associates with zero charge.
- For each simple particle, the mass is zero $\Leftrightarrow$ for the relevant one-some solution-pairs, $6 \notin \Gamma$ and $8 \in \Gamma$.
- For each simple particle, the following notions associate with the spin of the simple particle. Equation (37) and equation (38) define n'_Γ. In equation (37) and equation (38), each of Γ and n_Γ associates with each of the one (for simple particles other than quarks) or two (for quarks) relevant one-some solution-pairs. Equation (39) pertains regarding the spin S.

$$\delta_{6\in\Gamma} = 1, \text{ if } 6 \in \Gamma;\ \delta_{6\in\Gamma} = 0, \text{ if } 6 \notin \Gamma \tag{37}$$

$$n'_\Gamma = n_\Gamma - (1/2)\delta_{6\in\Gamma} \tag{38}$$

$$S = |n'_\Gamma - 4| \tag{39}$$

4.2.1. A catalog of simple particles

Table 5 catalogs simple particles.

4.2.2. Simple particles that our modeling suggests and people have yet to find

The aye boson (which associates with the family 0I) associates with popular modeling notions of an inflaton that might have played significant roles early in the evolution of the universe.

The jay boson (which associates with the family 1J) might be useful for modeling interactions between the two fermions in a pair of similar fermions. For such a pair, the two similar fermions do not necessarily need to be elementary fermions. The jay boson associates with popular modeling notions of Pauli repulsion.

The three arc fermions (which associate with the family 0.5R) would combine, via interactions mediated by gluons, to form hadron-like particles. Such hadron-like particles might associate with popular modeling notions of dark matter.

The three heavy fermions (which associate with the family 0.5M) might associate with popular modeling notions of dark matter.

4.2.3. Three-some solutions that associate with simple bosons

For this discussion, we consider that the two-word phrase can produce can associate with producing (the popular modeling notion of) virtual particles but might not necessarily need to associate with being able to produce only virtual particles.

The notion of $6 \in \Gamma$ for a three-some solution-pair pertains for each simple boson.

For each simple boson, we posit the following. The simple boson can produce a pair of similar simple fermions such that one of the simple fermions associates with left-solution (and, in the sense of popular modeling, is a left-handed fermion) and the other one of the simple fermions associates with right-solution (and, in the sense of popular modeling, is a right-handed fermion).

The notion of $8 \in \Gamma$ for a three-some solution-pair pertains for each simple boson, except the W boson.

Table 5: Simple particles. The one element symbol 1-some abbreviates the term one-some. The one element symbol 3-some abbreviates the term three-some. For each item in the second column or the third column, each of the integers is a member of a relevant Γ and the relevant Γ does not include other integers. The symbol n_{EP} denotes the number of elementary particles. For each one of some rows in table 5, three-some solutions cascade to become one-some solutions for the families to which the rightmost column alludes. This essay assumes that 32-pole solutions do not cascade. The symbol ‡ associates with the use - for two boson families - of the same set of one-some and three-some solution-pairs.

Families	$0 = \ldots$, re $0g\Gamma$ 1-somes	$0 = \ldots$, re $0g\Gamma$ 3-somes	n_{EP}	Name	Cascades to
0H	$\lvert +1-2-3+4 \rvert$	$\lvert +1-2-3-4+8 \rvert$; $\lvert -1+2-3-4+6 \rvert$	1	Higgs	1J, 1G; 0.5Q, 0.5M
1Z	$\lvert -1-3+4 \rvert$	$\lvert -1-3-4+8 \rvert$; $\lvert +1-3-4+6 \rvert$	1	Z	0I; 0.5N
$1W_1$	$\lvert -1-2+3 \rvert$	$\lvert -1-2-3+6 \rvert$	1	W	$0.5C_1$
0I	$\lvert -1-3-4+8 \rvert$	$\lvert +1-2-3-4+8 \rvert$; $\lvert -1+3-4-6+8 \rvert$	1	Aye	1J, 1G; 0.5R
1J	$\lvert +1-2-3-4+8 \rvert$ ‡	$\lvert +1-2+3-4-6+8 \rvert$, $\lvert -1-2-3+4-6+8 \rvert$	1	Jay	-
1G	$\lvert +1-2-3-4+8 \rvert$ ‡	$\lvert +1-2+3-4-6+8 \rvert$, $\lvert -1-2-3+4-6+8 \rvert$	8	Gluons	-
0.5N	$\lvert +1-3-4+6 \rvert$	$\lvert -1+3-4-6+8 \rvert$	3	Neutrinos	0.5R
$0.5C_1$	$\lvert -1-2-3+6 \rvert$	$\lvert -1+2-3-4+6 \rvert$, $\lvert -1+2-3-6+8 \rvert$	3	Charged leptons	0.5Q, 0.5M
0.5R	$\lvert -1+3-4-6+8 \rvert$	$\lvert -1-2-3+4-6+8 \rvert$, $\lvert +1-2+3-4-6+8 \rvert$	3	Arcs	-
$0.5Q_{y/3}$, $y = 1$ or 2	$\lvert -1+2-3-6+8 \rvert$; $\lvert -1+2-3-4+6 \rvert$	$\lvert -1-2-3+4-6+8 \rvert$, $\lvert +1-2+3-4-6+8 \rvert$	6	Quarks	-
0.5M	$\lvert -1+2-3-4+6 \rvert$	$\lvert -1-2-3+4-6+8 \rvert$, $\lvert +1-2+3-4-6+8 \rvert$	3	Heavy neutrinos	-

For each simple boson except the W boson, we posit the following. The boson can produce a pair of similar elementary bosons such that one of the bosons is a left-handed boson and the other one of the bosons is a right-handed boson.

For the W boson, no three-some solution-pair for which $8 \in \Gamma$ pertains. The lack of a three-some solution-pair for which $8 \in \Gamma$ associates with the popular notion that a W boson does not decay into a pair of bosons for which $Q = 0.5$ would pertain for each boson in the pair.

4.2.4. Cases for which n_{EP} (or, the number of simple particles) is three or six

Regarding simple fermions other than quarks, the notion - regarding one-some solutions - of $6 \in \Gamma$ associates with three flavors and, therefore, with three particles and with $n_{EP} = 3$. (See table 5.)

Discussion above regarding two values of Q - a Q of $(2/3)1 + (1/3)0$ and a Q of $(1/3)1 + (2/3)0$ - associates with $n_{EP} = 2 \times 3 = 6$ quarks.

4.2.5. Cases - other than the jay boson - for which n_{EP} is one

For each of the Higgs, Z, and W bosons, each relevant three-some Γ appears once in table 5. We posit that - for simple bosons - single appearances of three-some values of Γ do not point to values of n_{EP} that exceed one.

4.2.6. The jay boson and the gluons

Here, the same one one-some solution-pair pertains and the same two three-some solution-pairs pertain. One Γ (Γ = 1‘2‘3‘4‘6‘8) associates with the two three-some solution-pairs. Regarding particle count, we posit that a factor of three associates with each three-some solution-pair. Two factors of three multiply to yield nine. In popular modeling, $n_{EP} = 8$ pertains for gluons. We posit that, regarding the jay boson, n_{EP} equals nine minus eight. For the jay boson, $n_{EP} = 1$.

4.2.7. The notion of free and the notion of entwined

Popular modeling associates the word free with each known elementary particle other than the quarks and the gluons. This essay uses the word entwined to associate - in this context - with the opposite of free.

Table 6: Sets of solution-pairs that might associate with LRI (or, long-range interaction) elementary particles. For each relevant solution-pair, $\Sigma \in \Gamma$. The symbol n_{EP} denotes the number of elementary particles. Items that the table shows in parentheses might - depending on future data or on interpretations of vocabulary and modeling - associate with elementary particles. TBD denotes the three-word phrase to be determined.

Family	ΣgΓ for solution-pairs	n_{EP}	Boson
1L	1gΓ	1	Photon
(2L)	2gΓ	(1)	(Graviton)
(3L)	3gΓ	(1)	(TBD)
(4L)	4gΓ	(1)	(TBD)

We suggest that - across all simple particles - the following notions pertain. For each simple particle that associates with a one-some solution-pair for which $8 \in \Gamma$, the notion of entwined pertains. (For quarks, $8 \in \Gamma$ pertains for one of the two associated one-some solution-pairs.) For each simple particle that associates with no one-some solution-pairs for which $8 \in \Gamma$, the notion of free pertains.

Possibly, such a notion of entwined pertains regarding components - of LRI interactions - for which one-some uses associate with $8 \in \Gamma$.

4.2.8. The numbers of flavours, regarding elementary fermions

For each one-some use of a 0gΓ solution-pair that associates with elementary fermions, $6 \in \Gamma$. (For each one-some use of a 0gΓ solution-pair that associates with elementary bosons, $6 \notin \Gamma$.)

We posit that modeling based on a two-dimensional isotropic harmonic oscillator can associate with the pair $s_6 = -1$ and $s_6 = +1$. We posit that $SU(2)$ symmetry pertains. We posit that the three generators of the $SU(2)$ symmetry associate with three (fermion) flavors.

4.3. LRI elementary bosons

Table 6 alludes to sets of solution-pairs that might associate with LRI (or, long-range interaction) elementary particles. Elsewhere, we point to aspects that seem to associate with a limit - regarding ΣL - of $\Sigma \leq 4$. (See discussion related to table 20.) Elsewhere, we discuss some popular modeling notions that might associate with a limit of $\Sigma \leq 2$. (See discussions that cite reference [19].) For simple bosons, one-some solution-pairs do not associate with $6 \in \Gamma$. (See table 5.) This essay associates the acronym PNR with LRI solution-pairs for which $6 \in \Gamma$. (See table 3.) We retain, in our discussion, PNR LRI solution-pairs. However, if one assumes that PNR LRI solution-pairs do not pertain, our modeling might still explain all data that our modeling - including PNR LRI solution-pairs - explains.)

For each LRI boson, the notion of $6 \in \Gamma$ pertains for at least one three-some solution-pair. We posit that each LRI boson can produce a pair of similar simple fermions such that one of the simple fermions associates with left-solution (and, in the sense of popular modeling, is a left-handed fermion) and the other one of the simple fermions associates with right-solution (and, in the sense of popular modeling, is a right-handed fermion).

4.4. Properties of elementary bosons

We use the symbol S to denote spin, as in the popular modeling expression $S(S+1)\hbar^2$. We use the symbol Q to denote the magnitude (in units of the magnitude $|q_e|$ of the charge - q_e - of the electron). We use the symbol m to denote mass. We use the symbol m' to denote mass (in units of $m_{\mathrm{Z}}/3$, in which m_{Z} denotes the mass of the Z boson).

Equation (40) defines values for l_{ms}.

$$l_{ms} = 0, \text{if } m = 0;\ l_{ms} = -1, \text{ if } m > 0 \tag{40}$$

Equation (41) defines $(j_m)^2$.

$$(j_m)^2 \equiv (m')^2 + S^2 + Q(Q+1) + l_{ms} \tag{41}$$

Table 7 discusses relationships between properties of elementary bosons. (Reference [14] provides data regarding the masses of the Higgs, Z, and W bosons. Of the nonzero masses to which table 7 alludes, the most accurately known mass is that of the Z boson. Using the mass of the Z boson and numbers in table 7, one can calculate a nominal mass for the Higgs boson and a nominal mass for the W boson. The calculated mass for the Higgs boson differs from the experimentally determined mass by less than two (experimental) standard deviations. The calculated mass for the W boson differs from the experimentally

Table 7: Relationships between properties of elementary bosons. Q denotes the magnitude of charge, in units of $|q_e|$. m denotes mass, in units of $m_{\mathrm{Higgs}}/17^{1/2}$ or in units of $m_{\mathrm{Z}}/9^{1/2}$. S denotes spin, as in the expression $S(S+1)\hbar^2$. l_{ms} equals -1 for $m > 0$ and equals 0 for $m = 0$. $(j_m)^2$ is the sum of the numbers in the preceding four columns. Each sum is the square of an integer. For each nonzero-mass particle, j_m equals the one-some n_Γ. NYN denotes the three-word phrase not yet named.

Bosons	Family	$Q(Q+1)$	$(m')^2$	S^2	l_{ms}	$(j_m)^2$
Higgs	0H	0	17	0	-1	16
Aye	0I	0	0	0	0	0
Z	1Z	0	9	1	-1	9
W	$1\mathrm{W}_1$	2	7	1	-1	9
Jay	1J	0	0	1	0	1
Gluons	1G	0	0	1	0	1
Photon	1L	0	0	1	0	1
Graviton	2L	0	0	4	0	4
NYN	3L	0	0	9	0	9
NYN	4L	0	0	16	0	16

determined mass by less than four (experimental) standard deviations. To the extent that one uses the notion that ruling out an equality requires a difference of at least five standard deviations, experimental results do not seem to rule out relationships that table 7 states.)

Equation (42) suggests results regarding and possibly extending the popular modeling notion of weak mixing angle. (Equation (42) comports with data that reference [46] reports.)

$$(m_{\mathrm{W}})^2 : (m_{\mathrm{Z}})^2 : (m_{\mathrm{Higgs}})^2 :: 7 : 9 : 17 \tag{42}$$

This essay notes - but de-emphasizes the following discussion.

Each one of m' and S is always non-negative. Perhaps some modeling associates with two degrees of freedom - positive quantity and zero quantity. Regarding mathematics associated with Laplacian operators, for $D = 2$ and for each of $P = m'$ and $P = S$, the factor $P(P + D - 2)$ pertains regarding aspects of the math. (Compare with equation (28). The notion that P here associates with ν there pertains.)

Charge - which is a basis for Q - can be positive, zero, or negative. Perhaps some modeling associates with three degrees of freedom - positive quantity, zero quantity, and negative quantity. Regarding mathematics associated with Laplacian operators, for $D = 3$ and for $P = Q$, the factor $P(P + D - 2)$ pertains regarding aspects of the math.

Whether or not l_{ms} is nonzero associates with the popular modeling notion of whether longitudinal polarization can pertain. Regarding mathematics associated with Laplacian operators, for $D = 2$ and for $P = (l_{ms})^{1/2}$, the factor $P(P + D - 2)$ pertains regarding aspects of the math.

4.5. Properties of simple fermions

4.5.1. Known and hypothetical simple fermions for which $Q = 1$

For some value of mass, the gravitational attraction between two identical $Q = 1$ hypothetical simple fermions would equal the electrostatic repulsion between the two simple fermions. Elsewhere, our work shows that a mass - for which we use the expression $m(18, 3)$ - seems to have meaning beyond the notion that - for the mass $m(18, 3)$ - gravitational attraction between two $Q = 1$ identical simple fermions would be three-quarters of the electrostatic repulsion between the two identical simple fermions. (See table 9 and table 20.)

4.5.2. Known simple fermions

Table 8 discusses relationships between properties of known charged simple fermions. (Reference [14] provides the data that underlies table 8.)

Table 9 shows equations that underlie aspects of table 8. (Reference [14] provides the data that underlies table 9.)

The notion that d'' is nonzero might associate with a notion of anomalous mass and with the solution-pair 2g1'3'4 (which associates with the Z boson via $\Gamma = 1'3'4$ and with 2g").

Table 10 suggests rest energies that may pertain regarding the 0.5N neutrinos. This table extends aspects of table 8 and table 9. (Reference [14] provides data that underlies aspects of table 8, table 9, and table 10. Reference [47] discusses data and modeling regarding upper bounds for the sum of the masses of the three neutrinos. Reference [18] discusses the notion of neutrino mass mixing.)

Table 8: Values of $\log_{10}(m_{\text{particle}}/m_e)$ for known charged simple fermions. Regarding flavour, this table generalizes, based on popular modeling terminology that associates with charged leptons and with neutrinos. For example, popular modeling uses the term electron-neutrino. In table 8, the symbol l_f numbers the three flavours. The "l_f (0.5C$_1$)" terms pertain for fermions in the 0.5C$_1$ family. The symbol $0.5\mathrm{Q}_{>0}$ denotes the pair $0.5\mathrm{Q}_{1/3}$ and $0.5\mathrm{Q}_{2/3}$. The "l_f ($0.5\mathrm{Q}_{>0}$)" terms pertain for quarks (or, simple particles in the two families $0.5\mathrm{Q}_{2/3}$ and $0.5\mathrm{Q}_{1/3}$). l_m is an integer parameter. The domain $-6 \leq l_m \leq 18$ might have relevance regarding modeling. Q denotes the magnitude of charge, in units of $|q_e|$. The family 0.5C$_1$ associates with $Q = 1$. The family $0.5\mathrm{Q}_{2/3}$ associates with $Q = 2/3$. The family $0.5\mathrm{Q}_{1/3}$ associates with $Q = 1/3$. Regarding the rightmost four columns, items show $\log_{10}(m_{\text{particle}}/m_e)$ and - for particles that nature includes - the name of a simple fermion. For each † case, no particle pertains. Each number in the column with label $Q = 1/2$ equals the average of the number in the $Q = 2/3$ column and the number in the $Q = 1/3$ column. The notion of geometric mean pertains regarding the mass of the $Q = 2/3$ particle and the mass of the $Q = 1/3$ particle. Regarding each † case, a formula for $m(l_m, l_q)$ calculates the number. Regarding the formula, the domain $0 \leq l_q \leq 3$ pertains. Regarding table 8, $l_q = 3Q$ pertains. Table 9 shows the formula.

l_f (0.5C$_1$)	l_f ($0.5\mathrm{Q}_{>0}$)	l_m	$Q = 1$	$Q = 2/3$	$Q = 1/2$	$Q = 1/3$
1 (Electron)	1 (Up, Down)	0	0.00 Electron	0.66 Up	0.80 †	0.94 Down
-	2 (Charm, Strange)	1	1.23 †	3.36 Charm	2.83 †	2.29 Strange
2 (Mu)	3 (Top, Bottom)	2	2.32 Muon	5.52 Top	4.72 †	3.92 Bottom
3 (Tau)	-	3	3.54 Tau	-	-	-

Table 9: Equations that underlie aspects of table 8. This table shows equations that may pertain regarding all known charged simple fermions, the known 0.5N neutrinos, the suggested 0.5R arcs, and the suggested 0.5M heavy neutrinos. (Regarding 0.5N neutrinos, see table 10. Regarding 0.5R arcs, see table 11. Regarding 0.5M heavy neutrinos, see discussion related to equation (44).)

Topic	Note
Preliminary calculation	
	$\beta' = m_\tau/m_e$ - Defines β'. m_τ equals the mass of the tau particle (which is a charged lepton). m_e equals the mass of the electron. $(4/3) \times (\beta^2)^6 = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2)$ - Defines β. The right-hand side of the equation is the ratio of the electrostatic repulsion between two electrons to the gravitational attraction between the two electrons. The ratio does not depend on the distance between the two electrons. $\beta \approx 3477.1891 \pm 0.0226$ - This number results from data and the formula that defines β. The standard deviation reflects the standard deviation for G_N, the gravitational constant. $\beta' = \beta$ - We posit this equation. $m_{\tau,\ \text{calculated}} \approx 1776.8400 \pm 0.0115\ \text{MeV}/c^2$ - This number results from data and from $\beta' = \beta$. The standard deviation reflects the standard deviation for G_N, the gravitational constant.
Main calculation	
	These calculations produce numbers that table 8 shows. $l_q = 3Q$. $m(l_m, l_q) = m_e \times (\beta^{1/3})^{l_m + (j''_{l_m}) d''} \times (\alpha^{-1/4})^{g(l_q)\cdot(1+l_m) + j'_{l_q} d'(l_m))}$. $\alpha = ((q_e)^2/(4\pi\varepsilon_0))/(\hbar c)$ - Expression for α, the fine-structure constant. $j''_{l_m} = 0, +1, 0, -1$ for, respectively, l_m mod 3 = $0, 1, 3/2, 2$; with $3/2 \bmod 3 \equiv 3/2$. $d'' = (2 - (\log(m_\mu/m_e)/\log(\beta^{1/3}))) \approx 3.840679 \times 10^{-2}$. $g(l_q) = 0, 3/2, 3/2, 3/2, 3/2$, for, respectively, $l_q = 3, 2, 3/2, 1, 0$. $j'_{l_q} = 0, -1, 0, +1, +3$ for, respectively, $l_q = 3, 2, 3/2, 1, 0$. $d'(0) \sim 0.324,\ d'(1) \sim -1.062,\ d'(2) \sim -1.509$ - Based on attempting to fit data.

Table 10: Rest energies that may pertain regarding the 0.5N neutrinos.

Topic	Note
$l_m = -1$	$m(-1,3) = m(-1,3/2)$ - Comports with the equation underlying the main calculation regarding the masses of charged elementary fermions.
Assumption	$m(l_m, 3/2)$ pertains - regarding simple fermions - for $l_m \leq -1$.
Neutrinos	We suggest rest energies for the three 0.5N neutrinos. Popular modeling suggests - based on observations - that the sum of the three neutrino rest energies is at least approximately 0.06eV and not more than approximately 0.12eV. We note two possibilities. • $mc^2 = m(-4,3/2)c^2 \approx 3.4 \times 10^{-2}$ eV pertains for each of the three neutrinos. • $mc^2 = m(-4,3/2)c^2 \approx 3.4 \times 10^{-2}$ eV pertains for each of two neutrinos. For one neutrino, one of $m(-6,3/2)c^2 \approx 4.2 \times 10^{-6}$ eV and $m(-5,3/2)c^2 \approx 4.4 \times 10^{-4}$ eV might pertain.
Neutrinos	We suggest aspects regarding the popular modeling notion of possible differences between mass eigenstates and interaction eigenstates for the three 0.5N neutrinos. Interactions between 2L and a simple fermion conserve the mass of the simple fermion, but do not necessarily conserve the flavour of the simple fermion. (See table 12.) Interactions between 3L or 4L and a simple fermion do not necessarily conserve the mass of the simple fermion. Popular modeling Standard Model notions of mass mixing might associate with effects that associate - in our modeling - with 3g" solution-pairs or with 4g" solution-pairs. A QFT (or, quantum field theory) that includes gravity and that features a limit (regarding ΣL) of $\Sigma \leq 2$ might associate mass mixing with a notion - regarding our modeling - of anomalous gravitational properties and might successfully estimate aspects that popular modeling associates with mass mixing. (Compare with discussion - regarding anomalous magnetic moments - related to equation (54).) One such anomalous gravitational property might associate with the solution-pair 4g1‘2‘3, which associates with 4g" and does not associate with 4g'.

Table 11: Rest energies that might pertain regarding the suggested 0.5R arcs.

Topic	Note
Arcs	Our work suggests (but does not necessarily require) some specific masses for the three arc particles. $l_q = 0$ - This notion comports with the notion - for arcs - that $Q = 0$. $m(l_m, 0) = m(l_m, 1) \cdot (m(l_m, 1)/m(l_m, 2))$ - This essay assumes this equation. $m(0, 0)c^2 \approx 10.7$ MeV, $m(1, 0)c^2 \approx 6.8$ MeV, $m(2, 0)c^2 \approx 102$ MeV.

4.5.3. Masses for the would-be zero-charge quark-like simple fermions

Table 11 suggests rest energies that might pertain regarding the suggested 0.5R arcs. This table extends aspects of table 8 and table 9. (Reference [14] provides data that underlies aspects of table 8, table 9, and table 11.)

We explore two alternatives regarding values of $d'(0)$, $d'(1)$, and $d'(2)$. (See table 9.) Changing those numbers would impact the calculated masses for quarks and the calculated suggested masses for arcs. (Changing those numbers would not impact the calculated masses for charged leptons.) Regarding each of the two alternatives, if one excludes one of three methods for estimating the mass of the top quark, the calculated mass for each of the six quarks is within five standard deviations of the experimental mass. (Reference [14] discusses the three methods.) For the third method for estimating the mass of the top quark, the value that we calculate for the mass of the top quark would be less than eleven standard deviations below the mass that popular modeling has calculated.

One alternative has bases in the notions of $d'(-1) = 0^2/2^2$, $d'(0) = 1^2/2^2$, $d'(1) = -2^2/2^2$, and $d'(2) = -(2 \times 3)/2^2$. For this alternative, the three arc rest energies would, respectively, be ≈ 8.14 MeV, $m(1, 3)c^2$, and $m(2, 3)c^2$.

The other alternative has bases in the notions of $d'(0) \approx 0.264825$, $d'(1) = -2^2/2^2$, and $d'(2) = -(2\times3)/2^2$. For this alternative, the three arc rest energies would, respectively, equal $m(1, 3)c^2$, $m(1, 3)c^2$, and $m(2, 3)c^2$. Across the three 0.5C$_1$ elementary fermions and the three 0.5R elementary fermions, $m(0, 3)c^2$ would pertain once, $m(1, 3)c^2$ would pertain twice, $m(2, 3)c^2$ would pertain twice, and $m(3, 3)c^2$ would pertain once. Regarding $d'(0)$, one might consider the possibility that the following notions pertain. $d'(0) = +1/2^2$ pertains. The notion of anomalous property pertains. The anomalous property might associate with one-some use of the solution-pair 3g1‘2‘4 (which associates with 3g” and not with 3’g) or with one-some use of the solution pair 1g2‘3‘4.

4.5.4. Masses for the would-be heavy neutrinos

For purposes of estimating or calculating masses, the known neutrinos associate with a value of l_m for which $-6 \leq l_m \leq -4$. Quarks (and, our modeling suggests, arcs) associate with $0 \leq l_m \leq 2$. Charged leptons associate with $0 \leq l_m \leq 3$.

We posit that - for the purposes of our modeling regarding simple fermions other than charged leptons - one can consider that the ranges $-6 \leq l_m \leq -4$, $0 \leq l_m \leq 2$, and $6 \leq l_m \leq 8$ associate with left-handedness and that the ranges $-3 \leq l_m \leq -1$ and $3 \leq l_m \leq 5$ associate with right-handedness. (Discussion related to isomers and dark matter echoes these notions - regarding left-handedness and regarding right-handedness - regarding the ranges $0 \leq l_m \leq 2$, $3 \leq l_m \leq 5$, and $6 \leq l_m \leq 8$.

We posit that heavy neutrinos associate with left-solutions and with $6 \leq l_m \leq 8$.

Each one of equation (43) and equation (44) points to a possible lower bound regarding rest energies for heavy neutrinos.

$$m(6, 3)c^2 \sim 6 \times 10^3 \text{GeV} \tag{43}$$

$$m(6, 3/2)c^2 \sim 2.5 \times 10^9 \text{GeV} \tag{44}$$

The result that equation (43) shows might be large enough to comport with limits that associate with observations. (References [48] and [49] discuss limits that observations may set.) Use of the result that equation (44) suggests might be more appropriate than use of the result that equation (43) suggests, given the means above that we use to estimate rest energies for neutrinos.

5. Results - Dark matter

5.1. Five DM isomers of simple particles and one mostly OM isomer of simple particles

The acronym DM denotes dark matter. The acronym OM denotes ordinary matter.

We explore modeling that associates with the notion that r^0 for RSDP associates with $n_\Gamma = 0$ and with $\Gamma = \emptyset$. The modeling associates with K_0. (See equation 4.) The modeling associates with a next step in the series r^{-5} and 16-pole, $\cdots$, r^{-2} and dipole, and r^{-1} and monopole.

Within Newtonian physics, a property - of an object A - that would associate with the RSDP r^0 would not exert an LRI force on an object C. We associate this notion with the notion that object A surrounds object C, associates with an essentially unbounded extent of physical diameter, and has - for the purposes of relevant one-some modeling - uniform density of a relevant property.

Per equation (4), for any K_n that is relevant for our modeling, $k = 0$ is an element of K_n. Per equation (8), $k = 0$ is never an element of a list Γ.

For simple particles, we associate $s_0 = +1$ and $s_0 = -1$ with the ground state of one two-dimensional isotropic harmonic oscillator and with a corresponding $SU(2)$ symmetry. We associate each one of the three generators of the group $SU(2)$ with a notion for which we use the one-element term net-left-right.

For an object A, for each of the three net-left-right numbers, we associate the property that associates with r^0 with the relevant (to object A) number of left-solution simple particles minus the number of right-solution simple particles.

We posit that nature includes six isomers of each simple particle. The number six associates with three times two. The factor of three associates with three net-left-right numbers. The factor of two associates with the two aspects of left-solution and right-solution.

We posit that all ordinary matter simple particles associate with one isomer. For that isomer, we use the two-word phrase isomer zero. Regarding isomer zero, hadron-like particles made from arcs and gluons would measure - not as ordinary matter, but - as dark matter. Regarding isomer zero, heavy neutrinos would measure as dark matter.

For simple particles that have nonzero spin, left-solution associates with left handedness and right-solution associates with right-handedness. For the isomer that includes ordinary matter simple particles, popular modeling pertains and left-handedness associates with the word matter and right-handedness associates with the word antimatter.

For each of the three net-left-right numbers, we associate the one-element construct isomer-pair with the two relevant isomers.

We deploy the word instance. We say that our modeling points to six instances of simple particles. We anticipate extending the notion of instance to uses regarding ΣgΓ components of LRI.

We consider weak interactions. The interacting simple particles and the simple bosons that carry those interactions associate with just one isomer. This notion of just one isomer pertains for each of the six isomers (or, equivalently) for each one of the six instances of relevant simple particles.

Equation (45) pertains regarding simple particles and the weak interaction and also regarding simple particles and the strong interaction. The integer n_i denotes a number of instances of an aspect of nature. The integer ρ_I denotes the number of isomers that associate with one instance. Regarding ρ_I, we use the word reach. For each one of the weak interaction and the strong interaction, the reach of the simple bosons that mediate the interaction is one isomer. One instance of simple bosons does not reach (or, mediate interactions) between two (or more than two) isomers.

$$n_i \rho_I = 6 \tag{45}$$

Isomer zero underlies stuff that measures as ordinary matter (and some stuff - at least one of arc-plus-gluon hadron-like particles and heavy neutrinos - that would measure as dark matter). All stuff that associates with the other five isomers measures as dark matter. We name these five isomers isomer one, ..., and isomer five. We number the isomers so that each one of the isomer-pairs associates with a different one of the three following pairs - isomer zero and isomer three, isomer one and isomer four, and isomer two and isomer five.

Regarding ΣgΓ components of LRI, we anticipate the following. Positive integer values of other than six can pertain for n_i. Positive integer values of other than one can pertain for ρ_I. Equation (45) pertains. The notion of net-left-right does not pertain regarding components of LRI. For some popular modeling (but not popular modeling that features linear polarizations), notions of left circular polarization and right circular polarization pertain.

5.2. Reaches - and associated properties - for components of LRI fields

Work above does not address the topic of the number of instances of each component of 1L and of each component of 2L.

Popular modeling suggests that people do not (much) observe effects of electromagnetism that might associate with dark matter. For our modeling, the reach of a one-some use of an instance of 1g1 should be one isomer.

Popular modeling suggests that people observe effects of gravity that might associate with (all) dark matter. For our modeling the reach of a one-some use of one instance of 2g2 should be six isomers.

The following modeling points to an appropriate reach for 1g1 and an appropriate reach for 2g2. The modeling also points - for one-some uses of other components of 1L and 2L - to reaches that seem to help explain data. (Our discussion of the modeling may seem to lack a basis other than the notion that - eventually - the modeling seems to help explain data. However, the modeling seems to comport with popular modeling notions of Gauge symmetries. See discussion related to equation (55).)

We consider a one-some use of a Σg' solution-pair for which $1 \leq \Sigma \leq 4$. Equation (10) defines n_0. Our modeling considers that there are n_0 relevant instances of $s_k = -1$ for which a one-dimensional harmonic oscillator does not associate with arithmetic that computes Σ. Our modeling considers that there are n_0 relevant instances of $s_k = +1$ for which a one-dimensional harmonic oscillator does not associate with arithmetic that computes Σ. We also consider that each cascade-related three-some solution associates with one more instance of $s_k = -1$ and with one more instance of $s_k = +1$.

We consider the notion that modeling regarding each one-some use of one LRI solution-pair associates with mathematics that associates with a $2(n_0+1)$-dimensional isotropic harmonic oscillator. We set aside one one-dimensional harmonic oscillator to associate with excitations of the relevant Σg field. Modeling for each component of the Σg field associates with the ground state for a $(2n_0+1)$-dimensional harmonic oscillator and, thereby, if $n_0 \geq 1$, with a symmetry that associates with the group $SU(2n_0+1)$.

We posit the following regarding reaches of one-some uses of LRI solution-pairs.

If $n_0 = 0$, equation (46) pertains. This result comports with the notion that each isomer associates with its own instance of one-some 1g1 and associates with the notion that each isomer associates with its own instance of one-some 1g1‘2. Thus, each isomer does not (much) interact electromagnetically with other isomers.

$$n_i = 6,\ \rho_I = 1 \tag{46}$$

If $n_0 \geq 1$, equation (47) pertains. The result comports with the notion that one-some use of one instance of 2g2 mediates interactions between all six isomers.

$$\rho_I = \mathrm{gen}(SU(7))/\mathrm{gen}(SU(2n_0+1)) \tag{47}$$

Equation (46) and equation (47) associate with the following reaches regarding one-some uses of LRI solution-pairs - $\rho_I = 1$ for $n_0 = 0$, $\rho_I = 6$ for $n_0 = 1$, $\rho_I = 2$ for $n_0 = 2$, and $\rho_I = 1$ for $n_0 = 3$.

Table 12 shows the reach (ρ_I) for - and other information about - one-some uses of each one of some solution-pairs that table 3 lists. Compared to table 3, table 12 does not show solution-pairs for which NNC pertains. NNC solution-pairs do not serve in one-some roles.

Some aspects of our modeling regarding the notion of quadrupole extend the series position (monopole) and position and velocity (dipole) to include position and velocity and acceleration (quadrupole). In popular modeling, acceleration associates with interactions with other objects. In effect, quadrupole, octupole, and 16-pole aspects can associate with a loss of identity for an object. A notion of a region of space-time coordinates might be as useful as a notion of an object. The region would be bounded with respect to a temporal coordinate and with respect to three spatial coordinates.

We use notation of the form $\Sigma(\rho_I)\mathrm{g}\Gamma$ to denote a ΣgΓ solution-pair and the reach ρ_I that associates with one-some modeling use that features an instance of the solution-pair. For example, 2(2)g2‘4 pertains regarding the one-some use of the 2g2‘4 solution-pair. We extend use of such notation to simple particles. For simple particles, the reach is one and notation of the form $S(1)\Phi$ pertains.

We assume that - for each one-some use of a $\Sigma(2)\mathrm{g}\Gamma$ solution-pair - one instance of the solution-pair associates with interactions between isomer zero and isomer three. One instance of the solution-pair associates with interactions between isomer one and isomer four. One instance of the solution-pair associates with interactions between isomer two and isomer five.

We have yet to address the reaches of three-some uses of LRI solution-pairs. Suppose that $\Sigma\mathrm{g}\Gamma_1$ denotes a solution-pair for which there is a one-some use. Suppose that a one-some use of $\Sigma\mathrm{g}\Gamma_1$ and a three-some use of a solution-pair $\Sigma\mathrm{g}\Gamma_3$ associate with a four-some. Two choices pertain.

1. The reach of the three-some use of $\Sigma\mathrm{g}\Gamma_3$ equals the one-some reach of the $\Sigma\mathrm{g}\Gamma_1$ solution-pair.
2. The reach of the three-some use of $\Sigma\mathrm{g}\Gamma_3$ equals the one-some reach of the $\Sigma\mathrm{g}\Gamma_3$ solution-pair.

Table 12: Reaches and other information regarding one-some uses of Σg' solution-pairs that associate with electromagnetism, gravity, 3L, and 4L. ρ_I denotes reach. For each one of some of the solution-pairs, the table suggests an associated property of objects. In some sense regarding modeling, an object A with a nonzero property contributes to an LRI field via one-some aspects that associate with the associated solution-pairs and via three-some aspects that cascade (in one step) from those solution-pairs. For each of the solution-pairs 1g1‘2‘4, 2g2‘4, and 2g1‘2‘3‘4x, there is a three-some use that associates with rotation. We posit that, based on equation (41) and on a possible series 1g1 and Q (or, in some units, charge), 2g2 and m' (or, in some units, mass), 3g3 and net-left-right, $\ldots$, that one item in the series is 4g4 and S (or, in some units, spin). Regarding 1L and the notion of confined state, examples might include states of atoms, modes of laser cavities, and aspects of the Casimir effect. For other than monopole solution-pairs, the notion of associated property can vary between popular modeling kinematics models. Regarding the solution-pair 1g1‘2, we use the word nominal and we posit that - for some modeling - the solution-pair 3g1‘2 (which associates with 3g" and not with 3g') associates with the popular modeling notion of anomalous magnetic moment. Arithmetically, for some of the non-monopole one-some solution-pairs ΣgΓ, at least one Σ''g"Γ exists for which $4 \geq \Sigma'' \neq \Sigma$. The acronym SET abbreviates the two-element term stress-energy tensor and associates with popular modeling general relativity. TBD denotes the three-word phrase to be determined. PNR denotes the three-word phrase possibly not relevant. (See table 3.)

S	One-some solution-pairs	ρ_I	Solution-pair type	Associated property
1	1g1	1	Monopole	Charge
1	1g1‘2	1	Dipole	Nominal magnetic moment
1	1g1‘2‘4	6	Quadrupole	Rotating axis of magnetic moment
1	1g1‘2‘4‘8	2	Octupole	TBD (confined state might pertain, entwined pertains)
1	1g1‘2‘4‘6x	6	Octupole	PNR, TBD
1	1g1‘4‘6	2	Quadrupole	PNR, TBD (confined state might pertain)
1	1g1‘4‘6‘8	2	Octupole	PNR, TBD (confined state might pertain, entwined pertains)
2	2g2	6	Monopole	Rest energy (SET energy)
2	2g2‘4	2	Dipole	Rotating rest energy (SET pressure)
2	2g1‘2‘3	1	Quadrupole	Nominal moments of inertia (SET off-diagonal aspects)
2	2g1‘2‘3‘4x	1	Octupole	Rotating axis of moment of inertia
2	2g1‘2‘3‘4‘8x	6	16-pole	TBD (entwined pertains)
2	2g1‘2‘3‘6‘8x	6	16-pole	PNR, TBD (entwined pertains)
3	3g3	2	Monopole	Net-left-right
3	3g2‘3‘4	6	Quadrupole	TBD
3	3g2‘3‘4‘6	6	Octupole	PNR, TBD
3	3g2‘3‘4‘8	2	Octupole	TBD
4	4g4	1	Monopole	Angular momentum
4	4g1‘2‘3‘4x	1	Octupole	TBD
4	4g1‘2‘3‘4‘6x	1	16-pole	PNR, TBD
4	4g1‘2‘3‘4‘8x	6	16-pole	TBD (entwined pertains)

The case in which $\Sigma g\Gamma_3$ is 1g1‘2 provides no guidance regarding making a choice. Regarding this case, the one-some reach of the $\Sigma g\Gamma_3$ solution-pair 1g1‘2 equals one and equals the one-some reach of the $\Sigma g\Gamma_1$ solution-pair 1g1.

The case in which $\Sigma g\Gamma_3$ is 2g2‘4 and $\Sigma g\Gamma_1$ is 2g2 has relevance. For example, we consider an object A that is a non-rotating spherically symmetric object that contains equal amounts and equal distributions of all six isomers. (A notion of an idealized galaxy cluster might pertain.) We consider that object C is a non-rotating spherically symmetric object that contains only ordinary matter and that exists outside of the bounds of object A. The one-some reach of 2g2 is six. The one-some reach of 2g2‘4 is two. The popular modeling relativistic property (of object A) that associates with one-some use of 2g2 is energy. The popular modeling relativistic property (of object A) that associates with three-some use of 2g2‘4 is momentum. (The popular modeling relativistic property - of object A - that associates with one-some use of 2g2‘4 is - in the sense of a stress-energy tensor - pressure. See table 12.) For object C to observe that popular modeling based on special relativity pertains regarding object A, the number of instances of the three-some use of 2g2‘4 must equal the number of instances of the one-some use of 2g2 and the reach of one instance of a three-some use of 2g2‘4 must equal the reach of the one-some use of 2g2.

The following notion pertains. The reach of a three-some use of $\Sigma g\Gamma_3$ equals the one-some reach of the relevant $\Sigma g\Gamma_1$ solution-pair.

5.3. Interactions mediated by LRI fields

5.3.1. Dependences of LRI strengths on the isomeric compositions of objects

Above, this essay discusses aspects regarding objects A contributing to LRI fields. Here, we discuss notions regarding interactions between an object C and contributions - to LRI fields - that associate with an object A that is distinct from object C.

We discuss modeling associating with cases for which 2L (or, gravity) dominates. For this discussion, we assume that generally-Newtonian modeling suffices.

The symbol m_A denotes the mass of object A and the symbol m_C denotes the mass of object C. The symbol $f_{A,i}$ denotes the fraction of m_A that associates with isomer i. The symbol $f_{C,i}$ denotes the fraction of m_C that associates with isomer i. Our discussion here de-emphasizes effects that associate with the extents to which objects A and C model as having nonzero rotation. Our discussion here de-emphasizes the role of LRI fields in binding object A into an object and the role of LRI fields in binding object C into an object.

For a one-some use of a 2L component - associated with gravity caused by object A - for which the reach is ρ_I, the symbol F_{ρ_I} denotes a factor such that equation (48) provides a factor that pertains regarding the strength of the interaction between object A and object C.

$$F_{\rho_I} m_A m_C \tag{48}$$

For a reach ρ_I of six, equation (49) pertains.

$$F_6 = (\sum_{0\le i\le 5} f_{A,i})(\sum_{0\le i\le 5} f_{C,i}) = 1 \tag{49}$$

For a reach ρ_I of two, equation (50) pertains.

$$F_2 = \sum_{i=0,1,2} ((f_{A,i} + f_{A,i+3})(f_{C,i} + f_{C,i+3})) \le 1 \tag{50}$$

For a reach ρ_I of one, equation (51) pertains.

$$F_1 = \sum_{0\le i\le 5} (f_{A,i} f_{C,i}) \le 1 \tag{51}$$

For a case in which each $f_{A,i} = 1/6$ and each $f_{C,i} = 1/6$, equation (52) pertains.

$$F_{\rho_I} = 6/\rho_I \tag{52}$$

For a case in which $f_{A,0} = 1$, each other $f_{A,i} = 0$, $f_{C,0} = 1$, and each other $f_{C,i} = 0$, equation (53) pertains.

$$F_{\rho_I} = 1, \text{ for } \rho_I = 1, 2, \text{ or } 6 \tag{53}$$

We discuss some specific cases.

For interactions between the Sun (as an object A) and an ordinary matter planet (as an object C), equation (53) pertains. For interactions between the Sun (as an object A) and a photon (as an object C), equation (53) pertains. Our modeling does not suggest concerns regarding popular modeling that is based on general relativity.

For interactions between the Sun (as an object A) and a one-isomer planet (as an object C), results regarding components - of 2L - for which $\rho_I \neq 1$ can vary by the isomer that associates with object C. For example, the Sun rotates. If the one-isomer planet associates with isomer zero or isomer three, the one-some 2g2‘4 component of 2L (that associates with object A) affects the trajectory that associates with the orbit. If the one-isomer planet associates with isomer one, isomer two, isomer four, or isomer five, the one-some 2g2‘4 component of 2L (that associates with object A) does not affect the trajectory that associates with the orbit.

For interactions between two neighboring non-overlapping galaxies (one as an object A and one as an object C), some modeling might assume that each $f_{A,i} \approx 1/6$ and each $f_{C,i} \approx 1/6$. 2g2 associates with a reach of six. One-some 2g2‘4 associates with a reach of two. One-some 2g1‘2‘3 associates with a reach of one. Our modeling points to the notion that popular modeling would not necessarily be adequately accurate.

Regarding gravitationally-based observations pertaining to events in which a pair of small mass black holes merge to form one black hole, our modeling suggests that two sets of signatures might pertain. One set would associate with mergers for which the merging black holes associate with just one isomer-pair. The other set would associate with mergers for which each incoming black hole associates with an isomer-pair with which the other incoming black hole does not associate.

We discuss a general case of a point-like object C interacting with the 2L field that associates with an object A.

In effect, object C senses all 2gΓ solution-pair components that associate with the 2L field that associates with an object A. The weighting that associates with any one one-some solution-pair associates with the geometric factor of the pole (monopole, dipole, or so forth) that associates with the one-some solution-pair, with an orientation factor that associates with a tensor-like notion (scalar for monopole, vector for dipole, and so forth), and with an isomer composition factor F_{ρ_I}. (We use the word weighting to avoid possible conflation with popular modeling notions such as probability and amplitude. This essay does not operationally define the one-word term weighting.) For Newtonian modeling, the geometric factor associates with r^{-n_Γ}. Likely, effects that associate with one geometric factor or with two geometric factors dominate compared to effects that associate with other geometric factors.

We broaden our discussion to include aspects regarding ΣL fields other than 2L (or, gravity.)

Regarding 1L (or, electromagnetism), one-some 1g’ solution-pairs span the range of monopole through octupole. (See table 12.) Six instances - one per isomer - of charge pertain. For each instance of charge, values (of charge) for components of an object A can be negative, zero, or positive.

Regarding 3L, one-some use of the solution-pair 3g3 associates with the notion of monopole and a ρ_I of two.

Regarding 4L, one-some use of the solution-pair 4g4 associates with the notion of monopole and a ρ_I of one.

Generally, regarding modeling regarding LRI effects of an object A on an object C, effects that associate with each of the four ΣL fields pertain but effects associating with just a few solution-pairs may be adequately significant for purposes of modeling.

5.3.2. Possibilities for conversions between dark matter and ordinary matter

The following notions point to possibilities for conversions between dark matter and ordinary matter. Possibly, regarding recent times, such conversions might be at least one of improbable and hard to detect (except via precise measurements in controlled experiments).

3L bosons could catalyze conversions between isomers. For example, an isomer zero pair of elementary fermions for which one fermion is the antiparticle of the other fermion could annihilate to produce a 3L boson. The 3L boson could transform into one isomer zero left-solution simple fermion and one isomer three right-solution simple fermion.

Conversions between isomers might occur based on interactions mediated by quadrupole or octupole aspects of 1L or by monopole, dipole, or 16-pole aspects of 2L. (See table 12.) In particular, cross-isomer conversions might associate with effects of high-energy photons.

Some conversions between dark matter stuff and ordinary matter stuff might involve isomer zero dark matter. Conversions between heavy neutrinos and leptons might be possible. Conversions between hadron-like particles that contain 0.5R particles and hadron particles (such as neutrons) might be possible.

Table 13: Matches between masses and flavours, for isomers of charged elementary fermions. The symbol $0.5Q_{>0}$ denotes the pair $0.5Q_{1/3}$ and $0.5Q_{2/3}$. The symbol l_f numbers the three flavours. (See table 8.)

Isomer	l_m ($0.5Q_{>0}$)	Respective l_f ($0.5Q_{>0}$)	l_m ($0.5C_1$)	Respective l_f ($0.5C_1$)
0	$0, 1, 2$	1,2,3	$0, 2, 3$	1,2,3
1	$3, 4, 5$	1,2,3	$3, 5, 6$	3,1,2
2	$6, 7, 8$	1,2,3	$6, 8, 9$	2,3,1
3	$9, 10, 11$	1,2,3	$9, 11, 12$	1,2,3
4	$12, 13, 14$	1,2,3	$12, 14, 15$	3,1,2
5	$15, 16, 17$	1,2,3	$15, 17, 18$	2,3,1

5.4. Differences - between isomers - regarding properties of simple fermions

If the stuff that associates with each of the five all-dark-matter isomers evolved similarly to the stuff that associates with isomer zero, our suggestions regarding dark matter might not adequately comport with observations regarding the Bullet Cluster collision of two galaxy clusters. We anticipate that the isomers of simple particles differ in ways such that our suggestions regarding dark matter do not necessarily disagree with observations pertaining to the Bullet Cluster. (See discussion that cites reference [50].)

We use the symbol l_i to number the isomers. The notion of isomer l_i pertains.

Regarding each l_i that is at least one, we assume that the elementary particles in isomer l_i match - with respect to mass - the elementary particles in isomer zero.

For $0 \leq l_i \leq 5$, we associate the quarks in isomer l_i with three values of l_m. (See table 8 and table 9.) The values are $3l_i + 0$, $3l_i + 1$, and $3l_i + 2$. Across the six isomers, quarks associate with each value of l_m that is in the range $0 \leq l_m \leq 17$. Regarding quarks and flavours, we assume that - within isomer l_i - flavour 1 associates with $l_m = 3l_i$, flavour 2 associates with $l_m = 3l_i + 1$, and flavour 3 associates with $l_m = 3l_i + 2$.

Aspects of table 8 and table 9 point to the possibility that means for matching flavours and masses for charged leptons do not match means for matching flavours and masses for quarks. For charged leptons, isomer zero does not have a charged lepton that associates with $l_m = 1$ and does have a charged lepton that associates with $l_m = 3$. We assume that - for each l_i - a charged lepton associates with each of $l_m = 3l_i + 0$, $l_m = 3l_i + 2$, and $l_m = 3l_i + 3$.

We assume that - for each isomer l_i such that $1 \leq l_i \leq 5$ - the charged-lepton flavour that associates with $l_m = 3(l_i) + 0$ equals the flavour that associates with the isomer $l_i - 1$ charged lepton that associates with the same value of l_m and - thus - with $l_m = 3(l_i - 1) + 3$. We assume that across the six isomers, one cyclical order pertains regarding flavours for charged leptons.

Table 13 shows, for isomers of charged elementary fermions, matches between masses and flavours.

Beyond the topic of flavours, the topic of handedness exists. Ordinary matter associates with left-solution. Our modeling suggests the possibility - and we assume - that each one of isomer zero, isomer two, and isomer four associates with left-solution (and, therefore, for all fermion simple particles and most boson simple particles, with left-handedness) and that each one of isomer one, isomer three, and isomer five associates with right-solution (and, therefore, for all fermion simple particles and most boson simple particles, with right-handedness).

6. Results - Cosmology and astrophysics

6.1. Eras in the history of the universe

Concordance cosmology points to three eras in the rate of expansion of the universe. The eras feature, respectively, rapid expansion; continued expansion, with the rate of expansion decreasing; and continued expansion, with the rate of expansion increasing.

This essay suggests using the notion of eras regarding the separating from each other of clumps - that, today, people would consider to be large - of stuff. Examples of such clumps might include galaxy clusters and possibly even larger clumps. We discuss the notion that table 4 lists components of gravitational interactions that associate with some transitions between eras.

Table 14 discusses eras in the rate of separating of large clumps. (For discussion about possibilities leading up to a Big Bang, see reference [32].) For discussion about the possible inflationary epoch, see references [51], [16], [33], and [52]. For data and discussion about the two multi-billion-years eras, see references [53], [54], [55], and [56]. For discussion of attempts to explain the rate of expansion of the universe, see reference [31].)

Table 15 suggests details regarding eras to which table 14 alludes.

Table 14: Eras regarding the rate of separating of large clumps. The rightmost two columns suggest eras. (Table 15 discusses aspects that associate with each one of some eras.) In table 14, subsequent rows associate with later eras. The word inflation (or, the two word-term inflationary epoch) names the era that associates with the third row in the table. Regarding eras that would precede inflation, our modeling points to the possibility for the two eras that the table discusses. One-some solution-pair 0g1‘2‘3‘4‘8 associates with the jay boson. Concordance cosmology suggests inflation and the next two eras. Regarding inflation, popular modeling hypothesizes this era. Popular modeling suggests that the inflationary epoch started about 10^{-36} seconds after the Big Bang. Popular modeling suggests that the inflationary epoch ended between 10^{-33} seconds after the Big Bang and 10^{-32} seconds after the Big Bang. Possibly, no direct evidence exists for this era. Observations support the notions of the two billions-of-years eras. TBD denotes to be determined. The symbol † denotes a possible association between the relevant era and the notion of a Big Bang. The leftmost four columns describe phenomena that our modeling suggests as noteworthy causes for the eras. (Regarding phenomena that associate with gravitation, table 14 echoes aspects - including aspects regarding attraction and repulsion - that table 4 and table 12 show.) Generally, a noteworthy cause associates with notions of acceleration. Generally, an era associates with a range of velocities. The symbol $\rightarrow$ associates with the notion that a noteworthy cause may gain prominence before an era starts.

Force	One-some solution-pairs	Interaction	ρ_I	$\rightarrow$	Rate of separating	Duration
Attractive	2g1‘2‘3‘4‘8x	16-pole	6	$\rightarrow$	Is negative	TBD
Repulsive	0g1‘2‘3‘4‘8	-	1	$\rightarrow$	Turns positive †	TBD
Repulsive	2g1‘2‘3‘4x	Octupole	1	$\rightarrow$	Increases rapidly	Fraction of a second
Attractive	2g1‘2‘3	Quadrupole	1	$\rightarrow$	Decreases	Billions of years
Repulsive	2g2‘4	Dipole	2	$\rightarrow$	Increases	Billions of years
Attractive	2g2	Monopole	6	$\rightarrow$	Would decrease	-

Presumably, effects that associate with one-some uses of solution-pairs 2g1‘2‘3‘4x and 2g2‘4 associate with concordance cosmology notions of dark energy pressures.

Possibly, our notions regarding eras that start with and follow the inflationary epoch do not depend significantly on our notions regarding eras that might precede the inflationary epoch.

6.2. Baryon asymmetry

Per discussion above, an LRI boson that has a three-some component for which for $6 \in \Gamma$ can convert - within one isomer - a left-solution and right-solution pair of otherwise similar elementary fermions into a different left-solution and right-solution pair of otherwise similar elementary fermions.

If the related one-some component has a reach of at least two, the resulting left-solution and right-solution simple fermions need to associate with one isomer pair (that is, one of isomer zero and isomer three, isomer one and isomer four, and isomer two and isomer five) but not necessarily with just one isomer.

The solution-pair 3g2‘3‘4‘8 cascades to the solution-pair 3g2‘3‘4‘6‘8. The reach for a one-some instance of 3g2‘3‘4‘8 is two isomers. (See table 12.)

The notion of one-some use of 3g2‘3‘4‘8 and three-some use of 3g2‘3‘4‘6‘8 points to the possibility that a $\Sigma = 3$ LRI boson can decay into a left-solution simple fermion that associates with one isomer in an isomer-pair and a right-solution simple fermion that associates with the other isomer in the same isomer pair. For simple fermions, left-solution associates with left-handed and right-solution associates with right-handed.

Our modeling suggests the possibility that this notion of decays that produce opposite handedness products in two isomers underlies nature’s having - regarding isomer zero stuff - many more left-handed (or matter) simple particles than right-handed (or antimatter) simple particles. (This essay does not address the topic of the extent to which steps leading to a predominance in isomer zero of left-handedness - and not to a predominance of right handedness - have a basis - other than random chance - in modeling or nature.)

Popular modeling uses the two-word term baryon asymmetry to name the imbalance regarding isomer zero. (A term such as the two-element phrase matter-antimatter asymmetry can also pertain.) Isomer three stuff would have many fewer left-handed simple fermions than right-handed simple fermions. Popular modeling assumes that baryon asymmetry arose early in the history of the universe.

Possibly, widespread occurrence of interactions mediated by 3L contributed only negligible effects after stuff adequately dispersed sometime early in the history of the universe.

Alternatively, the following one-some solution-pairs might associate with mechanisms that lead to baryon asymmetry - 1g1‘2‘4 (for which PNR does not pertain), 1g1‘2‘4‘8 (for which PNR does not pertain), 1g1‘2‘4‘6x (for which PNR pertains), 1g1‘4‘6 (for which PNR pertains), 1g1‘4‘6‘8 (for which PNR pertains), 2g1‘2‘3‘4‘8x (for which PNR does not pertain), 2g1‘2‘3‘6‘8x (for which PNR pertains), 3g3 (for which PNR does not pertain), 3g2‘3‘4 (for which PNR does not pertain), 3g2‘3‘4‘6 (for which PNR pertains), and 4g1‘2‘3‘4‘8x (for which PNR does not pertain). (See table 12.)

Table 15: Details regarding eras regarding the rate of separating of large clumps. Table 14 discusses the eras. Our work does not necessarily specify the elementary fermions for which isomers form during the era that associates with the two-word phrase is negative. To the extent that the first significant appearance of most known elementary particles occurs during or just after the inflationary era, our work suggests that isomers of at least one of 0.5M and 0.5R form during the era that associates with the two-word phrase is negative. The symbol † associates with some aspects for which the involvement of 0.5M or 0.5R might pertain.

Rate of separating	Note
Is negative	Possibility: 2g1‘2‘3‘4‘8x and their compacting of "some form of energy" lead to conditions suitable for the universe to form and evolve. Possibility: The value of six for ρ_I (for 2g1‘2‘3‘4‘8x) associates with setting up a system for which roughly equal creation of isomers pertains. Possibility: Isomers of some simple fermions and of 1J form. † Possibility: The following interactions might characterize this era. For each interaction, the net circular polarization for each of before and after the interaction might be zero. † Presumably, the formation of gluons (or, 1(1)G) could associate with the formation of arcs (or, 0.5(1)R)). • 2(6)g1‘2‘3‘4‘8x + 2(6)g1‘2‘3‘4‘8x → 0.5(1)M + 0.5(1)M. • 2(6)g1‘2‘3‘4‘8x + 2(6)g1‘2‘3‘4‘8x → 0.5(1)R + 0.5(1)R. • 2(6)g1‘2‘3‘4‘8x + 2(6)g1‘2‘3‘4‘8x → 1(1)J + 1(1)J. Possibility: The six isomers of the relevant elementary fermions populate approximately equally. † Possibility: Some clumps of relevant elementary fermion stuff serve - eventually - as seeds for galaxies. †
Turns positive	0g1‘2‘3‘4‘8 associates with the 1J (or, jay) boson. The jay boson associates with the notion of Pauli repulsion. Possibility: 1J bosons stop the implosion of stuff that features relevant elementary fermion particles. † Possibility: Isomers of 0I form. The following interaction might characterize this era. Here, the net circular polarization for each of before and after the interaction might be two. 1(1)J + 1(1)J → 2(1)g1‘2‘3‘4x + 0(1)I. Possibility: The six isomers of 0I populate approximately equally. Possibility: Aspects of this era associate with notions of a Big Bang.
Increases rapidly	Some concordance cosmology modeling suggests that inflatons provide a major component of stuff. Possibility: The following interaction might characterize this era. Here, the net circular polarization for each of before and after the interaction might be two. 0(1)I + 2(1)g1‘2‘3‘4x → 0(1)I + 2(1)g1‘2‘3‘4x.
Decreases	-
Increases	-
Would decrease	This essay does not try to explore the possibility that (or to estimate a time at which) a transition - for the largest observable objects - from repulsion based on 2g2‘4 to attraction based on 2g2 might occur.

6.3. The evolution of stuff that associates with dark matter isomers

6.3.1. Notions that are common to all six isomers

We associate the symbol OMSE with all simple elementary particles except 0.5M and 0.5R simple fermions. OMSE abbreviates the three-element phrase ordinary-matter-similar elementary particles. We associate the symbol DMAI with the 0.5M and 0.5R simple particles. DMAI abbreviates the five-word phrase dark matter regarding all isomers. DMAI associates with the notion that - regarding isomer zero - these particles measure as being dark matter and do not measure as being ordinary matter.

We use the symbol l_i to number the isomers. We use the three-element term isomer l_i stuff to denote objects (including simple particles hadron-like particles, clumps of stuff, and stars) that associate with the isomer l_i set of simple particles.

0.5R particles model as entwined. We suggest that - at least after the inflationary epoch - 0.5R-based stuff consists of hadron-like particles. Each 0.5R-based-stuff hadron-like particle includes gluons and at least two arcs. Our work does not suggest an extent to which 0.5R-based stuff might form primordial black holes. Our work does not necessarily suggest that a two-or-three-hadron hadron-like particle can include both at least one quark and at least one arc.

0.5M particles model as free. Our work does not suggest an extent to which 0.5M-based stuff might form primordial black holes.

Regarding each one of the six isomers, we suggest that stuff made from DMAI behaves within bounds - for dark matter - that associate with concordance cosmology.

6.3.2. The evolutions of isomer 1, 2, 4, and 5 OMSE stuff

Here, we use the two-word term alt isomer to designate an isomer other than isomer zero and isomer three.

For each isomer, a charged baryon that includes exactly three flavour 3 quarks is more massive than the counterpart zero-charge baryon that includes exactly three flavour 3 quarks. (For example, two tops and a bottom have a larger total mass than do one top and two bottoms.) Alt isomer flavour 3 charged leptons are less massive than isomer zero flavour 3 charged leptons. (See table 13.) When flavour 3 quark states are much populated (and based on interactions mediated by W bosons), the alt isomer converts more charged baryons to zero-charge baryons than does isomer zero. Eventually, in the alt isomer, interactions that entangle multiple W bosons result in the alt isomer having more neutrons and fewer protons than does isomer zero. The sum of the mass of a proton and the mass of an alt isomer flavour 1 charged lepton exceeds the mass of a neutron. Compared to isomer zero neutrons, alt isomer neutrons scarcely decay. The IGM (or, intergalactic medium) that associates with the alt isomer scarcely interacts with itself via electromagnetism.

6.3.3. The evolution of isomer three OMSE stuff

The following two possibilities pertain. The evolution of isomer three OMSE stuff parallels the evolution of ordinary matter (or, isomer zero OMSE stuff). The evolution of isomer three OMSE stuff does not parallel the evolution of ordinary matter (or, isomer zero OMSE stuff). The second possibility might associate with - for example - a difference in handedness - with respect to charged leptons or with respect to W bosons - between isomer three and isomer zero. (See discussion related to table 13.)

This essay nominally assumes that the evolution of isomer three OMSE stuff parallels the evolution of ordinary matter (or, isomer zero OMSE stuff).

6.4. Tensions - among data and models - regarding large-scale phenomena

We suggest means to resolve tensions - between data and popular modeling - regarding the rate of expansion of the universe, regarding large-scale clumping of matter, and regarding gravitational interactions between neighboring galaxies.

6.4.1. The rate of expansion of the universe

Table 14 and table 15 discuss possible and known eras in the history of the universe.

People suggest that concordance cosmology modeling underestimates - for the second multi-billion-years era - increases in the rate of expansion of the universe. (See references [34], [35], [36], [37], [57], [58], [59], and [60].) Reference [61] suggests that the notion that dark matter is similar to ordinary matter might help resolve the relevant tension.

Our modeling suggests the following explanation for such underestimates.

When using modeling based on general relativity, people might try to extend the use of an equation of state (or use of a cosmological constant) that works well regarding early in the first multi-billion-years era. Regarding that time, our modeling suggests dominance by attractive effects that associate with one-some use of the 2g1‘2‘3 component of gravity. The notion of a reach of one pertains. The symbol 2(1)g1‘2‘3 pertains. Our modeling suggests that - later in the first multi-billion-years era - repulsive effects that associate with one-some use of 2(2)g2‘4 become significant. Dominance by 2(2)g2‘4 pertains by the time the second multi-billion-years era starts. However, popular modeling's use of an equation of state that has roots in the time period in which 2(1)g1‘2‘3 dominates would - at best - extrapolate based on a notion of 2(1)g2‘4 (and not based on a notion of 2(2)g2‘4). Popular modeling would underestimate the strength of the key gravitational driver - of expansion - by a factor of two. (See equation (52).)

Our modeling points - conceptually - to the following possible remedy.

People might change (regarding the stress-energy tensor or the cosmological constant) the aspects that would associate with repulsion and the 2g2‘4 component of gravity. The contribution - to the pressure - that associates with one-some use of 2g2‘4 needs to double (compared to the contribution that would associate with one-some use of 2(1)g2‘4).

6.4.2. Large-scale clumping of matter

People suggest that concordance cosmology modeling overestimates large-scale clumping of matter - ordinary matter and dark matter. (For data and discussion, see references [62], [63], [64], and [37].)

Our modeling suggests that concordance cosmology modeling associates with a repulsive component - 2(1)g2‘4 - of gravity. Our modeling suggests that 2(2)g2‘4 pertains. (That is, for each instance of 2g2‘4, a reach of two isomers pertains.) The additional (compared to concordance cosmology modeling) repulsion might explain the overestimating - of clumping, per concordance cosmology modeling - that popular modeling suggests.

6.4.3. Effects - within galaxies - of the gravity associated with nearby galaxies

People suggest that concordance cosmology modeling might not account for some observations about effects - within individual galaxies - of the gravity associated with nearby galaxies. (For data and discussion, see reference [41].)

Our modeling suggests that concordance cosmology modeling associates with a repulsive component - 2(1)g2‘4 - of gravity. Our modeling suggests that 2(2)g2‘4 pertains. The additional (compared to concordance cosmology modeling) repulsion might explain at least some aspects of the observations that people report.

6.5. Formation and evolution of galaxies

6.5.1. Mechanisms regarding the formation and evolution of galaxies

Reference [65] suggests that galaxies form around early clumps of stuff. The reference associates the word halo with such clumps.

Table 14 suggests that single-isomer stuff - such as stuff that features 0.5R particles - forms as early as during an era in which one-some solution-pairs 2g1‘2‘3‘4‘8x - which associate with attraction - dominate regarding prototype large clumps. Smaller-scale clumps might form before larger-scale clumps. Effects that associate with the one-some solution-pair 2g1‘2‘3 - which is attractive - might contribute to the formation of smaller-scale clumps. (See discussion related to table 4.) The reach that associates with 2g1‘2‘3 is one isomer. (See table 12.)

We suggest that each one of many early halos associates with one isomer. We associate with such early halos the three-element term one-isomer original clump. Clumping occurs based on gravitational effects. Differences - between the evolution of stuff associating with any one of isomer zero and isomer three and the evolution of stuff associating with any one of isomers one, two, four, and five are not necessarily significant regarding this gravitationally based clumping. The six isomers might form such clumps approximately equally.

Table 16 discusses suggestions regarding the formation and evolution of a galaxy for which a notion of a one-isomer original clump pertains.

Presumably, some galaxies form based on two or more clumps, for which all of the clumps associate with just one isomer. Possibly, some galaxies form based on two or more clumps, for which some clumps associate with isomers that are not the same as the isomers that associate with some other clumps.

6.5.2. Aspects regarding the evolution of galaxies

Table 16 suggests three eras regarding the evolution of galaxies. The first era associates with the first two rows in table 16. The second era associates with the 2g2 attractive force that associates with the third row in table 16. The third era associates with collisions between and mergers of galaxies.

Table 16: Stages and other information regarding the evolution of a galaxy for which a notion of a one-isomer original clump pertains. The table suggests stages, with subsequent rows associating with later stages. The next to rightmost column describes aspects of the stage. The leftmost four columns in the table describe a component of 2L that is a noteworthy cause for the stage. (Regarding phenomena that associate with gravitation, table 16 echoes aspects - including aspects regarding attraction and repulsion - that table 4 and table 12 show.) The one-element term 1-some abbreviates one-some. The one-element construct $\cdots$-pole alludes to the multipole notion that $\cdots$ associates with an integer that equals 2^{n_Γ}. The symbol $\rightarrow$ associates with the notion that a noteworthy cause may gain prominence before a stage starts. Table 16 associates with a scenario in which a galaxy forms based on one original clump and initially does not significantly collide with other galaxies. The galaxy might retain some stuff that associates with the repelled isomer. The rightmost column in table 16 suggests terminology regarding the evolution of galaxies. (A galaxy can include stuff from more than one earlier galaxy.)

Force	1-some solution-pair	$\cdots$-pole	ρ_I	$\rightarrow$	Stage	Aspects of the stage	Era
Attractive	2g1‘2‘3	4	1	$\rightarrow$	1	A one-isomer original clump forms.	First
Repulsive	2g2‘4	2	2	$\rightarrow$	2	The original clump repels (some) stuff that associates with the isomer that associates with the original clump and (most) stuff that associates with one other isomer.	First
Attractive	2g2	1	6	$\rightarrow$	3	The original clump attracts stuff that associates with the four not-repelled isomers and stuff that associates with the isomer that associates with the original clump.	Second
Attractive	2g2	1	6	$\rightarrow$	4	Another galaxy subsumes the original clump and might subsequently merge with yet other galaxies.	Third

Some galaxies do not exit the first era and do not significantly collide with other galaxies.

Many galaxies result from aspects associating with the 2g2 attractive force that associates with the third row in table 16. We discuss three cases. (Mixed cases and other cases might pertain.)

- Each one of some era one galaxies does not collide with other galaxies. Such a galaxy accumulates (via 2g2 attraction) stuff associating with various isomers that have representation in nearby IGM (or, intergalactic medium). The galaxy becomes an era two galaxy. The galaxy might include stuff that significantly associates with as many as five isomers.

- Each one of some era two galaxies merges (via 2g2 attraction) mainly just with galaxies that feature the same five isomers. The galaxy that merged, in effect, loses it status of being a galaxy. The resulting larger object is an era two galaxy. The galaxy might include stuff that significantly associates with as many as five isomers.

- Each one of some era one or era two galaxies merges (via 2g2 attraction) with other galaxies. The galaxy that merged, in effect, loses its status of being a galaxy. The resulting larger object is an era three galaxy. The galaxy might include stuff that significantly associates with as many as six isomers.

6.6. Explanations for ratios of dark matter effects to ordinary matter effects

6.6.1. Nominal explanations

Table 17 provides explanations for ratios - that pertain to galaxies - of dark matter effects to ordinary matter effects. (For data and discussion regarding observed early galaxies, see references [66] and [67]. Reference [66] influenced our choice of a time range to associate with the word early. For data and discussion regarding the combination of 0^{+}:1 and later, see references [68], [69], [70], [71], [72], and [73]. For data and discussion regarding observed dark matter galaxies, see references [65], [74], [75], and [76]. Current techniques might not be capable of observing early dark matter galaxies. References [77] and [78] suggest, regarding galaxy clusters, the existence of clumps of dark matter that might be individual galaxies. Extrapolating from results that references [65] and [79] discuss regarding ultrafaint dwarf galaxies that orbit the Milky Way galaxy might suggest that the universe contains many DM:OM

Table 17: Explanations for ratios - that pertain to galaxies - of dark matter effects to ordinary matter effects. DM denotes dark matter. OM denotes ordinary matter. DM:OM denotes a ratio of dark matter effects to ordinary matter effects. Inferences of DM:OM ratios come from interpreting data. Regarding galaxies, the notion of early associates with observations that pertain to galaxies that popular modeling associates with (or, would, if people could detect the galaxies, associate with) high redshifts. High might associate with $z > 7$ and possibly with smaller values of z. Here, z denotes redshift. The word later associates with the notion that observations pertain to objects later in the history of the universe. The three-word phrase dark matter galaxy denotes a galaxy that contains much less ordinary matter than dark matter. Possibly, people have yet to directly detect early dark matter galaxies. Table 16 provides information about the explanations.

Objects	DM:OM	Examples	Explanation
Some early galaxies	$0^+ : 1$	Reported	OM original clump. Stage 1 or 2.
Some later galaxies	$0^+ : 1$	Reported	OM original clump. Stage 1 or 2.
Some early galaxies	$1 : 0^+$	No known reports	DM-isomer(s) original clump. Stage 1 or 2.
Some later galaxies	$1 : 0^+$	Reported	DM-isomer(s) original clump. Stage 1 or 2.
Some later galaxies	$\sim 4 : 1$	Reported	Non-isomer-three original clump. Stage 3.
Many later galaxies	$5^+ : 1$	Reported	Any-isomer(s) original clump(s). Stage 4.

Table 18: Explanations for observed ratios - that pertain to larger-than-galaxies-scale - of dark matter effects to ordinary matter effects. DM:OM denotes a ratio of dark matter effects to ordinary matter effects. Inferences of DM:OM ratios come from interpreting data. The symbol DMAI associates with the 0.5M and 0.5R simple particles. DMAI abbreviates the five-word phrase dark matter regarding all isomers.

Aspect	DM:OM	Comment
Densities of the universe	$5^+ : 1$	DMAI stuff associates with the plus in DM:OM $5^+ : 1$. Stuff - other than DMAI stuff - that associates with isomers one through five associates with the five in DM:OM $5^+ : 1$.
Some galaxy clusters	$5^+ : 1$	We posit that galaxy clusters (that have not collided with other galaxy clusters) associate with DM:OM ratios that are similar to DM:OM ratios for densities of the universe.

$1 : 0^+$ later galaxies. Reference [80] discusses a trail of galaxies for which at least two galaxies have little dark matter. {Reference [80] suggests that the little-dark-matter galaxies result from a collision that would have some similarities to the Bullet Cluster collision.} For data and discussion regarding galaxies for which DM:OM ratios of ~4:1 pertain, see references [81] and [82]. For data and discussion regarding later galaxies for which DM:OM ratios of 5^+:1 pertain, see reference [65]. References [83] and [84] provide data about collisions of galaxies.)

Table 18 provides explanations for observed ratios - that pertain to larger-than-galaxies-scale stuff - of dark matter effects to ordinary matter effects. (For data and discussion regarding densities of the universe, see reference [14]. For data and discussion regarding galaxy clusters, see references [85], [86], [87], and [88].)

Table 19 lists ratios - that pertain to light that dates to about 400,000 years after the Big Bang - of observed effects to effects that popular modeling estimates. (For data and discussion regarding absorption of CMB, see references [89], [90], and [91]. For data and discussion regarding the amount of cosmic optical background, see reference [92].)

The following two paragraphs provide explanations for observations to which table 19 alludes. (Table 12 lists reaches for the relevant 1gΓ solution pairs.)

The four-word phrase some absorption of CMB associates with the notion that people measured some specific depletion of CMB (or, cosmic microwave background radiation) and inferred twice as much depletion as people expected based solely on hyperfine interactions with hydrogen atoms. Possibly, half of the depletion associates with DM effects. Our modeling suggests that isomer three hydrogen-like atoms

Table 19: Ratios - that pertain to light that dates to about 400,000 years after the Big Bang - of observed effects to effects that popular modeling estimates. The acronym CMB associates with radiation that - recently - measures as cosmic microwave background radiation. The three-word phrase cosmic optical background associates with radiation that - recently - measures as optical or close to optical radiation. DM:OM denotes a ratio of dark matter effects to ordinary matter effects that this essay posits.

Aspect	Reported : Expected	Measurement	Posited DM:OM
Some absorption of CMB	2 : 1	One reported measurement	1 : 1
Amount of cosmic optical background	2 : 1	One reported measurement	1 : 1

account for the half of the absorption for which isomer zero (or, ordinary matter) hydrogen atoms do not account. The reach of a one-some use of an instance of 1g1‘2‘4‘8 is two isomers. The reach of a one-some use of an instance of 1g1‘4‘6 is two isomers. The reach of a one-some use of an instance of 1g1‘4‘6‘8 is two isomers. We assume that at least one of 1g1‘2‘4‘8, 1g1‘4‘6, and 1g1‘4‘6‘8 associates with (at least) hyperfine states.

The three-word phrase cosmic optical background associates with now nearly-optical light remaining from early in the universe. An observation inferred twice as much light as people expected based on concordance cosmology. Our modeling suggests two possible sources of light and related explanations. To the extent that some of the light came from emissions that associate with states (such as atomic states) that associate with one-some use of 1g1‘2‘4‘8, with one-some use of 1g1‘4‘6, or with one-some use of 1g1‘4‘6‘8, half of that light came from isomer three stuff. To the extent that some of the light came from emissions that associate with one-some use of 1g1‘2‘4 or with one-some use of 1g1‘2‘4‘6x and that isomers one, two, four, and five evolved so as not to produce much such radiation, half of that light came from isomer three stuff.

6.6.2. A possible alternative explanation for DM:OM ratios of $5+:1$

We discuss a possible variation - regarding explanations - for DM:OM ratios of $5+:1$.

Here, we use the term alt isomer to refer to isomer one, isomer two, isomer four, and isomer five. We assume that the evolutions of alt isomer stuff deviate - compared to the evolution of isomer zero stuff - early enough that (nominally) isomer zero high-energy photons produce alt isomer stuff significantly more copiously than (nominally) alt isomer photons produce isomer zero stuff.

This variation can comport with each one of the DM:OM ratios of $5+:1$.

This variation could account for those ratios even if nature does not include arc (or, 0.5R) simple fermions and does not include heavy neutrino (or, 0.5M) simple fermions.

7. Discussion - General physics

Equation (41) and table 7 point to possibly deeper (than popular modeling might otherwise suggest) relationships between the physics properties of spin, mass, and charge and between properties of elementary bosons.

Table 8 and table 9 might point to possibly deeper (than popular modeling might otherwise suggest) relationships between the physics properties of mass, charge, and flavour and between properties of simple fermions.

7.1. Some known and possible conservation laws

One-some use of solution-pair 1g1 associates with the property of charge. Popular modeling includes the notion of conservation of charge. Popular modeling associates with only one isomer. Our modeling associates a reach ρ_I of one isomer for each one of the six instances of one-some 1g1. Our modeling suggests that conservation of charge pertains for each one of the six isomers.

One-some use of solution-pair 2g2 associates with the property of energy. Popular modeling includes the notion of conservation of energy. Popular modeling associates with only one isomer. Our modeling associates a reach ρ_I of six isomers for the one instance of one-some 2g2. Our modeling suggests that conservation of energy pertains for the set of six isomers.

Three-some use of solution-pair 2g2‘4 associates with the property of momentum. Popular modeling includes the notion of conservation of momentum. Popular modeling associates with only one isomer. Our modeling associates the reach ρ_I of three-some 2g2‘4 with the reach of one-some 2g2. Our modeling associates a reach ρ_I of six isomers for the one instance of one-some 2g2. Our modeling suggests that conservation of momentum pertains for the set of six isomers.

One-some use of solution-pair 3g3 associates with the property of net-left-right. Popular modeling includes the notion of conservation of number of matter charged simple fermions minus number of antimatter charged simple fermions, but associates with the notion that such a possible conservation law does not necessarily pertain early in the history of the universe. Popular modeling associates with only one isomer. Our modeling associates a reach ρ_I of two isomers for each one of the three instances of one-some 3g3. Our modeling suggests that conservation of net-left-right pertains for each one of the three isomer-pairs. Our modeling suggests that conservation of net-left-right might pertain throughout the history of the universe.

One-some use of solution-pair 3g3 (and the related three-some use of solution-pair 3g3‘6) does not associate with entwined. We posit that, for interactions that involve neither entwined simple particles nor entwined LRI components, conservation of net-left-right pertains for each fermion flavour.

We posit that one-some use of solution-pair 4g4 associates with the property of angular momentum. Popular modeling includes the notion of conservation of angular momentum. Popular modeling associates with only one isomer. Our modeling associates a reach ρ_I of one isomer for each one of the six instances of 4g4. Our modeling does not associate with isomers some aspects regarding LRI. This essay does not further discuss the notion of possible associations between 4g4 and conservation of angular momentum.

7.2. Geodesic motion

The reach that associates with a one-some use of an instance of 2g2 is six isomers. The reach that associates with a one-some use of an instance of 2g2‘4 is two isomers. The reach that associates with a one-some use of an instance of 2g1‘2‘3 is one isomer.

For situations in which only one isomer has significance, our work seems not to suggest changes to popular kinematics models. For example, modeling regarding deflection - by an ordinary matter star - of photons produced via ordinary matter would remain unchanged.

For modeling regarding situations in which - at each region in space - there is equal representation for each of the six isomers and modeling based on one-some use of 2g2 (and three-some use of 2g2‘4) is sufficiently accurate, our work seems not to suggest changes to popular modeling. For example, no change might pertain for interactions between two somewhat distant (from each other) galaxy clusters. However, for lesser distances, effects that associate with one-some use of 2g2‘4 might become significant. Popular modeling would overestimate effects - that associate with one-some use of 2g2‘4 - of one cluster on the other cluster by a factor of three.

Regarding modeling regarding the rate of expansion of the universe, popular models that feature a pressure that associates with an equation of state might not be adequately accurate. During the next to most recent era, the dominant component of gravity shifts from (one-some use of) 2g1‘2‘3, which has a reach of one isomer, toward (one-some use of) 2g2‘4, which has a reach of two isomers. A popular model that has an equation of state that is suitable for early in the next to most recent era would underestimate - for later times - the repulsive (pressure-related) force by a factor of up to two. Popular modeling would need to associate with an equation of state that varies over time.

Popular modeling includes notions of geodesic motion. Discussion just above might point to possible difficulties regarding notions of geodesic motion. Popular modeling might associate nonzero charge and nonzero electromagnetism with a combination of a notion of extra dimensions and the notion of geodesic motion. To the extent that such a concept pertains for cases of nonzero electromagnetism, perhaps the concept of geodesic motion can also pertain for modeling that takes into account more than one isomer. This essay does not explore the notion that notions of geodesic motion might pertain if modeling includes notions of extra dimensions.

8. Discussion - Elementary particles

8.1. Hypothesized elementary particles

We discuss possibilities regarding the existence and properties of some hypothesized elementary particles.

8.1.1. Axions

This essay seemingly does not suggest an elementary boson that would associate with notions of an axion. Popular modeling suggests that - under some circumstances - axions might convert into photons. We suggest that observations that popular modeling might associate with effects of axions might instead associate with the difference between our notion of 1(6)g1‘2‘4 and popular modeling notions that might associate with notions of 1(1)g1‘2‘4. Also, observations that popular modeling might associate with effects of axions might instead associate with interactions involving jay (or, 1J) bosons or aye (or, 0I) bosons. (See table 15.)

8.1.2. Magnetic monopoles

This essay does not suggest an elementary particle that would associate with notions of a magnetic monopole. Table 12 seems not to suggest a 1L interaction with a monopole other than an electric monopole.

8.1.3. Right-handed W bosons

Reference [93] discusses a fraction of decays - of ordinary matter top quarks for which the decay products include W bosons - that might produce right-handed W bosons. The fraction, f_+, is 3.6×10^{-4}. Reference [14] provides a confidence level of 90 percent that the rest energy of a would-be W_R (or, right-handed W boson) exceeds 715 GeV. (Perhaps, note also, reference [94].)

Our work suggests that W_R bosons associate only with isomers one, three, and five. Our work suggests possibilities for inter-isomer interactions and conversions. (See discussion related to table 14 and table 15.)

We explore a notion that aspects of our modeling might approximately reproduce the above result that Standard Model modeling suggests.

Aspects related to table 9 and table 13 suggest values of calculated masses that do not associate with masses of known or suggested elementary particles. For example, our modeling does not suggest that $m(5,3)$ associates with the inertial mass of an isomer one charged lepton. However, perhaps such mass-like quantities associate with some measurable aspects of nature. For charged leptons and $0 \leq l_i \leq 4$ and $0 \leq l_i' \leq 2$, $m(3(l_i+1)+l_i',3) = \beta m(3(l_i+0)+l_i',3)$. One might conjecture that isomer zero observations of some aspects of isomer one phenomena associate with notions of non-inertial mass-like quantities that are β times the inertial masses for isomer zero elementary particles (and that are β times inertial masses for the counterpart isomer one elementary particles).

Based on notions of scaling that might calculate non-inertial mass-like quantities, one might conjecture that our modeling suggests that $f_+ \sim e^{(\beta^{-1})} - 1 \approx \beta^{-1} \approx 2.9 \times 10^{-4}$. This estimate might not be incompatible with results that reference [93] discusses. A notion of $m_{\text{non-inertial,W}_R\text{isomer one}}c^2 = \beta m_{\mathrm{W}}c^2 \approx 2.8 \times 10^5$ GeV might pertain. Here, the notion of non-inertial mass-like quantity might associate with inferences that associate with interactions that associate with 1L or $1\mathrm{W}_1$. The interactions do not necessarily associate directly with 2L.

8.1.4. 3L and 4L elementary bosons

Reference [19] notes that modeling based on popular modeling QFT (or, quantum field theory) suggests that massless elementary particles cannot have spins that exceed two.

In our work, 3L might associate with nonzero anomalous magnetic moments for at least charged leptons. (See table 12 and discussion related to equation (54).) Modeling based on QFT suggests - without assuming elementary particles with spins of more than one - values for some anomalous magnetic moments.

In our work, 3L might associate with producing observed baryon asymmetry. This essay suggests alternatives, including an alternative that associates with one-some use of solution-pairs 2g1‘2‘3‘4‘8x, that might associate with a mechanism that leads to baryon asymmetry.

In our work, 3L or 4L might associate with notions - for at least neutrinos - of anomalous gravitational properties and mass mixing. (See table 10.) Table 10 notes the possibility that a QFT (or, quantum field theory) that includes gravity and that features a limit (regarding ΣL) of $\Sigma \leq 2$ might associate mass mixing with a notion - regarding our modeling - of anomalous gravitational properties and might successfully estimate aspects that popular modeling associates with mass mixing.

Our work is not necessarily incompatible with notions that popular modeling might not necessarily need to include zero-mass elementary bosons that have spins that exceed two.

8.2. Interactions involving the jay boson

We discuss interactions that involve jay bosons.

8.2.1. Interactions - before or during inflation - that involve jay bosons

We consider interactions in which two jay bosons move in parallel, interact, and produce one aye boson plus something else. We assume that conservation of angular momentum pertains. Here, we assume that one can de-emphasize angular momentum that is not intrinsic to the relevant elementary particles. We consider two cases. In the first case, the two jay bosons have the same (one of either right or left) circular polarization. Conservation of angular momentum allows an outgoing combination of one 2L particle and one 0I particle. The de-emphasizing of non-intrinsic angular momentum might - in effect - preclude producing one 1L particle and one 0I particle. In the second case, one jay boson has left circular polarization and the other jay boson has right circular polarization. Conservation of angular momentum allows the production of two 0I particles. The de-emphasizing of non-intrinsic angular momentum might - in effect - preclude producing one 1L particle and one 0I particle.

The two cases might comport with the notion that gravitation can be significant during inflation. The two cases might comport with the notion that jay bosons form before aye bosons form. (See table 15.)

The two cases might comport with a popular modeling notion that electromagnetism might become significant essentially only after inflation.

8.2.2. Pauli repulsion

Popular modeling includes the notion that two identical fermions cannot occupy the same state. Regarding some popular modeling, one notion is that repulsion between identical fermions associates with overlaps of wave functions. Another popular modeling notion features wave functions that are anti-symmetric with respect to the exchange of two identical fermions.

Our modeling might be compatible with such aspects of popular modeling and, yet, not necessitate - for kinematics modeling - the use of wave functions. Modeling based on jay bosons might suffice.

Modeling based on jay bosons might suggest that the prevention of two identical fermions from occupying the same state might associate with, in effect, trying to change aspects related to the fermions. Notions of changing a spin orientation might pertain. For elementary fermions, notions of changing a flavour might pertain.

8.2.3. Pauli crystals

Reference [95] and reference [96] report detection of Pauli crystals. We suggest that modeling based on the notion of jay bosons might help explain relevant phenomena.

8.2.4. Energy levels in positronium

Reference [97] and reference [98] discuss the transition - between two states of positronium - characterized by the expression $2^3S_1 \rightarrow 2^3P_0$. Four standard deviations below the nominal observed value of the energy that associates with the transition approximately equals four standard deviations above the nominal value of the energy that popular modeling suggests.

Perhaps, notions regarding jay bosons extend to explain the might-be discrepancy regarding positronium. Compared to popular modeling, a new notion of virtual charge exchange or a new notion of virtual flavour change might pertain.

To the extent that popular modeling does not suffice, modeling related to the jay boson might help to close the gap between observation and modeling.

8.3. Anomalous magnetic moments

In discussion above, the notion of Γ equals 1‘2 associates with the one-some property of magnetic moment.

Possibly, for some modeling, 1g1‘2 associates with the one-some property of nominal magnetic moment and 3g1‘2 associates with the one-some property of anomalous magnetic moment. Modeling based on this notion might provide an alternative to popular modeling for calculating anomalous magnetic moments. Such popular modeling features virtual photons (spin-1) and seems to match data regarding the electron and the muon. Modeling - based on 3g1‘2 - to which we allude might not be as fundamental (as the popular modeling might be), but might associate with useful extrapolations (to other particles) based on data from experiments.

We explore modeling regarding anomalous magnetic moments for $0.5\mathrm{C}_1$ elementary particles (or, charged leptons).

Two three-some solution-pairs associate with one-some use of the 3g1‘2 solution-pair. The 3g1‘2‘6 three-some solution-pair associates with $6 \in \Gamma$. We assume that the strength of 3g1‘2‘6 can vary based on elementary fermion flavour. The 3g1‘2‘4 three-some solution-pair associates with $6 \notin \Gamma$. We assume that the strength of 3g1‘2‘4 does not vary based on elementary fermion flavour.

We explore the notion that one can approximate a_{cl}, the anomalous magnetic moment for the cl charged lepton, via equation (54).

$$a_{cl} \approx a_4 + a_6 t_{cl} \tag{54}$$

Here, each one of a_4 and a_6 might be a constant with respect to a choice between $cl = e$ (for the electron), $cl = \mu$ (for the muon), and $cl = \tau$ (for the tau). We assume that t_{cl} is $(\log(m_{\mathrm{cl}}/m_e))^2$. (Perhaps, compare with table 8 and with aspects - that comport with squares of properties - of table 9. The notion of squares of properties might associate with notions of self-interactions.) Based on data that reference [14] provides regarding the electron and the muon, we calculate a_4 and a_6. Then, we calculate a value,

$a_{\tau,\mathrm{PM}}$, for a_τ. Here, PM denotes the two-word term proposed modeling. PM associates with our work. Reference [99] provides, based on popular modeling Standard Model modeling techniques, a first-order result - which we call $a_{\tau,\mathrm{SM}}$ - for a_τ. Here, SM denotes the two-word term Standard Model. The value of $a_{\tau,\mathrm{PM}}$ results in a value of $(a_{\tau,\mathrm{PM}} - a_{\tau,\mathrm{SM}})/a_{\tau,\mathrm{SM}}$ of approximately -0.00228. Each one of $a_{\tau,\mathrm{PM}}$ and $a_{\tau,\mathrm{SM}}$ comports with experimental data that reference [14] provides.

Regarding anomalous magnetic moments, this essay does not explore quantifying aspects that associate with higher-order popular modeling Standard Model terms or aspects that might associate with one-some use of the solution-pairs 3g1‘2‘4 and 3g1‘2‘6. Each of the solution-pairs 3g1‘2‘4 and 3g1‘2‘6 associates with 3g” and does not associate with 3g’.

8.4. Gauge symmetries

8.4.1. Popular modeling Gauge symmetries

Equation (55), equation (56), and equation (57) show popular modeling Gauge symmetries.

$$\text{Electromagnetic interaction: } U(1) \tag{55}$$

$$\text{Weak interaction: } SU(2) \times U(1) \tag{56}$$

$$\text{Strong interaction: } SU(3) \tag{57}$$

We discuss Gauge-like symmetries for the electromagnetic interaction, the weak interaction, and the strong interaction.

Based in part on information that table 5 shows, for the strong interaction and the weak interaction, the following steps might point to symmetries.

1. Determine the number of solution-pairs for each relevant three-some ΣgΓ. For gluons, the number is two. For each weak-interaction boson, the number is one.
2. Determine the number of k that are members of K_4 and not members of the relevant one-some Γ. For gluons, the number is zero. For each weak-interaction boson, the number is one.
3. Add the two numbers. For each of the strong interaction and the weak interaction, the resulting number is two.
4. Anticipate calculating a total number of one-dimensional oscillators by (based on the notion of pairs $s_k = +1$ and $s_k = -1$) doubling the above sum.
5. Determine the total number of associated harmonic oscillators. For each of the strong interaction and the weak interaction, the resulting number of one-dimensional harmonic oscillators is four.
6. Determine a number of associated harmonic oscillators that might be relevant regarding symmetry. Assume that one three-some-related oscillator associates with boson excitation and that the remaining oscillators associate with symmetry. (See discussion related to equation (36).) For each of the strong interaction and the weak interaction, the resulting number of (remaining) one-dimensional harmonic oscillators is three.
7. Posit symmetries.
   (a) For the strong interaction, one three-dimensional oscillator associates with the three one-dimensional oscillators. A ground state $SU(3)$ pertains.
   (b) For the weak interaction, the one remaining three-some-related one-dimensional oscillator associates with a ground state $U(1)$ symmetry and the two remaining one-some-related one-dimensional oscillators associate with broken ground state $SU(2)$. Here, $2 \notin \Gamma$ for the Z boson and $4 \notin \Gamma$ for the Z boson. Two possibilities pertain. One possibility features the notion that - for the W boson - K_3 is appropriate and K_4 is not appropriate. The break associates with the notion that $SU(2)$ symmetry has relevance regarding the Z boson and does not have relevance regarding the W boson. One possibility features the notion that the break associates with the notion that - for the W boson - K_4 is appropriate. The break associates with the difference between $2 \notin \Gamma$ for the Z boson and $4 \notin \Gamma$ for the W boson.

A similar set of steps points to a possible association between electromagnetism and a ground state $U(1)$ symmetry. Here, the relevant one-some solution-pair is 1g1, the set K_1 (and not K_4) is relevant, and the relevant three-some solution is 1g1‘2.

The above possible symmetries might associate with popular modeling notions of Gauge symmetries. This essay does not explore relationships - between details of our modeling and details of popular modeling - that might describe such an association.

8.4.2. The lack - for the Higgs boson - of Gauge-like symmetry

Popular modeling does not associate a Gauge symmetry with the Higgs field.

Discussion above regarding Gauge-like symmetries pertains to elementary bosons for which $S = 1$. For the Higgs boson, $S = 0$.

Use of just steps above (regarding Gauge-like symmetries regarding the strong interaction, the weak interaction, and the electromagnetic interaction) might suggest - for the Higgs boson - that relevant (that is, excitation-related and symmetry-related) aspects associate with two oscillators, that one oscillator associates with excitation of the Higgs boson, and that the other oscillator associates with $U(1)$ symmetry.

Regarding the popular modeling notion of handedness, neither left-handedness nor right-handedness pertains. (The Higgs boson has zero spin.)

For our modeling regarding simple particles, the notion of right-solution associates with a different isomer of the Higgs boson than does the notion of left-solution. (This notion pertains three times, based on three instances of net-left-right.)

We posit that the oscillator that might associate - regarding the Higgs boson - with a would-be $U(1)$ symmetry associates with the notion of left-solution and right-solution and does not associate with just one isomer. In the sense of the steps above (regarding Gauge-like symmetries regarding the strong interaction, the weak interaction, and the electromagnetic interaction), the total number of oscillators that associate with Gauge-like symmetries decreases from one to zero.

Our modeling seems to comport with the notion that, for the Higgs boson, no Gauge-like symmetry pertains.

8.5. The Higgs mechanism

We explore - as an extension to the above discussion about a lack of Gauge symmetry related to the Higgs boson - notions that might associate with popular modeling regarding the Higgs mechanism and with popular modeling regarding zero-point energy. (Reference [100] discusses the Higgs mechanism and discusses data relating the masses of some simple particles to the strength of interactions between the Higgs boson and those simple particles.)

For a two-dimensional isotropic harmonic oscillator, $D = 2$ and the base state associates with $\nu = -1$. Base states might associate with a popular modeling notion of zero energy.

We consider modeling based on one two-dimensional isotropic harmonic oscillator.

The modeling splits the two-dimensional isotropic harmonic oscillator into two one-dimensional isotropic harmonic oscillators. For each one of the one-dimensional isotropic harmonic oscillators, $D = 1$ pertains and the base state associates with $\nu = -1/2$.

We associate one base state with boson excitations for isomer zero and one base state with boson excitations for isomer three.

Popular modeling associates with isomer zero and not with isomer three. From the standpoint of popular modeling, the first $D = 1$ isotropic harmonic oscillator might associate with the lowest-energy state that associates with the Higgs mechanism and might associate with an opportunity to develop new modeling for which contributions - by boson ground states - to zero-point energy are zero.

8.6. Modeling regarding excitations regarding elementary particles

8.6.1. Modeling for excitations and de-excitations of elementary bosons

Discussion regarding equation (36), discussion regarding Gauge symmetry, and discussion regarding the Higgs mechanism point to modeling - for excitations and de-excitations of elementary bosons - based on a one-dimensional harmonic oscillator. Popular modeling also models excitations - of bosons - via mathematics that associates with a one-dimensional harmonic oscillator. For our notion of base states, our notion of point-like pertains.

8.6.2. Modeling for excitations and de-excitations of simple fermions

We parallel discussion regarding equation (36). Here, we use $j = 3$.

For a not-excited state, we propose that modeling based on $D_0 = 3$ and $\nu_0 = -3/2$ can pertain. Here, per equation (28), equation (58) pertains. Equation (58) comports with the popular modeling notions of $S = 1/2$ and $S(S + 1) = 3/4$. Our notion of point-like pertains.

$$\Omega = (-3/2)((-3/2) + 3 - 2) = 3/4 \tag{58}$$

For an excited state, we propose that modeling based on $D_1 = 1$, $\nu_1 = -1/2$, $D_2 = 2$, and $\nu_2 = -1$ can pertain. Regarding $D_1 = 1$, equation (59) pertains. Equation (59) comports with the popular modeling

Table 20: The possibility that - for LRI elementary particles ΣL - Σ might be no greater than four.

Topic	Note
$l_m = 18$	$((q_e)^2/(4\pi\varepsilon_0))/(G_N(m(18,3))^2) = 4/3$.
Monopole properties	A force strength factor of 4 seems to associate with 1g1 and a force strength factor of 3 seems to associate with 2g2. (See, above, the equation $(4/3) \times (\beta^2)^6 = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2)$.) Possibly, other force strength factors would be 2 for 3g3, 1 for 4g4, and 0 (or, zero) for 5g5. Possibly, the notion of zero force strength regarding 5g5 associates with a lack of relevance for (and a lack of monopole properties that would associate with) solutions ΣgΣ for which $\Sigma \geq 5$ and with a lack of LRI elementary particles ΣL for which $\Sigma \geq 5$.

notions of $S = 1/2$ and $S(S+1) = 3/4$. Regarding D_2, the $SU(2)$ ground-state symmetry associates with three generators and might comport with the popular modeling notion of three flavours. Our notion of point-like pertains.

$$\Omega = (-1/2)((-1/2) + 1 - 2) = 3/4 \tag{59}$$

Popular modeling does not necessarily associate harmonic-oscillator mathematics with fermions.

8.7. A possible limit regarding the spins of LRI elementary particles

Table (20) suggests the possibility that - for LRI elementary particles ΣL - Σ might be no greater than four.

A limit - for LRI elementary particles ΣL - of $\Sigma \leq 4$ seems to be consistent with other aspects of our modeling. (See discussion leading to equation (60).)

9. Discussion - Cosmology and astrophysics

9.1. Popular modeling constraints regarding dark matter

We discuss the extent to which our notion of dark matter comports with constraints - about the nature of dark matter - that people associate with data about dark matter or with popular models that have bases in assumptions about dark matter. (Reference [101] discusses aspects regarding popular modeling notions of possible types of dark matter.)

9.1.1. Aspects related to cosmological models

Reference [65] summarizes some thinking about constraints on dark matter and about notions of dark matter. Reference [65] notes that CDM (or, cold dark matter) might comport well with various models. Some popular models associate with the one-element term ΛCDM. Reference [65] notes that people have yet to determine directly whether nature includes CDM stuff. The article notes that people consider that notions of SIDM (or, self-interacting dark matter) might be appropriate regarding nature. Popular modeling also uses other terms, such as the three-word term warm dark matter, to note possible attributes of dark matter. For example, reference [102] suggests that notions of WDM (or, warm dark matter) might reduce discrepancies between data regarding clustering within galaxies and modeling that associates with CDM. Notions such as SIDM and WDM arose from popular modeling that differs from our modeling. We are reluctant to try to closely associate terms such as SIDM or WDM with our modeling. (We suggest that isomer zero 0.5R-based stuff, isomer zero 0.5M stuff, and all stuff associating with isomers one, two, four, and five might comport with some notions of CDM. We suggest that the remaining dark matter stuff - or, isomer three OMSE stuff - might associate with some notions of WDM and with some notions of SIDM.)

We suggest that our notion of dark matter is not necessarily incompatible with constraints - that have bases in popular cosmological models - regarding dark matter.

9.1.2. Aspects related to collisions of pairs of galaxy clusters

We discuss the Bullet Cluster collision of two galaxy clusters. (Reference [50] discusses the Bullet Cluster.) Presumably, observations regarding other such collisions might pertain.

Observations suggest two general types of trajectories for stuff. Most dark matter - from either one of the clusters - exits the collision with trajectories consistent with having interacted just gravitationally with the other cluster. Also, ordinary matter stars - from either cluster - exit the collision with trajectories consistent with having interacted just gravitationally with the other cluster. However, ordinary matter IGM (or, intergalactic medium) - from either cluster - lags behind the cluster's ordinary matter stars and dark matter. That ordinary matter IGM interacted electromagnetically with the other cluster's ordinary matter IGM, as well as gravitationally with the other cluster.

Our work suggests that - regarding each cluster - essentially all dark matter - except isomer three IGM - passes through without interacting significantly electromagnetically with stuff from the other cluster. Our work suggests that isomer three IGM that associates with each cluster might interact significantly with isomer three IGM that associates with the other cluster. Isomer three IGM might follow trajectories similar to trajectories for isomer zero IGM.

We are uncertain as to the extent to which observational data might suggest that the amounts of dark matter that lags the bulk of dark matter are sufficiently small that our nominal notions regarding isomer three IGM do not comport with observations.

Should the actual fraction of lagging dark matter be too small, we might need to reconsider the extent to which isomer three differs from isomer zero. We note some examples of possible reconsideration. For one example, possibly isomer three has right-handed elementary fermions but interactions involving such fermions model as retaining aspects of left-handed-centric interactions that associate with isomer zero. For another example, possibly isomer three does not evolve adequately similarly to isomer zero.

We suggest that our notion of dark matter is not necessarily incompatible with constraints - that have bases in observations of collisions of galaxy clusters - on dark matter.

9.2. Some phenomena that associate with galaxies

9.2.1. Some quenching of star formation

Some galaxies seem to stop forming stars. (See reference [103] and reference [104].) Such quenching might take place within three billion years after the Big Bang, might associate with a relative lack of hydrogen atoms, and might pertain to half of the galaxies that associate with the notion of a certain type of galaxy. (See reference [104].)

We suggest that the quenching might associate with repulsion that associates with 2(2)g2‘4. Quenching might associate with galaxies for which original clumps featured isomer zero stuff or isomer three stuff.

9.2.2. Some stopping of the accrual of matter

Reference [105] discusses a galaxy that seems to have stopped accruing both ordinary matter and dark matter about four billion years after the Big Bang.

The galaxy that reference [105] discusses might (or might not) associate with the notion of significant presence early on of one of isomers zero and three, one of isomers one and four, and one of isomers two and five. Such early presences might associate with a later lack of nearby stuff for the galaxy to accrue.

9.2.3. Aspects regarding stellar stream GD-1 in the Milky Way galaxy

Data regarding stellar stream GD-1 suggest the possibility of effects from a yet-to-be-detected non-ordinary-matter clump - in the Milky Way galaxy - with a mass of 10^6 to 10^8 solar masses. (For data and discussion regarding the undetected object, see references [106] and [107].) We suggest that the undetected object might be a clump of dark matter.

9.3. Zero-point energy and the cosmological constant

Discussion regarding equation (36), discussion regarding Gauge symmetry, discussion regarding the Higgs mechanism, and discussion related to excitations of simple bosons might associate with an opportunity to develop new modeling for which unoccupied states associate with zero energy.

Modeling based on notions that associate with zero zero-point energy might help resolve some concerns about popular modeling that suggests a cosmological constant that might be too large by a factor of something like 10^{120}.

9.4. Modeling that might point to a phase change regarding the universe

Popular modeling includes the two-word term phase change and sometimes suggests that the notion of a phase change might pertain early in the history of the universe. Possibly, such a notion of phase change might associate with - regarding 2L fields - a loss of significance (relative to one-some uses of 2gΓ solution-pairs for which $8 \notin \Gamma$ pertains for 2gΓ) for one-some uses of 2gΓ solution-pairs for which $8 \in \Gamma$ for 2gΓ.

10. Discussion - Our modeling

10.1. The notions that $5 \notin K_8$ and that $7 \notin K_8$

We consider the series $\text{gen}(SU(2n_0+1))$ for $n_0 = 1, 2, 3, 4, \cdots, 8, \cdots$. The series is $8, 24, 48, 80, \cdots, 288, \cdots$. For $1 \leq n_0 \leq 3$, each integer in the series evenly divides each larger integer in the series. Regarding $n_0 = 4$, $\text{gen}(SU(2n_0+1)) = 80$, 24 does not evenly divide 80, and 48 does not evenly divide 80. Regarding $n_0 = 8$, $\text{gen}(SU(2n_0+1)) = 288$ and each one of 8, 24, and 48 evenly divides 288.

We posit that all relevant solution-pairs cascade - via same-Σ cascading - from solution-pairs that have bases in K_4. (Equation (47) motivates our making the K_4 portion of this posit. Relevant solution-pairs include, for example, 3g1‘4.) Arithmetically, introducing one new odd integer cannot result in a same-Σ cascade. No solution-pair for which $5 \in K$ occurs in such cascades. No solution-pair for which $7 \in K$ occurs in such cascades.

This essay de-emphasizes notions such as the following. One-some use 1g1‘3‘5 might associate with electromagnetism and a property. One-some use of 0g2‘3‘5 might associate with one or more simple particles. Equation (60) might associate with a factor of six that is relevant to our modeling.

$$\text{gen}(SU((2 \times 8) + 1))/\text{gen}(SU((2 \times 3) + 1)) = 288/48 = 6 \tag{60}$$

We posit that a lack of $5 \in K$ associates also with the notion of an integer force-strength factor. (See table 20.)

10.2. Harmonic-oscillator mathematics that associates with 2ν being an odd integer

10.2.1. Some relationships between solutions to harmonic oscillator equations

Regarding solutions of the form that equation (26) shows, we develop a process for transforming fractional-integer-ν modeling into integer-ν modeling. (Popular modeling does not necessarily consider cases for which 2ν is a positive integer and ν is not an integer.)

We explore cases for which ν is not necessarily an integer, j_ν is an integer, and $j_\nu \nu$ is an integer.

We start with equation (61), which re-expresses equation (28).

$$\Omega = (1/j_\nu^2)(j_\nu \nu)((j_\nu \nu + j_\nu D_n - 2j_\nu) \tag{61}$$

Equation (62) defines, for integer n, D_{n+1} in terms of D_n. Equation (63) pertains. Equation (63) associates with an equivalent to equation (28). (Some uses of equation (63) may associate with, in effect, absorbing the factor - in the rightmost term in the equation - of $(j_\nu)^{-2}$ into the term $\xi'/2$.)

$$D_{n+1} = j_\nu(D_n - 2) + 2 \tag{62}$$

$$\Omega = (1/j_\nu^2)(j_\nu \nu)(j_\nu \nu + (j_\nu(D_n - 2) + 2) - 2) = (1/j_\nu^2)(j_\nu \nu)(j_\nu \nu + D_{n+1} - 2) \tag{63}$$

For the case $j_\nu = 2$, equation (64) pertains.

$$D_{n+1} = 2D_n - 2 \tag{64}$$

For the case $j_\nu = 2$ and $D_n = 3$, equation (65) pertains.

$$D_{n+1} = 2D_n - 2 = 4 \tag{65}$$

Table 21 shows, for $j_\nu = 2$, results D_{n+1} that associate with applying equation (62) once to some values of D_n. (Table 21 alludes to results that do not necessarily emphasize notions of angular coordinates, normalization, or relevance to physics modeling.)

Table 21: Some results of recursive applications of equation (62), assuming that $j_\nu = 2$.

D_1	D_2	D_3	D_4	D_5	$D_{\ldots}$
$\cdots$					
-1	-4	-10	-22	-46	$\cdots$
0	-2	-6	-14	-30	$\cdots$
1	0	$\cdots$			Note the case for which $D_1 = 0$.
2	2	$\cdots$			2
3	4	6	10	18	$\cdots$
4	$\cdots$				Note the case for which $D_2 = 4$.
5	8	14	26	50	$\cdots$
$\cdots$					

Table 22: Steps to avoid problems to which equation (67) seems to point.

Possible steps
• Use a transformation from $D_1 = 3$ to $D_2 = 4$. (See equation (65).)
• Split a set of four (as in, $D_2 = 4$) oscillators into two sets, each consisting of a pair of oscillators.
• Develop appropriate modeling that associates with at least one of the two sets of a pair of oscillators.

10.2.2. Angular coordinates and the case for which $D_1 = 3$, $j_\nu = 2$, and $\nu = 1/2$

Here, ν is positive and the possibly (that is, for example, for popular modeling) so-called total angular momentum $l\hbar$ associates with $l = \nu = 1/2$. Equation (66) shows the popular modeling angular factor in a solution $\Psi(r) = \phi_R(r)Y_{l,m}(\theta, \phi)$. Equations (67) and (68) pertain. Popular modeling uses notions of two-component spinors and four-component spinors to avoid problems to which the non-equality in equation (67) seems to point.

$$Y_{1/2,\pm 1/2}(\theta, \phi) = \exp(\pm i(1/2)\phi), \text{ for } 0 \leq \phi \leq 2\pi \tag{66}$$

$$Y_{l,m}(\theta, 2\pi) = \exp(\pm i\pi) = -1 \neq 1 = Y_{l,m}(\theta, 0) \tag{67}$$

$$Y_{l,m}(\theta, j_\nu(2\pi)) = Y_{l,m}(\theta, 0) \tag{68}$$

Table 22 list steps - other than deploying mathematics associating with spinors - that our modeling suggests to avoid problems to which equation (67) seems to point. (See discussion related to equation (58).)

10.2.3. Possible modeling regarding simple fermions

Our modeling includes solutions - of the form that equation (26) shows - for which 2ν is an odd integer. For the combination of $D = 3$ and $\nu = -3/2$, equation (28) yields $\Omega = (-3/2)((-3/2) + 3 - 2) = 3/4$. In popular modeling, the combination of $S = 1/2$ and $\Omega = S((S + 1) = 3/4$ associates with modeling regarding the spin $S\hbar = (1/2)\hbar$ that associates with elementary fermions.

10.2.4. Possible modeling that includes space-time coordinates

Equation (69) points to possible uses - of solutions for which $D = 1$ - regarding modeling for temporal aspects. The notion of $a \leftarrow b$ associates with the three-element phrase a becomes b (or, with the three-element phrase b replaces a. The symbol c denotes the speed of light. The symbol t denotes a temporal coordinate.

$$r \leftarrow ct \tag{69}$$

For our modeling, the domain $0 < t < \infty$ pertains. One choice of $t/\eta > 0$ or $t/\eta < 0$ might associate with the notion of before an event, with the event associating with $t \simeq 0$. The other choice of $t/\eta > 0$ or $t/\eta < 0$ might associate with the notion of after the event.

Table 23: Approximate relationships between modeling that can deploy elementary-particle properties and aspects of our modeling. n_I denotes a number - one or six - of isomers. Popular modeling associates with $n_I = 1$. Each one of some of the items in the symbol column does not associate with a popular symbol. CNC associates with charge-current 4-vectors and with Maxwell's equations. Compared to CNC, QED adds associations with magnetic fields created by other than charge currents and adds associations with anomalous magnetic moments. QCD associates with 1G, $0.5\mathrm{Q}_{1/3}$, and $0.5\mathrm{Q}_{2/3}$. We suggest the possibility that QCD might extend to associate with 0.5R. The symbol PEF associates with the three-word phrase Pauli exclusion force. We suggest that PEF associates with 1J, each 0.5Φ family, and fermions that are not elementary particles. WIP associates with $1\mathrm{W}_1$ and 1Z. The symbol † denotes a notion of a (currently) hypothetical analog to QED. Our modeling suggests that a modeling basis might need to encompass the notion of anomalous gravitational property and the notion of six isomers.

Modeling	Range of Σ	One-some $k \in \Gamma$	n_I	Symbol
Newtonian gravity	2	2	1	NEW
Moments of inertia	2	1 - 3	1	MOI
Electrostatics	1	1	1	EST
Charge-and-current 4-vectors	1	1	1	CNC
Quantum electrodynamics	1, 3	1, 2, 4, 6, 8	1	QED
Quantum chromodynamics	0	1 - 4, 8	1	QCD
Pauli exclusion force	0	1 - 4, 8	1	PEF
Weak-interaction phenomena	0	1 - 4	1	WIP
Suggested by our modeling	0 - 4	1 - 4, 6, 8	6	PRM
Gravitational analog to QED †	2, 4	1 - 4, 6, 8	6	QGD

10.3. Modeling that might associate with four space-time coordinates

Modeling above associates monopole with notions of property and position. Dipole associates with property, position, and velocity. That modeling does not associate a unique notion with position itself.

Popular modeling associates the RSDP r^{+2} with harmonic oscillators. Our modeling posits uses for the notion that k_{-2} associates with the RSDP r^{+2}. Our modeling includes the notion of s_{-2}.

The three spatial dimensions that associate with popular modeling might associate with the three generators of a group $SU(2)$ that would associate with a symmetry of the ground state of a two-dimensional harmonic oscillator that might associate with $s_{-2} = -1$ and $s_{-2} = +1$. The one temporal dimension that associates with popular modeling might associate with the one generator that associates with a $U(1)$ symmetry that might associate with a one-dimensional oscillator that might associate with $s_{-2} = 0$.

10.4. Modeling regarding physics properties

Table 23 discusses approximate relationships between modeling that can deploy elementary-particle properties and aspects of our modeling.

10.5. Connectedness within our modeling

Table 24 catalogs some - but not all - concepts that our modeling addresses. Some of the concepts and some of the notes associate with popular models. Some of the concepts and some of the notes associate with our proposed additions to physics modeling. The rightmost four columns tend to illustrate the notion that our modeling associates with each one of the concepts and each one of the notes.

11. Concluding remarks

Each of the following sentences describes a physics challenge that has persisted for the most recent eighty or more years. Interrelate physics models. Interrelate physics constants. Provide for elementary particles an analog to the periodic table for chemical elements. Describe bases for phenomena that popular modeling associates with the two-word term dark matter. Explain the overall evolution of the universe.

Physics amasses data that people can use as bases for developing and evaluating modeling aimed at addressing the challenges.

Our modeling addresses those physics challenges and has bases in the following mathematics - multipole expansions, Diophantine equations, and multidimensional harmonic oscillators.

Some of our modeling unites and decomposes aspects of electromagnetism and gravity. For each of those two long-range interactions, the decomposition seems to associate well with properties - of objects - that people can measure and that popular modeling features. For electromagnetism, the properties

Table 24: Concepts that our modeling addresses. Values in the column with the one-element label n_Γ pertain to one-some solution-pairs. The symbol † denotes that, for an object A, the amount is the sum of the amounts that associate with sub-objects of object A. NR denotes the two-word phrase not relevant. The three-element phrase anom mag mom abbreviates the three-word phrase anomalous magnetic moment.

Concept	Note	$\log_r r^n$	k	n_Γ	Σ
Position (coordinates)	1 temporal coordinate, 3 spatial coordinates	2	-2	-	-
Color charge	(For fermions:) 1 clear (or, white), 3 colors	1	-1	-	-
Net-left-right	NR for LRI particles, 3 for simple particles	0	0	-	-
LRI	Long-range interaction	-	-	-	≥ 1
Electromagnetic field	Range regarding components of the field	$-1,\cdots,-5$	1,2,4,6,8	$1,\cdots,4$	1
Gravitational field	Range regarding components of the field	$-1,\cdots,-6$	1,2,3,4,6,8	$1,\cdots,5$	2
Conserved properties	(That LRIs measure:) Charge, mass, net-left-right, intrinsic angular momentum	-1	$\geq 1, \leq 4$	1	≥ 1
Charge	(Within one isomer:) 3 "signs" : <0 †, 0 †, >0 †	-1	1	1	1
Mass	3 "types": simple fermion >0, 0, other >0	-1	2	1	2
Net-left-right	3 net-left-right numbers †	-1	3	1	3
Angular momentum	3 vector components	-1	4	1	4
Other properties	(That LRIs measure)	-	-	-	-
Magnetic moment	One-some: (nominal) magnetic moment	-2	1,2	2	1
Anom. mag. mom.	One-some: anomalous magnetic moment	-2	1,2	2	3
Stress-energy (SE)	Can include non-gravitational effects	≤ -3		≥ 3	≥ 1
SE tensor (SET)	(3 aspects: energy, pressure, off-diagonal)				2
SET energy	1 component: One-some: energy	-1	2	1	2
SET pressure	3 components: One-some: rotating energy	-2	2,4	2	2
SET off-diagonal	12 components (6 independent): One-some: (nominal) stress-energy	-3	1,2,3	3	2
Other SE	One-some: stress-energy	≤ -3		≥ 3	≥ 1
(LRI limit)	Zero force-strength factor for $k = 5$	-1	$\geq 1, < 5$		$\geq 1, \leq 4$
(Possible LRI limit)	Based on some popular models	-1	$\geq 1, \leq 2$		$\geq 1, \leq 2$
Velocity	3 vector components: Three-some, $n_\Gamma + 1$ with respect to n_Γ for a position	-	-	-	-
Acceleration	3 vector components: Three-some, $n_\Gamma + 1$ with respect to n_Γ for a velocity	-	-	-	-
Multipole range	n-pole ($1 \leq n \leq 6$)	$-1,\cdots,-6$	-	-	-
Weak interaction	Carriers: Z boson and W boson	-	-	-	-
Strong interaction	Carriers: Gluons	-	-	-	-
Pauli repulsion	Carrier: Jay boson	-	-	-	-
Simple particles (SP)	Simple (or, non-LRI) elementary particles	-	-	≥ 3	0
Isomers (of SP)	6 (= 3 isomer-pairs × 2 solutions)	-	-	≥ 3	0
Elementary fermion	SP: One-some $6 \in \Gamma \Leftrightarrow$ fermion	-	-	≥ 3	0
Flavours (generations)	3 flavours: One-some $6 \in \Gamma$	-	-	-	0
Charge (re SP)	A one-some solution-pair associates with zero charge $\Leftrightarrow 1, 3, 4 \in \Gamma$	-	1,3,4	≥ 3	0
Z and W bosons	For three-some: $n_\Gamma = 4$ and can have $6 \in \Gamma$	-	-	3	0
Gluons	For three-some: $n_\Gamma = 6$ and has $6 \in \Gamma$	-	-	5	0
Jay boson	For three-some: $n_\Gamma = 6$ and has $6 \in \Gamma$	-	-	5	0
Inflaton (aye boson)	For three-some: $n_\Gamma = 5$ and can have $6 \in \Gamma$	-	-	4	0
SP-boson decays	Three-some $6 \in \Gamma \Rightarrow$ can decay into two simple fermions	-	-	≥ 3	0
SP-boson decays	Three-some $8 \in \Gamma \Leftrightarrow$ can decay into two simple bosons	-	-	≥ 3	0
SP-boson m^2 term	$Q(Q+1)$ (unit for Q: magnitude of charge of the electron)	-	1	≥ 3	0
SP-boson m^2 term	$(m')^2$ (unit for m': 1/3 the Z-boson mass)	-	2	≥ 3	0
SP-boson m^2 term	S^2 (unit for S: $\hbar$)	-	4	≥ 3	0
Quantum transition	Excite or de-excite a boson field	-	-	-	-

include charge and magnetic moment. For gravity, the properties include mass and components of stress-energy. This modeling also points to all known elementary particles and to all would-be elementary particles that our work suggests.

Some of our modeling features isomers of elementary particles that do not mediate long-range interactions and features instances of components of long-range interactions. This modeling explains data regarding dark matter and points to possible resolutions of tensions - between data and popular modeling - regarding effects of dark energy.

We use our modeling to match data that popular modeling matches, to suggest explanations for data that popular modeling seems not to explain, to suggest results regarding data that people have yet to gather, and to point to possible opportunities to develop models that unite aspects of physics and physics modeling.

In summary, our work suggests augmentations - to popular modeling - that might achieve the following results. Extend the list of elementary particles. Predict masses for at least two neutrinos. Predict masses - that would be more accurate than known masses - for some other elementary particles. Describe dark matter. Explain ratios of dark matter effects to ordinary matter effects. Provide insight regarding galaxy formation. Describe bases for phenomena that popular modeling associates with the two-word term dark energy. Explain eras in the history of the universe. Link properties of objects. Interrelate physics models. Provide bases for further integrating and extending physics modeling.

Acknowledgments

The following people pointed to topics or aspects that we considered for inclusion in the scope of our work: Andrea Albert, Raphael Bousso, Lance Dixon, Persis Drell, Immanuel Freedman, Ervin Goldfain, Vesselin Gueorguiev, Kamal Melek Hanna, Wick Haxton, Richard B. Holmes, Nick Hutzler, William Lama, Tom Lawrence, Surhud More, Holger Muller, J. Xavier Prochaska, Martin Rees, Harrison Rose, and Mak Tafazoli.

The following people provided comments regarding the effectiveness of drafts that led to parts of this essay: Charles K. Chui, Mohamed Nassef Hussein Comsan, Immanuel Freedman, Ervin Goldfain, Vesselin Gueorguiev, Ali Khaledi-Nasab, William Lama, Tom Lawrence, and Mak Tafazoli.

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Manuscript date: August 4, 2022