Ultimate Acceleration in Quantum Mechanics: to Obtain Spin

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Abstract: In analogy with the ultimate speed *c*, there is an ultimate acceleration β, nobody's acceleration can exceed this limit β, for electrons and quarks, β=2.327421e+29(m/s^2). Because this ultimate acceleration is a large number, any effect connecting to β will become easy to test, including quantum gravity tests. In this paper, an approach is put forward to connect the ultimate acceleration with quantum theory, consequently, the spin concept can be derived out from the matter wave. This paper also carefully explains how the matter wave to display its spin effect in Stern-Gerlach experiments. It is completely a new aspect to quantum mechanics for the relativistic matter wave to contain spin.

1. Introduction

This year is 99th anniversary of the initiative of de Broglie's matter wave. In 1922, the Louis de Broglie considered blackbody radiation as a gas of light quanta [1], he tried to reconcile the concept of light quanta with the phenomena of interference and diffraction. In 1923 and 1924, the concept that matter behaves like a wave was proposed by Louis de Broglie [2,3]. It is also referred to as the de Broglie hypothesis, matter waves are referred to as de Broglie waves.

Using ultimate acceleration, this paper shows that matter wave has spin. In analogy with the ultimate speed *c*, there is an ultimate acceleration β, nobody's acceleration can exceed this limit β, for electrons and quarks, β=2.327421e+29(m/s^2). Because this ultimate acceleration is a large number, any effect connecting to β will become easy to test, including quantum gravity tests. In this paper, an approach is put forward to connect the ultimate acceleration with quantum theory, consequently, the spin concept can be derived out from the matter wave. This paper also carefully explains how the matter wave to display its spin effect in Stern-Gerlach experiments. It is completely a new aspect to quantum mechanics for the relativistic matter wave to contain spin.

2. How to connect the ultimate acceleration with quantum theory

In the relativity, the speed of light *c* is an ultimate speed, nobody's speed can exceed this limit *c*. The relativistic velocity *u* of a particle in the coordinate system ($x_1, x_2, x_3, x_4=ict$) satisfies

$$u_1^2+u_2^2+u_3^2+u_4^2=-c^2 \quad . \tag{1}$$

No matter what particles (electrons, molecules, neutrons, quarks), their 4-vector velocities all have the same magnitude: $|u|=ic$. All particles gain **equality** because of the same magnitude of the 4-velocity *u*. The acceleration *a* of a particle is given by

$$a_1^2 + a_2^2 + a_3^2 = a^2; \quad (a_4 = 0; \quad \because x_4 = ict) \tag{2}$$

Assume that particles have an ultimate acceleration β as limit, no particle can exceed this acceleration limit β. Subtracting the both sides of the above equation by β^2, we have

$$a_1^2 + a_2^2 + a_3^2 - \beta^2 = a^2 - \beta^2; \qquad a_4 = 0 \tag{3}$$

It can be rewritten as

$$[a_1{}^2 + a_2{}^2 + a_3{}^2 + 0 + (i\beta)^2]\frac{1}{1 - a^2/\beta^2} = -\beta^2 \tag{4}$$

Now, the particle subjects to an acceleration whose five components are specified by

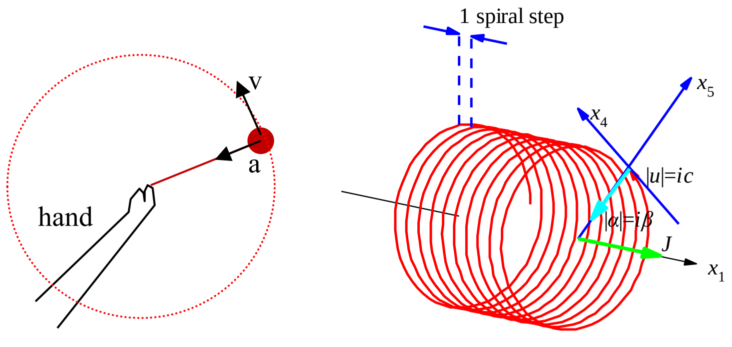
$$\begin{aligned} &\alpha_1 = \frac{a_1}{\sqrt{1-a^2/\beta^2}}; \quad \alpha_2 = \frac{a_2}{\sqrt{1-a^2/\beta^2}}; \\ &\alpha_3 = \frac{a_3}{\sqrt{1-a^2/\beta^2}}; \quad \alpha_4 = 0; \quad \alpha_5 = \frac{i\beta}{\sqrt{1-a^2/\beta^2}}; \end{aligned} \tag{5}$$

where α_5 is the newly defined acceleration in five dimensional space-time ($x_1, x_2, x_3, x_4=ict, x_5$). Thus, we have

$$\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2 + \alpha_5^2 = -\beta^2; \quad \alpha_4 = 0 \tag{6}$$

It means that the magnitude of the newly defined acceleration α for every particle takes the same value: $|\alpha|=i\beta$ (constant imaginary number), all particle accelerations gain **equality** for the sake of the same magnitude.

How to resolve the velocity u and acceleration α into x, y, and z components? In realistic world, a hand can rotate a ball moving around a circular path at constant speed v with constant centripetal acceleration a, as shown in Fig.1(a).



(a) (b) right hand chirality

Fig.1 (a) A hand rotates a ball moving around a circular path at constant speed v with constant centripetal acceleration a. (b) The particle moves along the x_1 axis with the constant speed $|u|=ic$ in the u direction and constant centripetal force in the x_5 axis at the radius iR (imaginary number).

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<Clet2020 Script>// Clet is a C compiler [17]
double D[100],S[2000];int i,j,R,X,N;
int main(){R=50;X=50;N=600;D[0]=-50;D[1]=0;D[2]=0;D[3]=X;D[4]=0;D[5]=0;D[6]=-50;D[7]=R;D[8]=0;
D[9]=600;D[10]=10;D[11]=R;D[12]=0;D[13]=3645;
Lattice(SPIRAL,D,S);SetViewAngle(0,80,-50);DrawFrame(FRAME_NULL,1,0xffffff);
Draw("LINE,0,2,XYZ,0","-150,0,0,-50,0,0");Draw("ARROW,0,2,XYZ,10","50,0,0,150,0,0");
SetPen(2,0xff0000);Plot("POLYLINE,0,600,XYZ",S[9]);i=9+3*N-6;Draw("ARROW,0,2,XYZ,10",S[i]);
TextHang(S[i],S[i+1],S[i+2],"   #if|u|=ic#t");TextHang(150,0,0,"   #ifx#sd1#t");SetPen(2,0x005fff);
Draw("LINE,1,2,XYZ,8","-50,0,50,-50,0,100");Draw("LINE,1,2,XYZ,8","-40,0,50,-40,0,100");
Draw("ARROW,0,2,XYZ,10","-80,0,100,-50,0,100");Draw("ARROW,0,2,XYZ,10","-10,0,100,-40,0,100");
TextHang(-50,0,110,"1 spiral step");i=9+3*N;S[i]=50;S[i+1]=10;S[i+2]=10;
Draw("ARROW,0,2,XYZ,10","50,0,0,50,80,80");TextHang(50,80,80," #ifx#sd5#t");
Draw("ARROW,0,2,XYZ,10","50,72,0,50,0,72");TextHang(50,0,72,"   #ifx#sd4#t");
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SetPen(3,0x00ffff);Draw("ARROW,0,2,XYZ,15",S[i-3]);TextHang(S[i],S[i+1],S[i+2]," #if|α|=iβ#t");
SetPen(3,0x00ff00);Draw("ARROW,0,2,XYZ,15","50,0,0,120,0,0");TextHang(110,5,5," #ifJ#t");
TextHang(-60,0,-80," right hand chirality");}#v07=?>A
```

In analogy with the ball in a circular path, consider a particle in one dimensional motion along the x_1 axis at the speed v, in the Fig.1(b) it moves with the constant speed $|u|=ic$ almost along the x_4 axis and slightly along the x_1 axis, and the constant centripetal acceleration $|\alpha|=i\beta$ in the x_5 axis at the constant radius iR (imaginary number); the coordinate system ($x_1,x_4=ict,x_5=iR$) establishes a cylinder coordinate system in which this particle moves spirally at the speed v along the x_1 axis. According to usual centripetal acceleration formula $a=v^2/r$, the acceleration in the x_4-x_5 plane is given by

$$a=\frac{v^2}{r} \quad \Rightarrow \quad i\beta=\frac{|u|^2}{iR}=-\frac{c^2}{iR}=i\frac{c^2}{R} \quad . \tag{7}$$

Therefore, the track of the particle in the cylinder coordinate system ($x_1,x_4=ict,x_5=iR$) forms a shape, called as **acceleration-roll**. The faster the particle moves along the x_1 axis, the longer the spiral step is.

As like a steel spring with elastic wave, the track in the acceleration-roll in Fig.1(b) can be described by a wave function whose phase changes 2π for one spiral step. Apparently, this wave is the de Broglie's matter wave for electrons, protons or quarks, etc.

Proof: The wave function phase changes 2π for one spiral circumference $2\pi(iR)$, then a small displacement of the particle on the spiral track is $|u|d\tau=icd\tau$ in the 4-vector u direction, thus this wave phase along the spiral track is evaluated by

$$phase=\int_0^\tau \frac{2\pi}{2\pi(iR)}icd\tau=\int_0^\tau \frac{c}{R}d\tau \quad . \tag{8}$$

Substituting the radius R into it, the wave function ψ is given by

$$\psi=\exp(-i\cdot phase)=\exp(-i\int_0^\tau \frac{c}{R}d\tau)=\exp(-i\frac{\beta}{c}\int_0^\tau d\tau) \quad . \tag{9}$$

In the theory of relativity, we known that the integral along $d\tau$ needs to transform into realistic line integral, that is

$$\begin{aligned} d\tau &=-c^2\frac{d\tau}{-c^2}=(u_1^2+u_2^2+u_3^2+u_4^2)\frac{d\tau}{-c^2} \\ &=(u_1dx_1+u_2dx_2+u_3dx_3+u_4dx_4)\frac{1}{-c^2} \end{aligned} \quad . \tag{10}$$

Therefore, the wave function ψ is evaluated by

$$\begin{aligned} \psi &=\exp(-i\frac{\beta}{c}\int_0^\tau d\tau) \\ &=\exp(i\frac{\beta}{c^3}\int_0^x (u_1dx_1+u_2dx_2+u_3dx_3+u_4dx_4)) \end{aligned} \quad . \tag{11}$$

This wave function may have different explanations, depending on the particle under investigation. If the β is replaced by the Planck constant for electrons, the wave function is given by

$$\begin{aligned} &assume: \quad \beta = \frac{mc^3}{\hbar} \\ &\psi = \exp\left(\frac{i}{\hbar}\int_0^x (mu_1 dx_1 + mu_2 dx_2 + mu_3 dx_3 + mu_4 dx_4)\right) \end{aligned} \quad . \tag{12}$$

where $mu_4dx_4=-Edt$, it strongly suggests that the wave function is just the de Broglie's matter wave [4,5,6].

In Fig.1(b), the acceleration-roll of particle moves with two distinctions: right-hand chirality and left-hand chirality. The direction of the angular momentum J would be slightly different from the x_1 due to spiral precession. It is easy to calculate the ultimate acceleration β , the radius R and the angular momentum J in the plane x_4-x_5 for a spiraling electron as

$$\begin{aligned} &\beta = \frac{c^3 m}{\hbar} = 2.327421\text{e+}29\ (\mathrm{M/s^2}) \\ &R = \frac{c^2}{\beta} = 3.861593\text{e-}13\ (\mathrm{M}) \\ &J = \pm m\,|\,u\,|\,iR = \mp\hbar \end{aligned} \quad . \tag{13}$$

```
<Clet2020 Script>// Clet is a C compiler [17]
double beta,R,J,m,D[10];char str[200];
int main(){m=ME;beta=SPEEDC*SPEEDC*SPEEDC*m/PLANCKBAR;
R=SPEEDC*SPEEDC/beta;J=PLANCKBAR;Format(str,"beta=%e, #nR=%e, #nJ=%e",beta,R,J);
TextAt(50,50,str);ClipJob(APPEND,str);}#v07=#t
```

Considering another explanation to ψ for planets in the solar system, no Planck constant can be involved. But, in a many-body system with the total mass M, the data-analysis [14] tells us that the ultimate acceleration can be rewritten in terms of **Planck-constant-like constant** h as

$$\begin{aligned} &assume: \quad \beta = \frac{c^3}{hM} \\ &\psi = \exp\left(\frac{i}{hM}\int_0^x (u_1 dx_1 + u_2 dx_2 + u_3 dx_3 + u_4 dx_4)\right) \end{aligned} \quad . \tag{14}$$

The constant h will be determined by experimental observations. The papers [15,16] showed that this wave function is applicable to several many-body systems in the solar system, the wave function is called as the **acceleration-roll wave**.

Consider third explanation to ψ for atoms in cells and viruses. Typically, sound speed in water is v=1450m/s, according to $v=f\lambda$, the sound wavelength λ for frequency 1kHz---1Mhz is 1.5m---1.45mm. In general, the sound wavelength is larger than cell size, because cell size is about 1 micro. Thus, almost all cells and viruses live in a smaller space which is not sensitive to the sound. The acceleration-roll can provide a kind of wave with a shorter wavelength and lower frequency for various cells and viruses beyond human-sensitive sound wave [14].

Of cause, there is "ultimate distance" [14] from which the Hubble law can be derived out.

Tip: actually, ones cannot get to see the acceleration-roll of a particle in the relativistic space-time ($x_1,x_2,x_3,x_4=ict$) ; only get to see it in the cylinder coordinate system ($x_1,x_4=ict,x_5=iR$).

3. Dimension and wave

Dimension is defined as the number of independent parameters in a mathematical space. In the field of physics, dimension is defined as the number of independent space-time coordinates. 0D is an infinitesimal point with no length. 1D is an infinite line, only length. 2D is a plane, which is composed of length and width. 3D is 2D plus height component, has volume.

In this section we at first discuss how to measure dimension by wave. In Fig.2(a), one puts earphone into ear, one gets 1D wave in the ear tunnel.

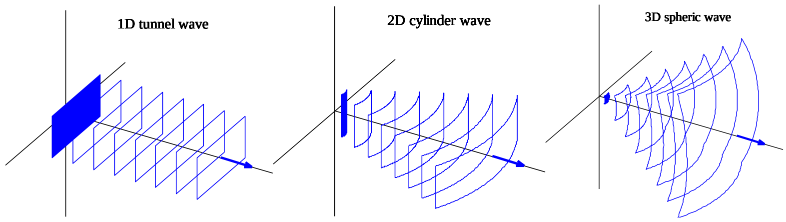
$$1D: \quad y = A\sin(kr - \omega t) = \frac{A}{r^0}\sin(kr - \omega t)\,. \tag{15}$$

where r is the distance between the wave emitter and the receiver. In Fig.2(b), one touches a guitar spring, one gets 2D cylinder wave.

$$2D: \quad y = \frac{A}{r^{1/2}}\sin(kr - \omega t)\,. \tag{16}$$

In Fig.2(c), one turns on a music speaker, one gets 3D spherical wave.

$$3D: \quad y = \frac{A}{r}\sin(kr - \omega t)\,. \tag{17}$$



(a) 1D tunnel wave (b) 2D cylinder wave (c) 3D spherical wave

Fig.2 The wave behavior in various dimensional spaces.

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<Clet2020 Script>// Clet is a C compiler [17]
int i,j,k,type,nP[10]; double D[20],S[1000];
int main(){SetViewAngle("temp0,theta60,phi-30");SetAxis(X_AXIS,0,0,200,"X;0;200;");
DrawFrame(FRAME_LINE,1,0xafffaf); type=2;SetPen(1,0x00ff);
for(i=10;i<160;i+=20){D[0]=i;D[1]=0;D[2]=0;D[3]=i+5;D[4]=0;D[5]=0;D[6]=i;D[7]=10;D[8]=0;
if(type==0) {D[9]=4;D[10]=40;D[11]=20;D[12]=i;TextHang(50,0,100,"1D tunnel wave");k=CARD;}
else if(type==1) {D[9]=200;D[10]=i/2;D[11]=20;D[12]=i;TextHang(50,0,100,"2D cylinder wave");k=50;}
else {D[9]=200;D[10]=i/2;D[11]=i/2;D[12]=i;TextHang(50,0,100,"3D spheric wave");k=40;}
Lattice(k,D,S);nP[0]=POLYGON;nP[1]=0;nP[2]=200;nP[3]=XYZ;
if(i==10) nP[1]=3; if(type==0) nP[2]=4; Plot(nP,S[9]);}
j=30;D[3]=D[0]+j*S[0];D[4]=D[1]+j*S[1];D[5]=D[2]+j*S[2];
SetPen(3,0x00ff);Draw("ARROW,0,2,XYZ,10",D);}
#v07=?>A
```

In general, we can write a wave in the form

$$y = \frac{A}{r^w}\sin(kr - \omega t)\,. \tag{18}$$

It is easy to get the dimension of the space in where the wave lives, the dimension is D=2w+1. Nevertheless, wave can be used to measure the dimension of space, just by determining the parameter w.

Waves all contain a core oscillation (vibration invariance)

$$\frac{d^2y}{dr^2} + k^2 y = 0\,, \tag{19}$$

Substituting y into the core oscillation, we obtain the radial wave equation

$$\frac{d^2y}{dr^2}+\frac{2w}{r}\frac{dy}{dr}+(k^2+\frac{w(w-1)}{r^2})y=0 \cdot \tag{20}$$

This equation expresses the wave behavior modulated by the spatial dimension parameter w. For 1D wave w=0, it is trivial, but for 2D wave w=1/2, it reduces to the Bessel equation in a cylinder coordinate system (r,φ)

$$\frac{d^2y}{dr^2}+\frac{1}{r}\frac{dy}{dr}+(k^2-\frac{1}{4r^2})y=0 \quad \text{(2D wave)}$$

comparing to the Schrodinger's equation: . (21)

$$\frac{d^2R(r)}{dr^2}+\frac{2}{r}\frac{dR(r)}{dr}+[k^2-\frac{l(l+1)}{r^2}]R(r)=0$$

In quantum mechanics, y is an electronic wave function, comparing to the Schrodinger radial wave equation in textbooks [9,10,11], we find that the $-1/4r^2$ term represents the electronic spin effect. However, here according to the above radial Bessel equation we can simply conclude: sound wave, electromagnetic wave, or any wave can have spin effect in 2D space! Let us use $\boldsymbol{k}$ denote the wave-vector, then the above 2D wave equation tells us

$$k_r^{\ 2}=k^2-k_\varphi^{\ 2}; \quad k=\frac{2\pi}{\lambda}; \quad k_\varphi=\pm\frac{1}{2r} \cdot \tag{22}$$

The k_φ causes the 2D wave-vector $\boldsymbol{k}$ to spin little by little as illustrated Fig.3. The positive and negative k_φ corresponds to spin up and spin down respectively; as r goes to the infinity, the spin effect vanishes off.

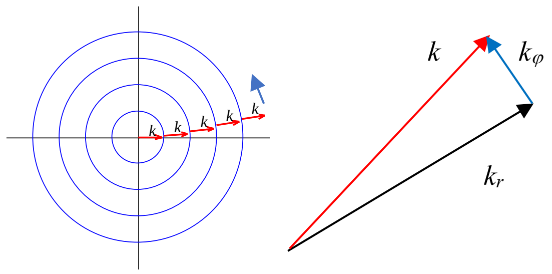

Fig.3 2D wave-vector $\boldsymbol{k}$ spins little by little in the cylinder coordinates (r,φ).

```
<Clet2020 Script>// Clet is a C compiler [17]
int i,j,k;double r,x,y,a,D[100];
int main(){DrawFrame(FRAME_LINE,1,0xafffaf); SetPen(1,0x0000ff);
for(i=0;i<90;i+=20) {D[0]=-i;D[1]=-i;D[2]=i;D[3]=i; Draw("ELLIPSE,0,2,XY,0",D);}
for(i=0;i<90;i+=20) {a=0.2*(i-i*i/200)*PI/180;r=i;D[0]=r*cos(a);D[1]=r*sin(a);
r+=18;D[2]=r*cos(a);D[3]=r*sin(a); SetPen(2,0xff0000);
Draw("ARROW,0,2,XY,8",D);TextHang(D[2]-10,D[3]+5,0,"#ifk");}
}#v07=?>A
```

If the 2D wave is the de Broglie matter wave for a particle beam, in a cylinder coordinate (r,φ), then the matter wave has a spin angular momentum given by

$$k_r=\frac{p_r}{\hbar}; \quad k_\varphi=\frac{p_\varphi}{\hbar}=\frac{J_\varphi}{r\hbar}; \quad J_\varphi=\pm\frac{1}{2}\hbar \cdot \tag{23}$$

According to the angular momentum formula in general physics, it is recognized that the particle total momentum p is a constant given by

$$\left(\frac{p}{\hbar}\right)^2=\left(\frac{p_r}{\hbar}\right)^2+\left(\frac{p_\varphi}{\hbar}\right)^2 . \tag{24}$$
$$k^2=k_r^{\ 2}+k_\varphi^{\ 2}=const.$$

Since the particle total wave vector k is a constant, the wave-vector k_r must vary as r changes. The wave-vector in the radial direction would change as the wave attenuates.

For 3D wave in a spherical coordinate system (r, θ, φ), the modulated wave reduces to

$$\frac{d^2y}{dr^2}+\frac{2}{r}\frac{dy}{dr}+(k^2+0)y=0 \quad \text{(3D wave)} \cdot \tag{25}$$

It gives us an impression there is not spin effect, actually the impression is not true. Because spin effect belongs to 2D rotation in a plane, it is required for us to transform the 3D spherical wave into a 2D platform for observation, in the plane x-y as shown in Fig.4, we re-arrange the terms to fit the 2D cylinder Bessel equation as

$$\frac{d^2y}{dr^2}+\frac{1}{r}\frac{dy}{dr}+(k^2+\frac{1}{ry}\frac{dy}{dr})y=0 \quad . \tag{26}$$

(2D plateform for 3D wave, radial equation)

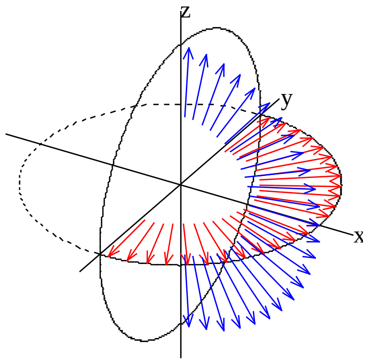

Fig.4 In the plane x-y as a 2D platform for observing the 3D spherical wave.

```
<Clet2020 Script>// Clet is a C compiler [17]
int i,j,k; double r,rot,x,y,z,dP[10],D[1000];
int main(){k=100;SetViewAngle("temp0,theta60,phi-30");
DrawFrame(FRAME_LINE,1,0xafffaf);Overlook("2,1,80", D);
r=80;k=12;rot=0.04*PI;SetPen(1,0xff0000);
for(i=-k;i<=k;i+=1) {x=r*cos(rot*i);y=r*sin(rot*i);z=0;
D[0]=x/2;D[1]=y/2;D[2]=z/2;D[3]=x;D[4]=y;D[5]=z;
Draw("ARROW,0,2,XYZ,10",D);} SetPen(1,0x0000ff);//k=5;
for(i=-k;i<=k;i+=1) {y=0;x=r*cos(rot*i);z=r*sin(rot*i);
D[0]=x/2;D[1]=y/2;D[2]=z/2;D[3]=x;D[4]=y;D[5]=z;
Draw("ARROW,0,2,XYZ,10",D);}r+=20;
TextHang(r,0,0,"x");TextHang(0,r,0,"y");TextHang(0,0,r,"z");
}#v07=?>A
```

Substituting expected 2D wave y=$(1/r^{1/2})$exp(ikr-$i\omega t$) into the newly separated term, we could get the wave equation in the radial direction

$$\begin{gathered}\frac{d^2y}{dr^2}+\frac{1}{r}\frac{dy}{dr}+(k^2-\frac{1}{2r^2}+\frac{ik}{r})y=0 \\ k_r^{\ 2}=k^2-\frac{1}{2r^2}+\frac{ik}{r}=(k+\frac{i}{2r})^2-\frac{1}{4r^2}\end{gathered} \quad . \tag{27}$$

The observed 2D wave in the 2D platform for the spherical wave is approximately given by

$$y \simeq \frac{A}{\sqrt{r}}\exp(ir[\sqrt{k_r^2+\frac{1}{4r^2}}-\frac{i}{2r}]-i\omega t)+O(\frac{1}{r^3})$$
$$k+\frac{i}{2r}=\sqrt{k_r^2+k_\varphi^2};\quad k_\varphi=\pm\frac{1}{2r} \tag{28}$$

It contains an imaginary number $i/2r$ term which approximately represents the attenuation of the wave. The k_φ causes the 2D wave-vector to spin little by little as shown Fig.3, this is a kind of visualization of the spin effect. The positive and negative k_φ corresponds to spin up and spin down respectively; as r goes to the infinity, the spin effect vanishes off.

If the 3D wave is the de Broglie matter wave for a particle beam, in a cylinder coordinate (r,φ) as a 2D platform for observation, then the matter wave has a spin angular momentum given by

$$k_r=\frac{p_r}{\hbar};\quad k_\varphi=\frac{p_\varphi}{\hbar}=\frac{J_\varphi}{r\hbar};\quad J_\varphi=\pm\frac{1}{2}\hbar\cdot \tag{29}$$

According to the angular momentum formula in general physics, it is recognized that the particle total momentum p holds

$$\left(\frac{p}{\hbar}\right)^2=\left(\frac{p_r}{\hbar}\right)^2+\left(\frac{p_\varphi}{\hbar}\right)^2,\quad (k+\frac{i}{2r})^2=k_r^2+k_\varphi^2\cdot \tag{30}$$

Finally, spin visualization belongs to 2D world.

4. Stern-Gerlach experiment and Lander factor

In general, it is hard in macroscopic scale to separate the left-turn electrons and right-turn electrons in a 2D matter wave, except under certain conditions in some deliberately designed magnetic apparatus. In Stern-Gerlach experiments, as shown in Fig.5(a), silver (Ag) atoms are heated in an oven, the oven has a small hole through which some silver atoms escape. The silver vapor busts out the oven and go through a slit as the collimator and is then subjected to an inhomogeneous magnetic field. In this experiment, the single valent election of silver atom moves in its Bohr orbit, as shown in Fig.5(b), we adopt a cylinder coordinates (r,φ) with the origin at the sliver center.

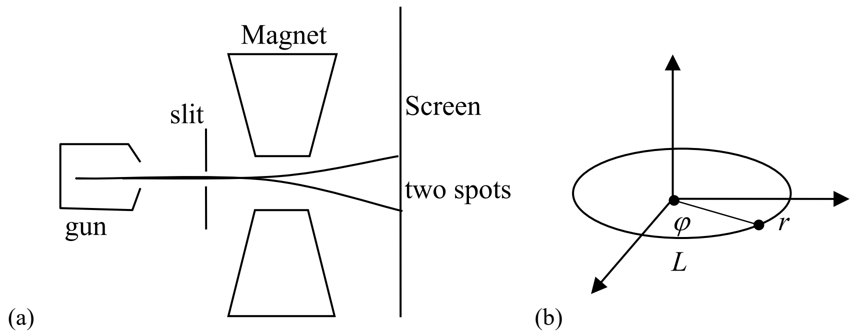

Fig.5 The Stern-Gerlach experiment apparatus.

As shown in Fig.6(a), if the coherent length of the electron in a hydrogen atom is long enough, it will wind around the time axis, the matter wave in the circle will overlap, and interfere with itself at every location. The overlapping number N depends on the coherent length L; if $N=\infty$, the matter wave has the interference like the Fabry-Perot interference in optics.

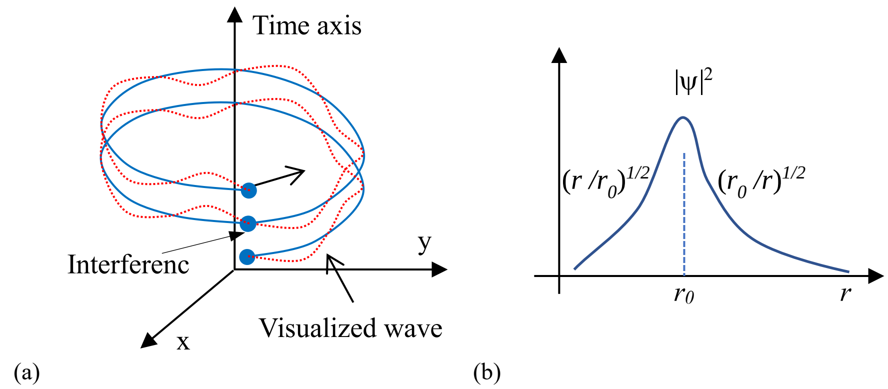

Fig.6 (a)The matter wave winds around the time axis, overlap and interfere with itself. (b)The distribution of overlapped 2D matter wave about its Bohr' radius.

The overlapped matter wave is given by

$$\psi = w + we^{i\delta} + we^{i2\delta} + ...we^{i(N-1)\delta} = w\frac{1-\exp(iN\delta)}{1-\exp(i\delta)} . \quad \delta = \frac{1}{\hbar}\oint_L pdl \tag{31}$$

where, δ is the phase shift after one circle retardation for the matter wave. Obviously, the Bohr orbits can survive only if the denominator of the above equation is satisfied by

$$\delta = 2\pi n; \quad n = 1, 2, 3, ... \tag{32}$$

For a 2D Bohr orbit, in the φ direction and in the r direction, the 2D wave function satisfies ψ=$\Phi(\varphi)R(r)$, consider a simplest case as shown in Fig.6(b), the overlapped matter wave with a finite N is approximately given by

$$\Phi(\varphi) = \exp(\frac{ip_\varphi}{\hbar}r\varphi); \quad R(r) = \begin{cases} \sqrt{\frac{r_0}{r}}\exp(\frac{ip_r}{\hbar}r) = \sqrt{\frac{r_0}{r}} & p_r \simeq 0 \quad r > r_0 \\ \sqrt{\frac{r}{r_0}}\exp(\frac{ip_r}{\hbar}r) = \sqrt{\frac{r}{r_0}} & p_r \simeq 0 \quad r < r_0 \end{cases} . \tag{33}$$

Although p_r=0, the fact was omitted by almost all of us, the wave function in a 2D cylinder wave (r>r_0) satisfies

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + [k^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)}{r^2}]R = 0 . \tag{34}$$

Because the single valent election of silver atom lives in a 2D orbital space, acquiring the extra spin angular momentum, the Ag atom has a magnetic moment

$$J_{spin} = \pm\frac{1}{2}\hbar \quad \Rightarrow \quad \mu = g\frac{e}{2m}J_{spin} . \tag{35}$$

where g is the Lander factor, g=2. Because the interaction energy of the magnetic moment with the magnetic field is just -μB, the z-component of the force experienced by the silver atom is given by

$$f_z = \frac{\partial}{\partial z}(\mu B) = \mu \frac{\partial B}{\partial z} \cdot \tag{36}$$

where we have ignored the components of B in direction other than the z- direction. In the z direction, the silver atom beams, 50% atoms experience an upward force, and other 50% atoms experience a downward force, thus on the screen we get the view there are two spots, in agreement with the theoretical prediction.

Why did physics introduce the Lander factor? In fact, there are two kinds of angular momentum we should consider in the φ direction

$$\begin{aligned} J_{total} &= J_\varphi = \pm\hbar \\ J_{spin} &= \pm\frac{1}{2}\hbar \end{aligned} \cdot \tag{37}$$

The spin needs to occupy the half of the total angular momentum in this case, in other words the total angular momentum contains the spin angular momentum, that is

$$J_{total} = J_\varphi = \pm(\frac{1}{2}\hbar + \frac{1}{2}\hbar) = \pm(\frac{1}{2}\hbar + |J_{spin}|) \cdot \tag{38}$$

In other words, the spin angular momentum is enveloped by the total angular momentum. Thus, we obtain the normal magnetic moment which should be

$$\begin{aligned} &\mu = \frac{e}{2m} J_{total} = \frac{e}{2m} J_\varphi = \frac{e}{2m} 2J_{spin} = \frac{e}{2m} gJ_{spin} \\ &\therefore \quad g = 2 \end{aligned} . \tag{39}$$

The external magnetic field can only probe the total magnetic moment, but fails to directly detect the enveloped spin angular momentum, we need the Lander factor to grip on the spin concept.

5. Conclusions

This year is 99th anniversary of the initiative of de Broglie's matter wave, it is a good time for rediscovering the matter wave. In analogy with the ultimate speed *c*, there is an ultimate acceleration β, nobody's acceleration can exceed this limit β, for electrons and quarks, β=2.327421e+29(m/s^2). Because this ultimate acceleration is a large number, any effect connecting to β will become easy to test, including quantum gravity tests. In this paper, an approach is put forward to connect the ultimate acceleration with quantum theory, consequently, the spin concept can be derived out from the matter wave. This paper also carefully explains how the matter wave to display its spin effect in Stern-Gerlach experiments. It is completely a new aspect to quantum mechanics for the relativistic matter wave to contain spin.

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